InScight

Patterns In Primes Via Probability
Dixit & Pathak

My Scientific Journey With Nobel Laureate Omar Yaghi
Prof. Rahul Banerjee

Against All Odds: The Man Who Brought IVF To India
comic by Kajori & Afreen

art by Lipsamayee Mishra

Reflections on a Growing Scientific Dialogue

*Foreword by **Prof. Asok K. Nanda***
Department of Mathemetical Sciences, IISER Kolkata

The initiative taken by our young minds to publish the Science Magazine *InScight* to inculcate interests among the people in academic community to have better understanding in different interesting topics with the help of people of varied interests in science disciplines gives me immense pleasure to see that it has completed one year of its successful journey starting in January 2025. Now, in January 2026 issue, *InScight* has brought to you the discussion of science in different disciplines including Mathematics which, in my opinion, is not a science rather a tool for developing science. This issue includes discussion on the topic of 2025 Nobel Prize in Medicine for discoveries on peripheral immune tolerance, opening a new field of research for treatments of auto-immune diseases. A discussion on Diophantine Equations, connecting Recreational Mathematics to Number Theory, will give ample scope of thinking to the mathematics lovers. You will also get discussion on prime numbers, showing how irregularities in practice follow statistical regularity.

If you are truly motivated towards something, no obstacles can stop you reaching the destination, as can be seen through an inspiring article on how Nikolai Koshliakov developed Generalized Zeta Function from a Soviet Labour Camp. The discussion on how Niels Bohr's work on Quantum Revolution was inspired from Bohr-Van Leeuwen theorem which raises a fundamental paradox, is an article worth reading. This issue also gives some very interesting mathematics quiz along with their correct answers. Along with some more interesting topics, one will get a wonderful discussion, through a comic, on how Dr. Subhash Mukhopadhyay was deprived of his recognition for his discovery on IVF technique, leading to his untimely demise.

The discussions are not only by the members of the IISER Kolkata community but also by the students and the faculty members of different premier institutions, showing the gain in popularity of *InScight* in just one year period. It is mostly observed that the people at large do not think out of the traditional way because of lack of time due to different obligations, or because of lack of interest. However, there is a necessity of out-of-the-box thinking in order to promote science towards its proper utilization in human interest. You will all agree with me that any science without mathematics works like a fish out of water, and hence understanding and development of mathematics for solving different real-life problems is a must.

Through this platform we invite students, researchers, educators and all interested citizens to get involved in different scientific discussions, and *InScight*, I am confident, will serve the purpose of bringing different domains of science under the same umbrella to handle any issue of this universe by providing a platform for critical discussion and analyses of different problems. I wish *InScight* to keep its upward journey through the radder of success with time, as it has been doing for last one year.
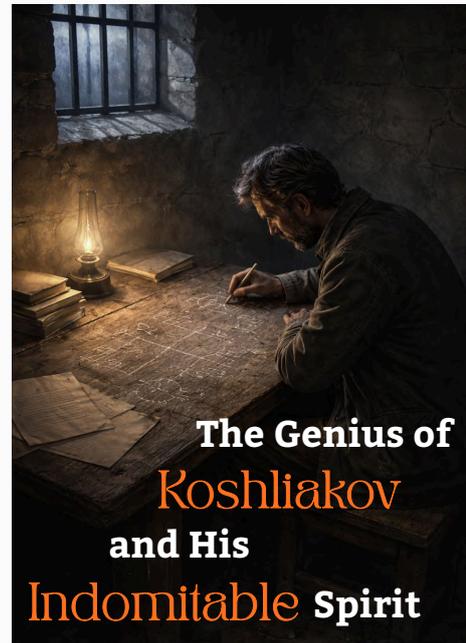
The Genius of **Koshliakov** and His **Indomitable** Spirit

From a Soviet labor camp, Nikolai Koshliakov developed a striking generalization of the Riemann zeta function under conditions that defy imagination. Read more.

