COGITO137

The Thought Capsule

Third issue
August 2020

Comictious

Art, illustrations and toons themed around COVID19

How 'cool' is Absolute Zero?
Simli Mishra

What do scientists really know?
Varun Srivastava

Love paves the path for a strong dog-human bond
Debottam Bhattacharjee

Hilbert's Quarantine Centre: To infinity and beyond!
Debmalya Bandyopadhyay

Quantum Entanglement: Our ignorance or the Universe's ghost?"
Magare Sourabh Suryakant

Cogito137
The Thought Capsule

Photo by Alex Andrews from Pexels

Editor's Note

Cogito137 - The thought capsule is perhaps the youngest students' initiative in IISER Kolkata. Shortly after we released our first issue, a nationwide lockdown was declared. The better part of our lives has since shifted to online platforms, and that has stalled the bustling campus life at IISER Kolkata.

However, Team Cogito137 used it to their advantage. As we grow, we are still in the process of expanding our team and the scope of our platform. I thank all our writers and contributors of the August issue, who decided to use some idle lockdown hours to write for us and extend a warm welcome to all the new members of our team.

This issue begins with a cardinal opinion piece about the perception of science and scientists in our society and how distant it is from reality, by Varun Srivastava from batch '16. Magare Sourabh Suryakant, a recent graduate, gives you a simplistic sneak-peek into the ‘quantum world’. Next is a piece by another recent graduate, Simli Mishra, about ‘absolute zero’ - the lowest theoretically possible temperature. In her piece, she also writes about an anecdotal moment when she had the chance of witnessing this 'theoretical temperature' being attained within research equipment. Dr. Debottam Bhattacharjee who just defended his PhD thesis, writes about one of his studies which proves that love paves the way for the age-old dog-human bonding. Debmalya Bandyopadhyay from batch '17 takes you through a thought experiment of building a quarantine centre, containing infinite number of rooms, in his very lucid piece about a certain infinity paradox.

This issue also contains a section from “Comictious” - a series of sci-art and sci-illustrations themed around the COVID19 pandemic. The deadline for submission to this series has been extended till the 15th of September.

Alongside, our August issue has brought some good tidings with it. We had promised to open up submissions to other languages with availability of editors. It is a great pleasure to announce that Cogito137 will now be accepting written content in three languages and video content in eight languages. Also, we are open to submissions from anyone and everyone who wants to communicate science and engage with the scientific community and the public at large.

In our effort to be a platform for researchers and scientists to engage in fruitful conversations with the society, we have added a ‘Forum’ to our website. We invite all faculty and students from IISER Kolkata and beyond, to initiate and participate in conversations related to science, society and scientific research in general.

We are striving to rise to a national level multimedia science communication platform and we require maximum support from our home institution. I hope that you will stand by us in the process, through your valuable feedback and consider submitting content of your choice to Cogito137.

Arunita Banerjee
Chief Editor, Cogito137

What do scientists really know?

-Varun Srivastava

"Science is not a perfect tool, but it's the best that we have" -Carl Sagan

As a child interested in science, I used to love reading different books that seemed to explain many remarkable (and complicated) ideas in a way that the general public could understand. After reading them, I used to feel a sense of superiority over others. I could go about telling random "facts" to people- "You know time slows down for things that move fast"; "The world at small scales is a world of probabilities"; "We live in an 11-dimensional world!". All this sounds very interesting and "sciencey", and I used to think that I know so much, just because I could recite facts from a science book.

> *Do the concept of science, and the perception of science disagree? Is science a field that deals with the complete and absolute knowledge of Nature, or should we take scientific 'breakthroughs' with a pinch of salt, knowing that it is only valid till it's not? This opinion piece looks at how science in popular culture is at odds with science in reality.*

My perception was not a unique case. In fact, the education system, especially at the elementary level, has conditioned us into thinking that if we know facts we are knowledgeable, because mugging up facts earns us rewards in the form of good marks. Being right is more important than understanding. So we have learned to value this kind of learning.

Ironically, this is how myths start. Myths and unfounded beliefs propagate through the society because we have been trained not to question. That is perhaps why inherently curious children stop questioning and start accepting facts. That is also perhaps why learning science becomes a burden for most, rather than an enjoyable experience.

Later in my life, I gradually realized the importance of being comfortable with the fact that most of the things I believe to be true may not be so and it is okay to not know. This is a crucial skill that every scientist learns at some point in his or her training. This idea was captured beautifully by Richard Feynman when he said, "I don't mind not knowing. It doesn't scare me."

Scientists are perceived in society as people who know a lot of stuff. It's true; you have to know a lot of stuff to be a scientist. But what most people fail to recognize is that a scientist's job starts here. It is not enough to know a lot of things. It is much more important to recognize areas that you don't know, the gaps in your knowledge, and the ability to ask the right questions to fill these gaps and discover something new. The remarkable thing is, every discovery in science has led to even more difficult questions. It's like finding a key to a closed-door, opening it, and entering a room only to realize that there are ten more doors to open, and each one leads to ten more.

In a way, scientists are doomed to an eternity of exponential uncertainty. However, is being uncertain really that bad? Should we abandon all hope and stop doing science altogether? The answer is, of course, not! Science, through its methods of inquiry, has provided us with a view of the universe so remarkably beautiful and wonderfully bizarre that none of the other human-made stories about gods and miracles come close.

Most people, even researchers and scientists, find the fact that we may never reach a final answer, scary. Looking at the history of science, we can say that this might be true, and if we think carefully, it is not that surprising.

According to our current understanding the Universe started about 14 billion years ago and has been "human-free" for almost the entirety of its existence. Furthermore, the period we have been doing science is miniscule compared to the period our species has been present on the planet. Our brains primarily evolved to find food, hunt animals and make social bonds as opposed to visualizing abstract mathematical concepts. So, it is reasonable to think that Nature may never reveal all her secrets to us, or that our brains might not even be capable of comprehending nature in her full glory. Despite our obvious biological disadvantages, our intellectual curiosity still drives us forward to look up at the night sky and try to ponder our place in a vast universe filled with billions of galaxies, just like our own.

Albert. A. Michelson is claimed to have said in 1894, “It seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice.” In hindsight, the claim seems almost preposterous!

Think of a ripple in a pond. Each discovery of ours causes a ripple of unanswered questions to propagate, and we are left with ever more uncertainties than before. Scientists are always on an active lookout for evidence that may say that their theory isn’t quite right. They are always looking to fill the gaps in their knowledge. Thus we need to redefine the way scientists and science are perceived in society. Science, currently, isn’t an ‘all- knowing body of knowledge’, and in fact, may never be. It is a body of ideas and methodologies that have always evolved to make sure that what we claim to know about nature is as close to the truth as possible.

I would like to end with another one of Feynman’s stories, about the time he came across Descartes’ argument. Descartes philosophized that because he could conceive of a god which was perfect, therefore a perfect god must exist in reality. Feynman argued that in science, everything is imperfect and known only to a certain degree of approximation. What Descartes failed to understand is that his idea of perfection was probably based on the belief that something perfect should exist.

Science doesn’t guarantee perfection. Science is, in fact, the most perfectly imperfect tool developed by us to view nature as she is.

Varun Srivastava is a final year BS-MS student majoring in Physics at IISER Kolkata. He has always found science communication to be a fascinating field and is hence trying it out through this platform. Apart from science he enjoys reading history and philosophy, and is also into sports, especially playing and following football.

Quantum Entanglement: Our ignorance or the Universe's ghost?"

-Magare Sourabh Suryakant

> *A simplistic overview of one of the most startling quantum mechanical phenomena known to exist, one which greatly disconcerted Albert Einstein himself; and a discussion on its consequences regarding our knowledge about the world.*

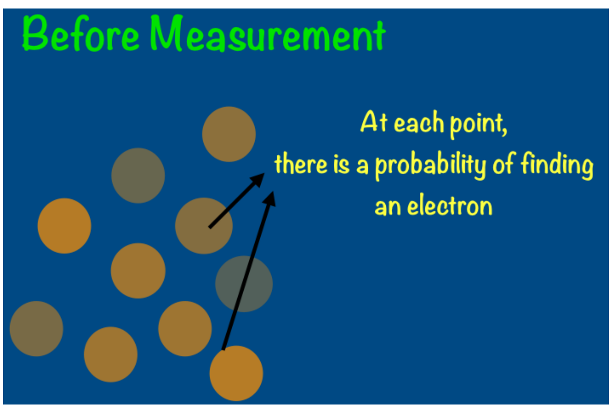
We live in a 'Classical' world. We call it classical because it follows Newton's Maths.

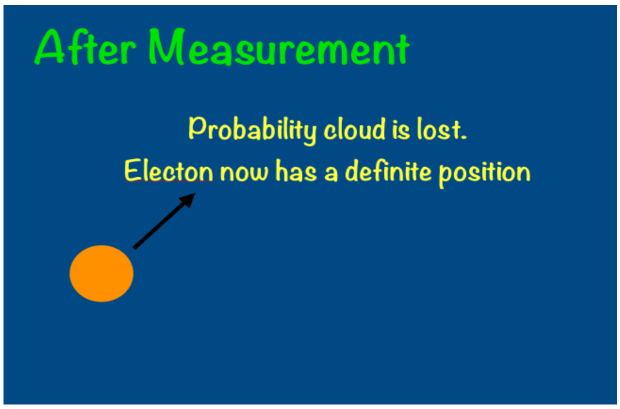
If you throw a ball at a solid wall, it hits the wall and bounces back at you. If you throw that ball as a projectile, it will follow a particular trajectory in the air before it falls.
Using Newton's laws, one can accurately calculate the trajectory or path of that ball. However, the laws of physics are different on a small scale. Particles like electrons, protons and neutrons which make up an atom, behave very differently from anything that we see around us. They don't obey Newton's laws; they follow the rules of the quantum world.

In our classical world, a ball has a definite position. It is precisely present at a point. But the same is not correct for an electron. There is some chance for an electron to be found at one point, and some for it to be found at another. One cannot say with absolute certainty that it is at that point; one can only talk about its probability. Furthermore, when you try to measure its position, its probability cloud is lost, and it takes a definite place. Now, the electron is present at one point precisely. Thus, making a measurement makes the electron lose its probability halo and choose an exact location.

If you think in terms of an analogy, it appears to be like throwing a dice. Before you throw a dice, there is a probability of getting any one of the six possible numbers. After you throw the dice, it will give you a particular number. Is the act of electron position measurement the same as that of throwing a regular dice? The answer is No.

The weird thing about 'Quantum Objects' like an electron is that before measurement, it behaves as if it is present in many places at the same time, like a ghost.

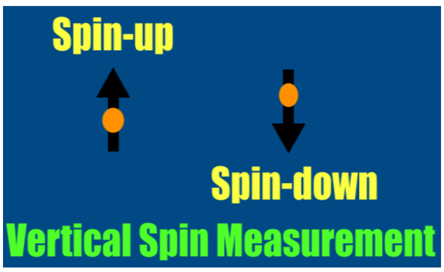


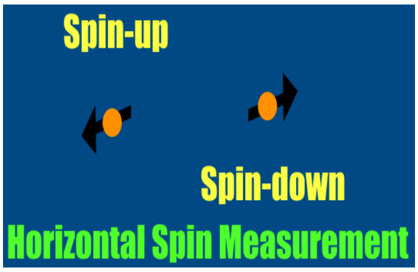


Physicists call it by the jargon 'Quantum Superposition'. And when you attempt to take a measurement, it jumps to one of those many positions. Albert Einstein called this "a spooky action at a distance".

To see this spooky action at play, let's understand the 'spin of an electron', which also shows the same ghostly properties. Spin is a property of the electron, which is measured in two directions: Vertical and Horizontal. After the measurement, there is a 50% chance of getting a spin-up state and a 50% chance of getting a spin-down state. But remember, electron spin has nothing to do with an electron spinning like a ball. Spin is a property of the electron itself, just like the mass and charge of an electron. It's just that scientists are not very good at naming things.