From ISI to **Princeton** and Back

Conversation with Ritabrata Munshi
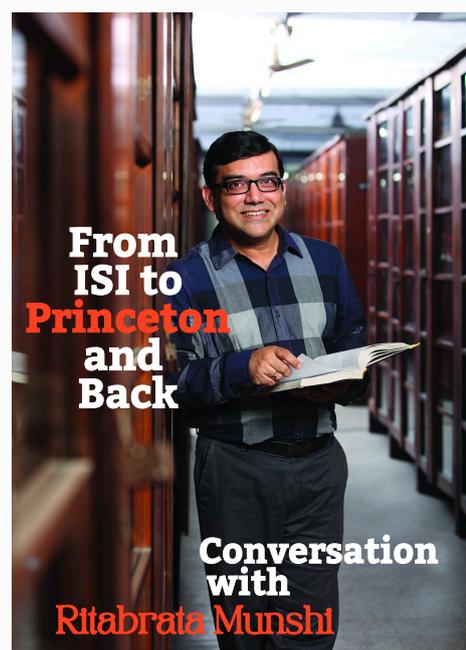
In this interview, Prof. Munshi reflects on a life shaped by mathematics—from early fascination with symbols to research at Princeton and a return to ISI Kolkata as a faculty member.

A Word from the Editors

Puppala Ainesh Kumar
Editor at InScight, IISER Kolkata

Hello fellow readers, we are back with the latest issue of *InScight*-The IISER Kolkata Science Magazine. Before I go any further I would like to thank all the readers for their support that helps us continue our journey.

We are back again with a variety of interesting articles over different subjects. To kick it all off we have an article on regulatory T-Cells. The 2025 Nobel Prize in Physiology or Medicine was awarded to Shimon Sakaguchi, Mary E. Brunkow, and Fred Ramsdell was awarded for the elucidation of the cellular and genetic basis of the same. The article 'The peacekeepers within', delves deeper into the story that redefined modern immunology as we know it.

To quote Albert Einstein "In the middle of difficulty lies opportunity", we have a fascinating article on Nikolai Sergeevich Koshliakov who developed a striking generalization of the Riemann zeta function, while in a soviet labour camp. His story serves as inspiration for all of us who believe nothing good can come out of even the most belligerent times. We have a very interesting article regarding the nature of primes, called 'patterns in primes via probability' written by Anup Dixit and Sidhi Pathak.We also have a creative comic by Kajori Barman, Afreen Chowdhury highlighting the incredible story and hardships faced by Dr. Subhash Mukhopadhyay, in bringing IVF to India.

To end it all, we have a few fun science inspired games and a quiz at the end of the magazine.

This issue of *InScight* comes packed with a lot of inspirational stories that we hope will help you keep moving forward, despite of how bad everything gets, despite of how dark the path may seem, for 'it is the path through the darkness, that brings us closer to the light'

The **Peacekeepers** Within

A small group of calm "guardian" act as peacekeepers to prevent the immune system from turning against itself and maintain harmony among cells and organs.

Regulatory T cells are a unique subset of cells that play a crucial role in suppressing harmful autoreactive T cells in the body. Read the rest in this article.

In This Issue

Macroscopic Quantum Phenomena : The Story of 2025 Physics Nobel

Raka Dasgupta   *(Dept. of Physics, University of Calcutta)*

December 22, 2025

We grow up learning that the everyday world obeys Newton's laws, while quantum mechanics belongs to the fascinating realm of atoms and electrons—but what if that boundary isn't so strict after all? Inspired by the 2025 Nobel Prize in Physics, this article explores how carefully engineered electrical circuits can behave like "giant atoms," exhibiting quantum tunneling and discrete energy levels on a scale you can hold in your palm.

EDITED BY: Shibaraj Sahu, Sukalyan Deb

Raka Dasgupta is a theoretical physicist, engaged in both teaching and research. Presently she is an Assistant Professor at Dept. of Physics, University of Calcutta. Her broad research interests include quantum many-body physics and statistical physics. She is also an acclaimed poet and has been honored with the Krittibas Puraskar, the Bangla Academy Award, and the Yuva Puraskar from the Sahitya Akademi of India.

Giants and Lilliputians

Remember how Gulliver ended up first in the land of tiny people, and then in the land of giants? The two countries had completely different social norms. Well, the same is there in the physical world, too! in our everyday life, everything is governed by laws of Classical Mechanics. Starting from how a stone thrown upwards falls back to the ground, to how an accelerated car moves, to planetary motion, satellite launching: the gospel of Newton has it all! At the opposite end of the spectrum, there is the world of the small — atoms and its constituents: electrons, protons, neutrons. In that world, the laws of Quantum Mechanics rule supreme. The quantum particles often show behavior that would be deemed strange, or even impossible in the classical world.

Can the twain ever meet? Can the quantum laws govern classical systems, if they are tailored suitably? Let's take a look at the Nobel prize for Physics, 2025. It was awarded to John Clarke, Michel H. Devoret, and John M. Martinis. As per the press release of the Nobel Committee, the recognition is "for the discovery of macroscopic quantum tunneling and energy quantization in an electrical circuit." So there are three distinct key-words (or, maybe key phrases!) involved : "macroscopic", "quantum tunneling", and "energy quantization". "Macro" means large, as opposed to the small – "Micro". Evidently, the work behind this prize somehow bridges the world of the giants (i.e., "Macro") and the Lilliputians (i.e., "Micro"). Intriguing, isn't it?

What really are quantum tunneling and energy quantization? And what does it mean to have them on a macroscopic scale? In this article, we'll try to find answers to these questions.

Quantum Tunneling

Imagine a ping pong ball lying at the bottom of a big circular tub. Lightly shake the tub, and the ball starts rolling inside it. Even then, the ball cannot escape the tub: because before it stands a tall barrier - the wall of the tub. The kinetic energy of the ball is not sufficient for it to climb over that wall.

Now replace the ball by a tiny particle like an atom, and the tub by a potential well where it is trapped. There is a certain probability that the quantum particle might cross the barrier, and land outside. This is more like the apparition or magical transportation you come across in Harry Potter books or our very own Gupi Gayen Bagha Bayen movies. This is not guaranteed that it will definitely cross the wall, but yes, there is a high chance of it. So if you have hundreds of such particles, maybe ten or twenty will "Break on through the other side".

This is quantum tunneling.

FIG 1 : *Left: A classical particle cannot escape from the tub. Right: A quantum particle can tunnel to the other side.*

Energy Quantization

And Energy quantization?

Quantum mechanics says that such tiny particles cannot have arbitrary energy values. They have specific allowed energies — like the steps of a staircase. Just like you cannot stand in the intermediate region between two steps of a stairway, atoms, too can stay on the distinct energy levels only : not midway.

But all of these are familiar textbook material: essentially, the primer of quantum mechanics. The very ideas of quantum tunneling and energy quantization date back to a century. In fact, the pioneers of quantum theory: Planck, Einstein, Bohr, de Broglie, Heisenberg, Schrödinger, Dirac, Pauli — all were honored with Nobel Prizes long back.

So what is the big deal this time? What led to another Nobel prize?

The novelty lies in the word "macroscopic." It takes us back to the giant vs. Lilliputian conflict again. The Lilliputian particles obey quantum mechanics, and the giant particles do not.

What if one fine morning you see a giant macroscopic system to follow quantum rules? Will not that be astonishing... something like a new world order?

Superconductivity, Superfluidity, and Condensates

Interestingly, macroscopic quantum behavior is not, in itself, new either. At very low temperatures, some metals suddenly lose electrical resistance, an effect known as superconductivity. Liquid helium becomes a superfluid at 2.17 Kelvin, and demonstrates a flow without viscosity. If we think carefully, what superconductivity is for charged particles, superfluidity is for charge-neutral ones. That they support dissipation-less low (current flow in the former, particle flow in the later) can be attributed to the collective movement of the constituent particles. When cooled enough, these tiny balls all start moving in perfect synchrony, like the lockstep marching of soldiers. As a result, there are no collisions, no scattering of particles (nobody is going to bump into a fellow soldier in that parade) : only streamlined motion ahead.