Now let's take two electrons and pass them through a machine we call 'Entangler'. This machine creates a 'connection' between these two electrons. Now, we call these electrons Entangled. In this state, when one electron is measured to have spin-up, then the other electron measured in the same direction, will have spin-down. On measuring the spins of two particles in the same direction, we find that their spins will always be opposite.

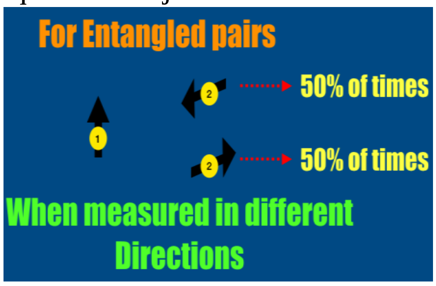


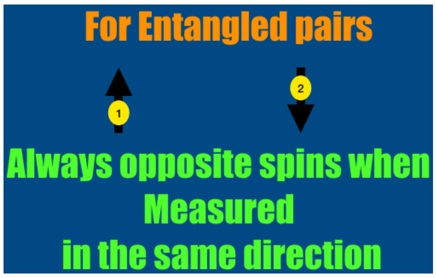


So let's consider an experiment with our two scientists- Alice and Bob with each carrying a particle of an entangled pair. We take them hundreds of kilometers away. Now, they decide to measure their spins. Remember, spin can be measured in two directions. Suppose that Alice decides to measure her electron's spin in the vertical direction and finds it to be spin up. Now, by the property of entangled electrons, Bob's electron will be in a spin-down state when measured in a vertical direction. It appears as if Bob's electron has collapsed into a spin-down state, instantly after Alice measured her electron's spin. But this means the two particles are communicating' at speed faster than speed of light. "Spooky action at a distance !"

But not so fast. Let's consider the same experiment, but this time with a coin instead of electrons. Suppose that we put a coin into either Alice's or Bob's bag. There is a 50% chance that it is in Alice's bag and a 50% chance that the coin is to be found in Bob's bag. When they are far apart, Alice checks her bag and finds the coin; this immediately fixes that it is not in Bob's bag. Isn't this coin behaving like a Quantum Particle? Isn't the coin also showing the same spooky action at a distance? The answer is No.

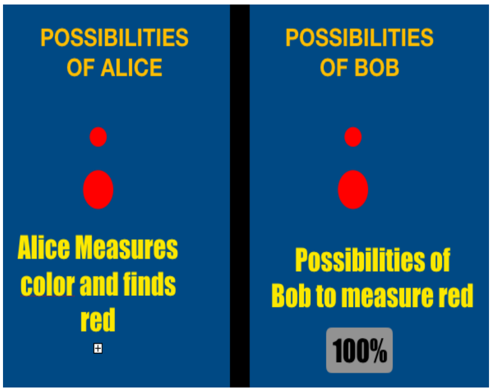
To demonstrate this, suppose that Bob always measures spin in the vertical direction, and Alice can measure spin in both directions. If Alice measures spin in the vertical direction and finds it to be spin-up, Bob will always find his electron to be spin-down. On the other hand, if Alice measures spin in the horizontal direction, Bob will measure his electron spin to be spin-up 50% of times and spin-down 50% of times. It is actually the choice of measurement, horizontal or vertical, by Alice, that is affecting Bob's electron's outcome. In the case of a coin, there is no such choice affecting the other partner's result. So I can safely say that coins are not quantum objects.

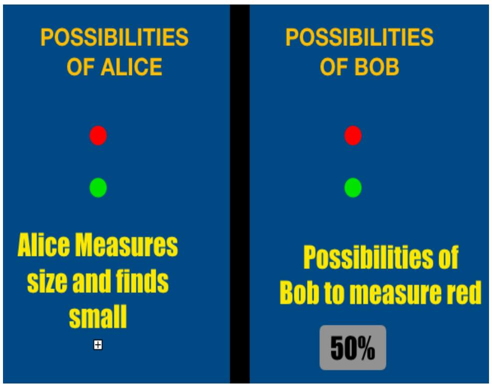




The two entangled particles don't have spin-up or spin-down in the beginning. They were in the probability halo of up-down spin. Only when one of the spins is measured, is their probability halo lost, and the system takes a particular state. But what makes the quantum particles behave that way? To understand that, let's try a 'Classical Entanglement Experiment.'

Just as before, we have Alice and Bob, but now there is a central machine that throws identical balls towards Alice and Bob. The machine throws four types of balls- Small Red, Small Green, Big Red, and Big Green.





We now make our observers Handicapped: Bob can only know the ball's color and not size. Alice can know both color and size, but only one at a time.
If Alice measures the color of the ball and finds it red, she knows Bob's ball is red with 100% certainty. But if Alice measures the size and finds it small then, there is a 50% chance for Bob to find the ball to be red and 50% chance to be green.
We can now ask the same question again- How does Alice's choice of measurement affect Bob's outcomes?

Our Classical Entanglement Experiment works only when our observers - Alice and Bob - have a limited knowledge of the system, that is, they are handicapped. They can know either color or size, but not both. Now, since quantum particles behave in the same way by Quantum Entanglement, does it mean that our understanding of the quantum world and quantum particles is limited? Is our knowledge of the quantum world is handicapped, just as Alice and Bob can measure only the size or color?

This was Einstein's take on Quantum Physics. He stated that quantum physics is incomplete, and one needs a different theory of physics to explain this incompleteness. This started the famous debate between physicists Niels Bohr and Albert Einstein, and paved the way to Quantum Information and Quantum Computation.

Magare Sourabh Suryakant is a 5th year BS-MS student at IISER Kolkata majoring in Physical Science. He enjoys attempting to explain difficult and complicated concepts in a simple way in order to help improve his own understanding. He also enjoys making science videos and uploads them on his YouTube channel, 'Straight Outta Science'.