Bose–Einstein condensation, (named after Satyendra Nath Bose, and Albert Einstein) is another fascinating phenomenon. Bosons , i.e., particles with integer spins, condense to the lowest momentum state collectively. In spirit, these condensates belong to the same family as

that of superconductors and superfluids. These all are quantum phenomena, observed on a macroscopic scale. Unlike the electrons and protons that you cannot see with your bare eyes, you can see how a superconducting circuit functions or a superfluid rises up the capillary well in our lab environment.

Here is a catch again. These are undoubtedly manifestations of quantum physics, emerging in a macro system. However, the real culprits behind such occurrences are micro again : electron pairs (average distance between those electrons: 10-100 nm), bosonic atoms (radius : fraction of a nm), etc. What if it could be demonstrated that a large object itself can behave the way a quantum particle does? **If we find that the effect arises not merely because the constituent particles follow quantum mechanics, but because the large system itself, as a whole,show clear quantum signatures? That would be even more remarkable. Clarke, Devoret, and Martinis precisely did this.**

It's all about Scales

The terms "Giants" and "Lilliputians": or even, "Large" and "Small" are, in fact, misnomers. Classifying a system as large immediately raises the question: large with respect to what? Similarly, calling a system small is meaningful only relative to a chosen reference. So one
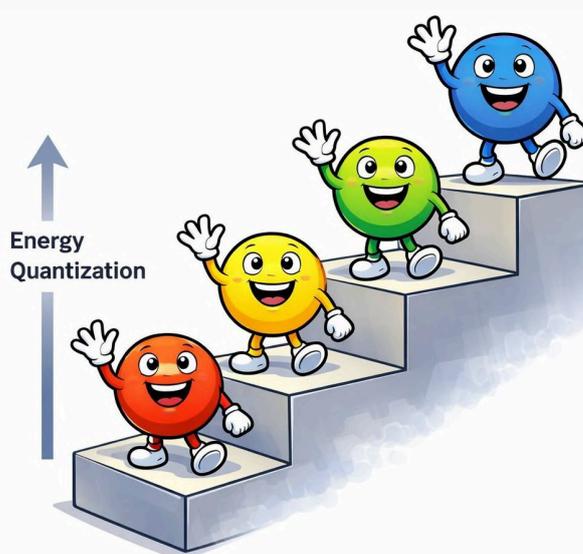

FIG 2 : *Quantum particles can stay on distinct energy levels only: like the steps of a staircase.*

needs to ask: is it larger or smaller than some predefined scale or object?

So if we say, micros are quantum, and macros are classical : that would be an oversimplified picture. Rather, let's talk about energy scales. A system exhibits quantum behavior when the relevant quantum energies exceed the competing classical scales. For a single particle (in the quantum domain, it is capable of displaying both particle and wave natures) : the comparison should be between the de Broglie wavelength (the wavelength associated with the "wave" nature of a particle), and system's other length scales. For a collection of particles, one might need to check the competition between quantum fluctuations, and thermal fluctuations.

Thermal fluctuations result from random motion of particles : a temperature effect. Quantum fluctuation has its roots in Heisenberg's uncertainty principle : there will always be some fluctuation in position, momentum of the particle even if you freeze it to absolute zero of temperature. If in a system the thermal fluctuation overpowers its quantum counterpart, the system is predominantly classical. **If, on the other hand, the quantum fluctuation overrides the classical ones (this is what happens for small particles, at very low temperatures : that is why superconductivity, superfluidity and Bose Condensation all occur at near-zero temperature ranges), we see the live demonstration of quantum mechanics.**

Clarke and team used a classical circuit to mimic a quantum system like an atom, and engineered it such a way that the corresponding "quantum-like" energy scales dominate over the thermal scale. This is the key to the macroscopic quantum behavior they observed.

Simulating an Atom

How a system can mimic another?

Simple. If two systems, however distant, follow same form of mathematical equation, one can be used to "simulate" the other. Meaning, by studying one of them, we can deduce the properties of the other. Take, for example, an inductor-capacitor circuit follows the same equation as that of a spring-mass harmonic oscillator system. If the parameters (inductance, capacitance, temperature) can be chosen accordingly, it can even yield a "quantum harmonic oscillator" : if the discrete level spacings (distance between the steps of the energy

staircase) is higher, compared to the thermal energy scale.

Similarly, the differential equation describing the dynamics of a Josephson junction with self-capacitance can be shown to be equivalent to the equation for a point mass in the one-dimensional tilted cosine potential. A Josephson junction consists of two superconductors are separated by a thin insulating barrier. Wonder of wonders: through this junction, an electrical supercurrent can flow even without any applied voltage! This miraculous effect was predicted by Brian Josephson using laws of quantum mechanics, and was later realized experimentally.

This junction was an essential ingredient in the experiment conducted by Clarke, Devoret, and Martinis.

The Experiment

Around 1984–1985, the trio carried out a series of laboratory experiments. At that time, Clarke was the supervising professor, Devoret a postdoctoral researcher, and Martinis a PhD student. Their experiments unveiled a new generation of quantum-mechanical behaviors: that on a macroscopic scale.

How did they come up with the idea? Well, Science never advances through sudden, unprepared miracles. Clarke's work was grounded in decades of theoretical and experimental studies. **Theoretical physicist Anthony Leggett (he won Nobel prize for his works on superfluidity) suggested this possibility in 1980, distinguishing two kinds of macroscopic quantum phenomena: (1) where a large object behaves strangely because its microscopic constituents obey quantum mechanics (like superconductivity, superfluidity), and (2) where the large object itself shows quantum behaviour**. Clarke's 1985 experiments attempted to demonstrate the second. The team numerically simulated the experiment beforehand with help from Daniel Estève to identify the exact range of parameters. Thus, theory, experiment, and computater simulation — the three pillars of modern science — all played essential roles in this discovery.

Clarke and his team fabricated Josephson junctions on oxidized silicon chips by layering specific materials. They connected the junction to a current source, a microwave radiation source of fixed frequency, and a voltage meter. The entire experiment was conducted at extremely low temperatures — in the milliKelvin range.
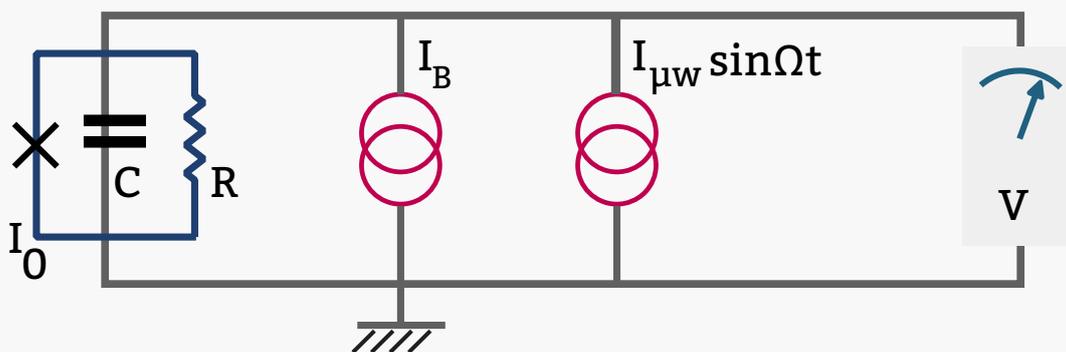

FIG 3 : *Circuit diagram of the experiment : taken from [4]. The Josephson junction is marked by a cross, and is shunted by a capacitance C. R is the resistance. The junction is connected to two current sources : static bias (I_B) and microwave ($I_{\mu w}$) . The voltage V across the junction is measured by a low-noise amplifier.*

Because it was a Josephson junction, they observed current flowing even without any applied voltage. This zero-voltage state is the junction's initial and most stable configuration — like the ball trapped at the bottom of a parabolic potential, or, in this particular case, at one of the minima of the cosine potential.