Suggested reading:
1)Quantum Physics: A First Encounter, by Valerio Scarani
2)What is reality?,By Ganeshan Venkataraman

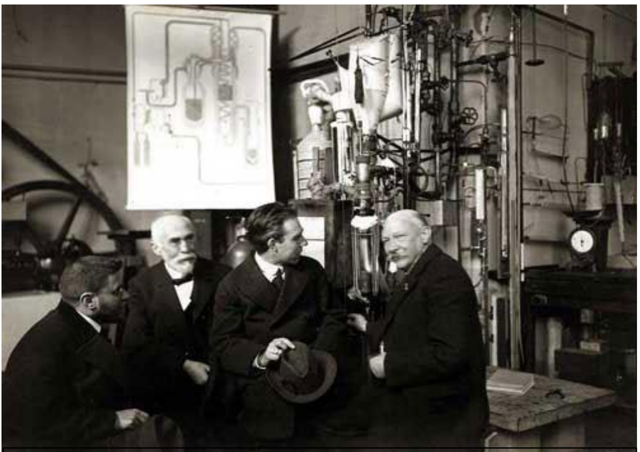
Image References : https://www.hup.harvard.edu/catalog.php?isbn=9780674001817

How 'cool' is Absolute Zero?

- *Simli Mishra*

'Absolute zero' is where there is no motion of molecules. An ideal gas (a hypothetical gas whose molecules occupy negligible space, have no interactions and obey the gas laws) attains zero volume if they are cooled to -273.15 °C or 0 K at constant pressure. However, reaching this temperature would require an infinite amount of energy. "Quantum physics states that it is impossible for a particle to be fully at rest in a specific location," says Alessandro Toschi, Associate Professor at TU Wien. "Heisenberg's uncertainty principle tells us that position and momentum cannot be ascertained with total precision. Therefore, a particle's position and momentum can still change at absolute zero." [1]

> *Absolute zero is the lowest temperature theoretically possible which is also practically impossible to achieve. It is the temperature which the Universe is tending to in its theorised eventual heat death. A number of interesting phenomena occur near this temperature which have intrigued scientists for a century, and it also holds the key to future engineering.*



The cryogenic laboratory in Leiden, Netherlands where Helium was liquified and superconductivity was observed for the first

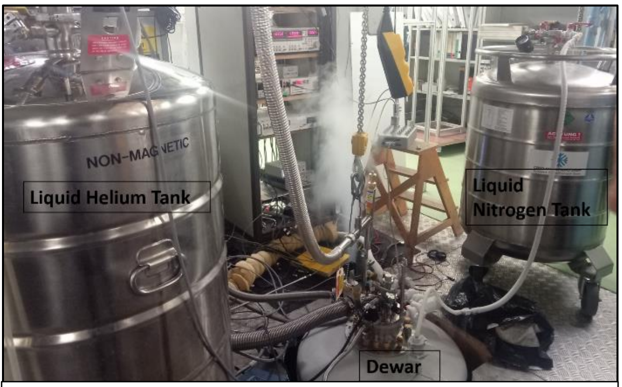
The scientific journey of cooling and liquefaction of gas in laboratories began long back to understand the physical properties of different materials and different kinds of interactions happening in a system. In 1845, Michael Faraday became one of the first scientists to liquefy gases. Following his work, many gases, including oxygen, nitrogen, and hydrogen, were also liquified. As Kamerlingh Onnes finally liquifie the last remaining gas, helium at 4 Kelvin (-269°C) in 1908, it revolutionized experimental research and unraveled hundreds of interesting phenomena, happening at that new extreme.

The use of liquid hydrogen as a fuel for rockets or liquid helium in orbiting infra-red telescopes are just a few examples from an entire pile of applications of cooled and liquefied gases. Perhaps the greatest application in modern times is in the study of superconductivity, where resistance that a material offers to the flow of current, drops down to zero. This means current can flow through superconductors without any loss of power at all.

At a very low temperature liquid helium behaves as a 'superfluid', that is, it flows on a surface without any friction. If you put some of it in a circular channel and get it flowing, it will do so forever, not slowed down by friction.

To give a perspective of how low that temperature is: average body temp is 310 K (or 37°C) and at 33°C you will suffer from amnesia. If your body temperature drops to 294K (or 21°C), you might die of hypothermia. The coldest habitable place on earth in Oymyakon, Siberia, experiences a temperature of around -60°C in the winter months.[2] The minimum temperature measured on earth, at the ice-covered southern continent is -89°C. It was measured at Soviet Vostok station, Antarctica on

1 It's never too cold for quantum: The peculiar characteristics demonstrated by 'quantum critical points' at absolute zero remain one of the great unsolved mysteries of science https://www.sciencedaily.com/releases/2017/08/170801094401.htm
2 Welcome to Oymyakon, the coldest inhabited place on Earth https://www.telegraph.co.uk/travel/destinations/europe/russia/galleries/oymyakon-the-coldest-inhabited-place-on-earth/



A typical setup in a modern-day cryo-lab to attain mili kelvin temperatures.

21st July 1983 [3]. In the solar system, Uranus holds the title for the lowest recorded temperature of -224°C. Boomerang nebula, 5000 light-years away from earth is known to be the coldest place in nature with a measured temperature of just -272.1 °C or 1K, that is just one degree above the absolute zero.[4]

I had the pleasure of observing this phenomenon myself. We used a sophisticated assembly of huge containers, pipes, vacuums, electronics inside a huge container called a "Dewar." It started from something around 250 K and slowly went down to 77K. After a couple of people went and changed a few settings and added a liquid into the huge 'Dewar', the temperature dropped again to 4K (Liquid Helium), only 4 degrees above absolute zero. As the minutes flew by, the temperature further dropped to 1.5K, and finally settled at an astounding 0.027K.

A temperature much lower than what exists in nature was achieved in a complicated laboratory equipment right beside me. I stood there watching, awestruck - in a medium-sized laboratory in a small town of Europe.

It is amusing how technology has taken us so close to the absolute zero, to an asymptotic limit, which is theoretically impossible to achieve. The lowest temperature measured in a laboratory was recorded to be 0.00036K at the National Institute of Standards and Technology (NIST) in Boulder, Colorado.[5]

What is even more astonishing is that such a low temperature can be attained by using an elementary principle, the same principle that causes us to sweat more on a hot day, that evaporation causes cooling. Except in this case we use a mixture of two types of helium in their liquid state, instead of sweat.[6]

A simple analogy to understand this process would be our habit of blowing air into hot water to cool it down, before we can drink it. Instead of air and water, this process uses a lighter (concentrated with 3He) and a heavier (dilute with 3He) form of a 3He and 4He mixture. And, instead of blowing air, we use a vacuum. The evaporation of 3He from concentrated to dilute form causes the temperature to fall down drastically.

For someone who is looking forward to performing experiments at such low temperatures, these dewars, refrigerators, and cryostats hold the key to delightful scientific surprises. It is indeed pretty cool to find answers to what happens in the extreme temperatures near absolute zero.

Simli Mishra, just finished final year BS-MS with a major in Physics, from IISER Kolkata. Her research interest lies in the field of experimental condensed matter physics, with particular focus on exploring correlated electron systems at very low temperatures. She will be joining Max Planck Institute in Dresden for her PhD.

3 https://doi.org/10.1029/2009JD012104
4 Why the coldest place in the universe is so special http://www.bbc.com/earth/story/20140916-the-coldest-place-in-the-universe
5 Sideband cooling beyond the quantum backaction limit with squeezed light https://doi.org/10.1038/nature20604
6 Hitchhiker's Guide to the Dilution Refrigerator http://www.roma1.infn.it/exp/cuore/pdfnew/Fridge.pdf

Love paves the path for a strong dog-human bond

- *Debottam Bhattacharjee*

The public perception of street dogs in India seems to be divided. While some have a very humanistic approach towards these creatures, a recent nationwide survey that I conducted suggested that 50% of the people interviewed, considered them to be a menace on the street. While the allegations against "man's best friend" may seem justified, no one till date has tried to judge it through a scientific lens.

> *A recent study from the Dog Lab, IISER Kolkata challenges the notion that dogs associate themselves with humans solely for seeking food. It shows that affectionate behaviour paves the way for dog-human bonding.*

In a Developing country like ours, street dogs are an integral part of the society. They are found everywhere, from remote villages to busy metropolitans. They have evolved as scavengers, generally depending on human left-over food, but are also found to be 'begging' from people by short or prolonged gazing, while standing or sitting in close proximity. A recent study by VonHoldt et al. (2017) published in Science establishes these activities as dogs' display of extremely 'social' behaviour. Unfortunately, this begging behaviour , so far, has only been linked to their need for food and researchers seem to have overlooked affection and love as the drivers behind it.

Several studies have concluded that pet dogs are remarkably great at communicating with humans. For example, they can follow human pointing gestures or cues to locate hidden food rewards. My studies on the street dogs in India revealed that they are no different, but the most striking discovery was their flexible[1] nature of this particular behaviour.

In a recent experiment, I provided the dogs with two covered opaque plastic bowls, one of which had a food reward inside and tested their responsiveness in multiple trials by pointing randomly at one of the bowls. Puppies ranging from four to eight weeks of age were very fast at approaching the bowls and following pointing cues. Juveniles (ranging from 13 – 18 weeks of age) showed hesitation to approach less of them followed the pointing cue, as compared to puppies. Interestingly, in the case of adult dogs (> 1 year old), after they obtained the hidden food reward, the chances of following my cue again in the next trial increased. But when a dog followed my cue and did not obtain the food reward, the chances of following the cue in the next trial decreased.

My study concluded that such behaviour might have been generated due to dogs' regular interactions and extent of socialization with humans. Puppies at their young age remain mostly protected by their mothers, and have very low exposure to human socialization. Also, people find dog puppies adorable, thus making most of their interactions highly positive ones. When these puppies grow up and become juveniles, they tend to forage in and around human habitations and start to receive negative human impacts like harassment, chasing and beating. So, they become hesitant in terms of approaching an unfamiliar human being. By the time they are adults they have gathered ample life experience on

1 https://journals.plos.org/articplosonele?id=10.1371/journal.pone.0180643

how to approach and respond to a human stranger. Needless to say, they learn to respond differently and in a situation-specific manner, taking into account the previous encounters with human beings.

In order to understand the current status of dog – human relationship in India, my supervisor, Anindita Bhadra of the Dog Lab, Indian Institute of Science Education and Research - Kolkata (IISER- K) and I conceptualized and designed another study[2]. We tested a large number of Indian street dogs to see whether they would choose to pick a food reward from an unfamiliar human hand or from the ground . The idea was to investigate dogs' intentions to make direct physical contact with strangers.

We found that 63% (majority) of the dogs avoided making direct physical contact with the unfamiliar human experimenter and chose ground as the preferred place. Since it was impossible for us to quantify the magnitude of positive and negative interactions these dogs have had with humans previously, we went on to test two subsets of this population in short and long-term conditions.

In canine scientific literature, social contact or petting is considered as a reward which is comparable in importance to food. We re-checked some of the dogs' preferences immediately after providing them with positive social contact, which was defined by petting three times on their heads. We found very little influence as they did not change the preference of obtaining food from ground to human hand. Only the approach time shortened, suggesting a faster response to the short-term social rewarding. In the long-term condition, we checked for the effects of social petting. An additional food reward was provided respectively to two different groups of dogs from a separate subset on specific day intervals from Day 0 to Day 15.

What we found was very surprising – dogs showed less willingness to socialise with the human experimenter even when they received an additional piece of food reward, while provision of a social reward like petting, resulted in dogs' higher exhibition of socialisation with the human experimenter.

Dogs that received social petting became more friendly compared to the group which received an additional piece of food every time. Thus we concluded that dogs rely on affection from humans rather than food for building trust.

Indian streets abound in dog – human conflicts. Previous investigations identified great slack in the dog population management and lack of awareness among people about the dos and don'ts of treating animals as the underlying reasons for such conflicts. Our findings suggest that a more loving approach by humans could reduce these conflicts to a great extent.

So, next time you see a dog on the streets, try being a little compassionate rather than hitting it or shooing it away because affection seems to have a greater impact than food in their lives. What's better than becoming best friends again?

Dr. Debottam Bhattacharjee has recently completed his PhD from IISER Kolkata. He will be joining Utrecht University as a Marie Curie Fellow for his postdoctoral studies. His research interests range from behavioural ecology to comparative cognition.