Now recall that ping pong ball. Classical mechanics forbids it from climbing out. Yet a quantum version could escape via tunneling.

In the Clarke–Devoret–Martinis experiment, the junction did escape from the zero-voltage state to a finite-voltage state — by tunneling through the energy barrier, exactly like a quantum particle. How much of that tunneling occurred? Since the "trapped" system corresponds to the zero-voltage state, any non-zero voltage appearing in the voltage measuring amplifier implies a tunneling of the system from it, and can directly measured this "escape rate".

So non-zero voltage meant that quantum tunneling was occurring — the state of a system has tunneled.

This is the quantum-like state arising from a macroscopic object, right? The Josephson junction itself was small (10 micrometer X 10 micrometer), but the full circuit was of the scale of centimeter: something that you can comfortably hold on the palm of your hand. The temperature was not too cold either: certainly not the nanoKelvins or picoKelvins one requires for realizing Bose-Einstein condensation and related stuff.

But the story doesn't end there. Remember energy quantization, i.e., he staircase of energy levels? The zero-voltage state is like the lowest step. In principle, the junction could occupy higher steps too — step 1, step 2, and so on. If energy is supplied externally, the system can climb the steps. And once it climbs higher, escaping the potential well become even easier.

Martinis applied microwaves. Remarkably, the voltage, i.e., the tunneling probability, sharply increased at exactly three specific current values, and not at intermediate ones. **This meant the system had climbed to particular higher energy levels before the tunneling took place**. Just like atoms, the Josephson junction circuit behaved as if it possessed discrete quantum energy states. In effect, it acted like an artificial atom.

What's Next?

The range of applications is vast. If you can design a circuit that can mimic the properties of an atom, you end up having highly controllable "artificial atoms". Whatever you aspire to do using an atom by exploiting its quantum mechanical features, you can do it now using the circuit, and with much better precision. An atom that can stay in either of its two lowest energy levels can function as a qubit: with states |0> and |1>. The same can be achieved with this Josephson junction circuit, utilizing its ground state and the first excited state. Only now, the controlling knobs are much more efficient: one can simple change the resistance or capacitance , or the microwave frequency to change the state of the system as per need. **Thus, such a Josephson junction circuit can be employed as versatile platforms for quantum computation.**

These systems also hold potential for more application-oriented areas such as quantum teleportation (a protocol that transfers quantum information across a distance) and quantum cryptography (ensuring secure communication using principles of quantum mechanics).

Even if we put aside the application aspects, Clarke and team's work does not lose its significance. For, this work is also about the fundamental nature of physics. It springs from the very premise quantum mechanics was built upon, and probes our basic understanding of matter. The demonstration that a quantum effect can be harnessed and controlled in an otherwise classical circuit marks a profound step forward, and adds to humankind's eternal quest for deciphering the grammar of this universe.

Some schematic illustrations in this article were created using generative AI tools (ChatGPT, OpenAI).