2 https://jeb.biologists.org/content/220/24/4654

Hilbert's Quarantine Centre: To infinity and beyond!

- Debmalya Bandyopadhyay

This article revisits a classic thought experiment in Mathematics. It explores some of the intricacies of countable infinity and how its definition can be used to achieve seemingly impossible results, and contemplates how useful these results may be in the current pandemic.

While I write this sitting in the privilege of my home, the pandemic keeps raging outside, the hunt for a vaccine continues, and the number of active cases keep skyrocketing towards the figures in Buzz Lightyear's memorable line, "To infinity and beyond". As health departments around the world scramble to find space for isolating potential victims of the virus, perhaps it is time to turn to math for possible solutions, or rather to a curious thought experiment developed by one of the most fantastic German mathematicians to have ever lived- David Hilbert himself.

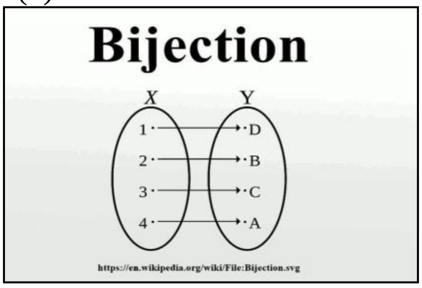

Image 5.1: David Hilbert

Hilbert was one of those prolific polymaths who emerged towards the end of the 19th century. His genius was everywhere and in abundance. He had made contributions in not just a number of mathematical disciplines, but he also built the fundamentals of mathematical logic itself - a field of study now known as 'proof theory'. He kickstarted several arenas of research by presenting a collection of 23 unsolved problems in the year 1900, that kept mathematicians busy throughout the last century. After 120 years, only 9 of these problems have well agreed solutions, some of the others have solutions which are only partially accepted in the mathematical community and some (including a cheeky little question called the Riemann hypothesis) remain unsolved to this date.

But fret not, we would not be discussing any of those problems in this article. We would much rather focus on the pandemic at hand, and explore a hypothetical quarantine centre which can not only hold an infinite number of residents, but (with a little belief in Buzz Lightyear from Toy Story) something far beyond that number.

Hilbert had introduced this idea in a 1924 lecture, and it was later popularized by George Gamow in his book "One Two Three...Infinity". This paradox is popularly referred to as "Hilbert's infinite hotel" or "Hilbert's grand hotel paradox". Since the current pandemic disallows us the luxury of a hotel and we have lives to save, we shall adopt its structure to that of a quarantine centre. But before we jump onto the jaw dropping properties of this place, let us first explore a little about what *infinity* really means.

Mathematics mostly deals with two kinds of infinities, one *countable* and the other *uncountable*. For most practical purposes the concept of a number being infinite is understood as something which is *unbounded*, or greater than any finite number one can imagine. When the *cardinality* (fancy term for number of elements) of a set is said to be countably infinite, it means that we can establish a one to one correspondence between elements of the set, and the set of natural numbers. This basically implies that for each natural number we can identify a unique element of the set such that no element in the set is left unidentified. Thus we can count the elements of the set (even if there are infinitely many of them) using our natural intuition for counting and that is where the name comes from. This unique correspondence between two sets is called *bijection*. As an example, a bijection between the set of all positive integers and the set of all negative integers is f(x) = -x.

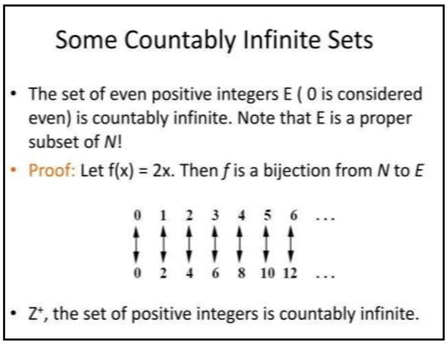


On the other hand, uncountable infinity is something far larger and messier (like the set of all real numbers) and we cannot count them because given any element from an uncountable set it is impossible to determine its immediate successor or predecessor in the set. (As a small example for the real numbers, one might think 0.51 comes right after 0.5, but then 0.501 sits in between them, contradicting the thought!). The cardinality of an uncountable set is thus strictly greater than the set of natural numbers, and one cannot establish a bijection between the two. (John Green had probably meant this when he wrote "Some infinities are bigger than other infinities.") To save ourselves from mental exhaustion, for the rest of this article we will be dealing with and referring to only countable infinity, which is well behaved and kind enough to allow us tricks that we would use here.

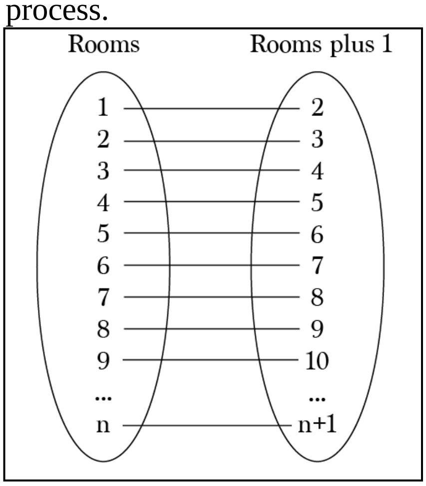
Now, consider the set of even natural numbers. This is a *strict subset* of the set of natural numbers. In spite of this, it turns out that one can establish a unique correspondence between it and the set of natural numbers (with n corresponding to 2n)!

How is it possible that the cardinality (or size) of a set is the same as that of another which is entirely contained in it? As an equally surprising result, it turns out that the set of all integers (positive, negative and zero) which strictly contains the natural numbers, has its cardinality same as that of the natural number set itself! There is nothing wrong with this apparent contradiction. The way we defined our (countable) infinity, our infinite sets may hold subsets that are also the same size as that of natural numbers. Hence, well defined bijections are completely possible between a set and it's subset, if both are infinite. In fact, any infinite subset of a countably infinite set is bijective to it because both are bijective to the set of natural numbers.

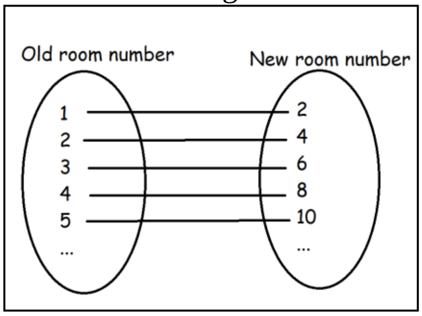
Thus, the following sets have the same cardinality (size) : Even natural numbers, odd natural numbers (since they only differ from the evens by 1), natural numbers, and integers, for we can establish bijections between any two of them! One can also prove that countability is preserved under (countable) unions and finite *cartesian products*, a fact we would use later on. (Finite cartesian product of a set with itself (in our case, the set of natural numbers since we are only concerned with cardinality) can be thought of as taking finitely many layers of the same set at the same time.) Mathematics is indeed a land of strange miracles, and we will unearth stranger phenomena in this article itself!



Now that we are somewhat versed in the basic definitions, it's time to visit Hilbert's quarantine centre, which (as you must have guessed already), has infinitely many rooms. For ease of understanding we shall name the rooms as 1, 2, 3 and so on with natural numbers. On a given night, say we have a full house, and a suspected patient arrives asking for a room. For any other quarantine centre, we would have to turn the visitor away, but we don't do that here. Even though we have a full house, all we need to do is to shift the occupant of room 1 to room 2, that of room 2 to room 3, and so on. Thus by shifting the occupant of room n to room n+1, we free up room 1 for our new guest. But doesn't this mean that the person who resided in the last room would be thrown out?. That is precisely the catch, because in this quarantine centre, there is no last room! There is always a next room to any given room, but since there are infinitely many of them, there is no end to them. Thus you know what to do if any ‘finitely many’ covid suspects arrive, say k many of them. All we need to do in order to find them space is move the resident of room n to room n+k, freeing up the first k rooms for our new guests in the process.

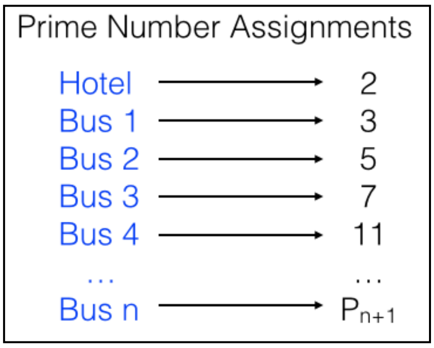


But on a particular night, the government’s worst nightmare comes true, and *a coach of infinitely many* covid suspects arrive at our quarantine centre. Instead of vilifying whoever sent them together in a single coach, we need to quickly arrange rooms for them in our centre. Now we will take advantage of the weird fact I mentioned earlier. All we need to do is move the resident of room n to room 2n, thus moving the entire set of natural numbers to the set of even natural numbers (since their cardinalities are the same!) freeing up all the odd numbered rooms in the process. Since the set of all odd natural numbers also have the same cardinality as that of natural numbers, we have no trouble with assigning rooms to all of our new guests.



But due to the exponential growth rate of infections, soon the situation gets even worse sending the government into a blind panic attack, and we are sent an *infinite number of buses, each carrying infinitely many suspects*. This is a situation none of us were prepared for, but we have to somehow accommodate all of them in our quarantine centre. We turn to the natural numbers that mark our rooms for a solution, to find a way to pull off this apparently impossible task, and a very special set of numbers called the prime numbers come to our rescue.

We first claim that the set of all prime numbers is infinite. If primes were indeed finite in number, we could have taken the product of all the primes, added 1 to this product and formed a “new” prime, because no prime (hence no number except 1) can divide its own multiple and an immediate successor of that. Thus by comments made earlier they are also of the same size as the set of all natural numbers. Moreover, no prime interferes in another prime’s powers (for example, no power of 2 is divisible by any other prime other than 2). Hence for each prime, we have an infinite chain of rooms which are its powers. Voila!



We first move our existing set of residents to rooms which are *powers* of the first prime, 2. Once this is done, we take the first bus and move its infinite number of passengers to rooms that are powers of the next prime, 3. For the second bus, we consider powers of the next prime, 5. Since both the set of all primes and the set of all possible powers of each prime are the same size as the natural number set, we have no trouble in accommodating infinitely many buses each filled with infinitely many passengers!

What is even more surprising is that once we have moved all the people into their rooms, we have also vacated infinitely many rooms, because any composite numbered room that is not a power of a prime is now vacant! Note that this is not the only way to accommodate these many people. There are other ways to assign them room, for example we could have assigned the person in the a^{th} seat of the b^{th} bus the room 2^a3^b (after moving our existing residents to the room numbers where b=0 in the above formula). Since prime factorization of any natural number is unique, no two persons would be allotted the same room and we would have infinitely many rooms vacant in this case as well.

We can similarly adapt our above algorithm(s) to a situation when we are sent an infinite number of ships, each carrying an infinite number of buses, each of which carries infinite passengers (The government has basically given up at this point, all hope is lost, except that of mathematicians). We now have 3 layers of infinity, but we don't get intimidated by such figures anymore. All we have to do to fit this enormous amount of people is raise our prime powers as powers of another layer of primes. If that confuses you, think of it as assigning the person in seat a, of bus number b, of ship number c the room $p_a^{p_b^c}$ where p_a and p_b are the a^th and b^th primes.

If we want to extend the alternate method that we described above, we move the person seated in the above mentioned way to room number $2^a3^b5^c$. We can keep adapting the procedure(s) similarly as long as we have *finitely many layers of infinity*, because finite cartesian products (or layers) of countable infinities are countable infinity in return, and as long as we have our set of guests to be countably infinite, we can find them rooms in our quarantine centre. Unfortunately, the same is not true if we have countably infinite layers of infinity, because it can be proved to be uncountable- the other dreaded infinity where none of this wholesomeness is preserved.

Of course, we are only assigned the task of planning who gets what room, and we leave the other details to the staff. We need not worry ourselves about moving possible patients in and out of rooms, changing their allotment almost every other night and ensuring safe distancing and sanitization in the infinite corridors. We also leave the task of building this architectural marvel to our engineer friends, and once they are done we can use our math to save the world. If you know someone who might be up for the job, make sure you send them some motivation with this article and a copy of Toy Story!