References

1. Press Release, Nobel Prize in Physics 2025
2. M.H. Devoret, J.M. Martinis, & J. Clarke (1985), Physical Review Letters, 55(18), 1908.
3. M.H. Devoret, J.M. Martinis, & J. Clarke (1987), Physical Review B, 35(10), 4682.
4. J.M. Martinis, M.H. Devoret, & J. Clarke (2020), 16(3), 234−237.
5. A.J. Leggett (1980), 69, 80−100.
6. Johanna L. Miller (2025), Physics Today
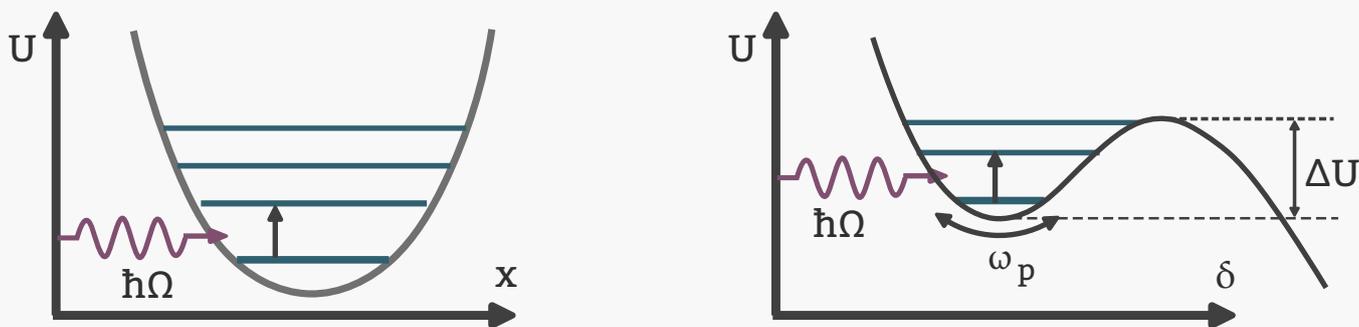
FIG 4 : *Left: Energy levels for a quantum particle in a harmonic potential, and induced transitions. Right (from [4]): The same for a cosine potential in the experiment being discussed (δ is the phase difference of the Josephson junction). Here ω_p is the natural frequency of oscillation of the Josephson junction, and Ω is the frequency of the externally applied microwave, causing the transition.*

Metal–Organic and Covalent Organic Frameworks

My Scientific Journey with

Prof. Omar Yaghi

Metal–Organic Frameworks (MOFs), Covalent Organic Frameworks (COFs), and My Scientific Journey with Professor Omar Yaghi

Rahul Banerjee (*Department of Chemical Sciences, IISER Kolkata*)

December 22, 2025

Prof. Rahul Banerjee of IISER Kolkata weaves the scientific rise of MOFs and COFs with a personal journey shaped by Professor Omar Yaghi, tracing how a single paper sparked a career-long engagement with reticular chemistry. It reflects on mentorship, perseverance, and bold ideas, showing how foundational experiences in MOFs continue to inform cutting-edge research on COFs and next-generation functional materials.

EDITED BY: Suman Halder

Metal–Organic Frameworks (MOFs) are a class of crystalline porous materials constructed from metal ions or metal clusters connected by organic linkers through coordination bonds. This modular "reticular" approach allows precise control over pore size, shape, surface chemistry, and functionality. As a result, MOFs possess exceptionally high surface areas and tunable architectures, making them uniquely suited for applications such as gas storage and separation, carbon capture, catalysis, sensing, drug delivery, and energy-related processes, including hydrogen storage and water harvesting. My introduction to MOFs dates back to 2005, during my PhD studies in structural chemistry under the guidance of Professor Gautam R. Desiraju at the University of Hyderabad. At that time, MOFs were still an emerging concept, unfamiliar primarily to researchers in India, including myself. Quite by accident, I came across a research paper by Professor Omar Yaghi describing these materials.

While I did not initially grasp the full chemical depth of the work, I was immediately drawn to the conceptual elegance of MOFs and the striking visual representations of their crystalline architectures. That single paper had a profound impact on me; it convinced me that I wanted to pursue postdoctoral research in this new class of materials and, more importantly, to work with the

Prof. Banerjee is an organic chemist and a professor at the department of chemical sciences of the Indian Institute of Science Education and Research Kolkata. A fellow of the Royal Society of Chemistry, he is known for his studies in the field of Metal–organic framework designing.