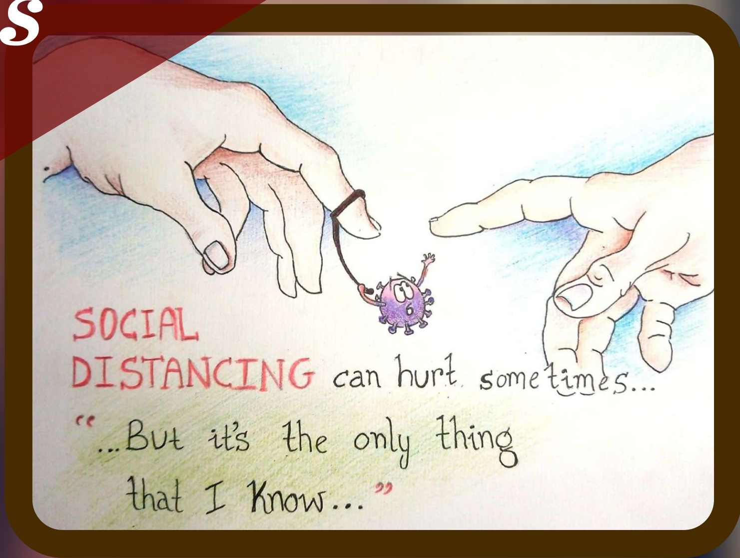
“ *Debmalya Bandyopadhyay is a fourth year undergraduate student pursuing Integrated BS-MS in Mathematics and Statistics from IISER Kolkata. Apart from math, he is likely to be found adrift in poetry, cinema and music.* ”

---

www.slideserve.com
http://mathandmultimedia.com/2014/05/26/grand-hotel-paradox/
www.images.google.com
https://medium.com/i-math/hilberts-infinite-hotel-paradox-ca388533f05
https://www.youtube.com/watch?v=Uj3_KqkI9Zo
The horror of my sketches: Microsoft paint

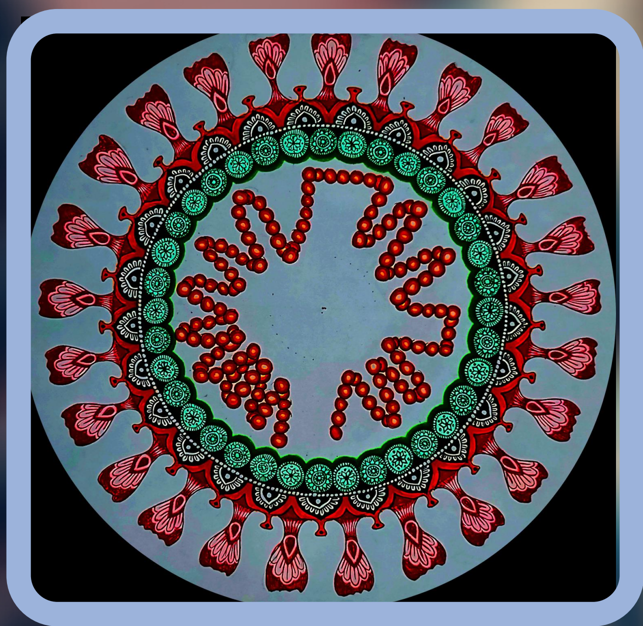
Comictious

submissions



Anchit Bhair
IISER Kolkata

Pavithra Ashok
IISER Thiruvananthapuram



Ananya Arvind
IISER Thiruvananthapuram

Harsh Bardhan
IISER Kolkata

Send your COVID-19 related Sci-art and illustrations by 15 september on scicomm@iiserkol.ac.in

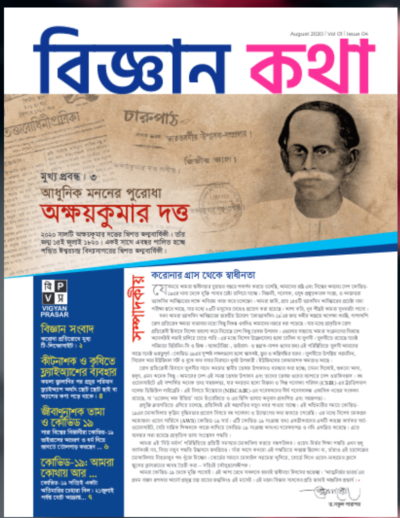
Cogito137
The Thought Capsule

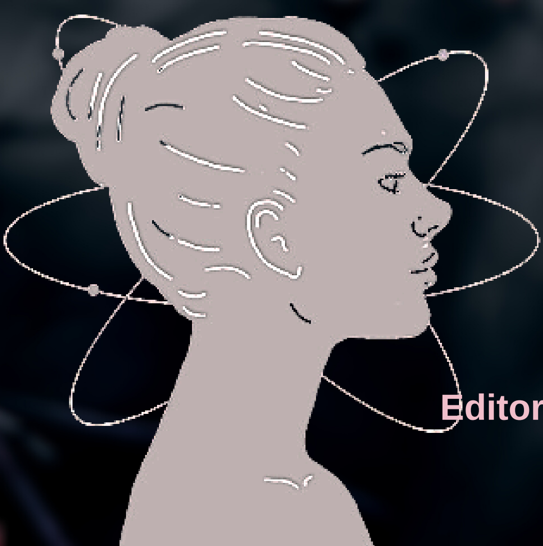
Women in Science
IISER

#WIISER

Cogito137-The thought capsule, is proud to host a showcase of amazing women, excelling at various STEM fields in IISER.

_FEATURED__





Team Cogito137

Chief Editor: Arunita Banerjee
Faculty advisor: Prof. Subhajit Bandyopadhyay
PR Manager: Dakshesh Vasan
Website Manager: Rahul Bordoloi, Prajwal Padmanabha
Publicity team: Bhagwat Isha Paresh, Ananyapam Dey
Editors: Abhigyan Saha, Nandana Goswami, Srujana Mohanty, Subhayu Bagchi, Shrestha Chowdhury, Aditya Dwarkesh
Designer: Shreyash Borkar