FIG 1 : *With Professor Yaghi at the Nobel Symposium at Alfred Nobel's house in Karlskoga, Sweden.*
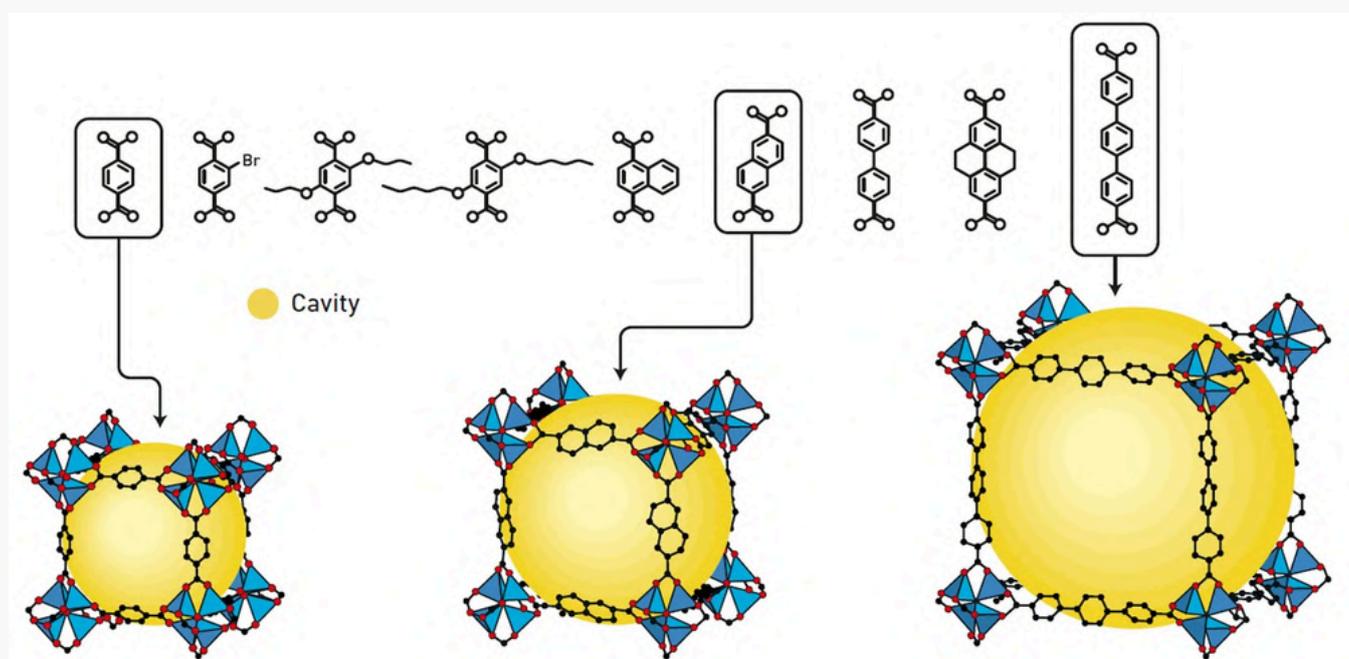
FIG 2 : *A series of Isoreticular MOFs. These MOFs can be modified through rational design, endowing them with new and desirable properties. Photograph: Kind courtesy @NobelPrize/X.*

scientist who had pioneered them. Encouraged by my PhD mentor, I wrote directly to Professor Yaghi. To my surprise, he responded within hours, expressing interest in hosting me as a postdoctoral researcher, subject only to a recommendation letter. Shortly thereafter, I joined his group at the University of California, Los Angeles (UCLA) in August 2006. This marked not only my first time outside India, but also the beginning of an intense and formative phase of my scientific career.

At that time, Professor Yaghi was already recognized as a visionary scientist; however, MOFs themselves were far from being universally accepted. Many in the broader materials and chemistry community were skeptical, questioning the stability, relevance, and utility of these new porous frameworks. In contrast, Professor Yaghi had absolute clarity of purpose. His philosophy was uncompromising: research must be fundamentally original, conceptually transformative, and driven by long-term vision rather than short-term validation. He often said that a good paper should make readers feel, "I wish I had thought of this." The working culture in his group reflected this philosophy. Long hours, relentless experimentation, repeated failure, and intellectual self-reliance were part of everyday life. Professor Yaghi believed strongly that students must find their own scientific paths through trial and error. His role as a mentor was not to provide ready-made solutions, but to ensure that one did not lose direction. This approach profoundly shaped my own thinking about research and mentorship. My work in the Yaghi group focused on Zeolitic Imidazolate Frameworks (ZIFs), a subclass of MOFs formed by the self-assembly of transition metal ions (such as zinc or cobalt) with imidazolate linkers. ZIFs combine the structural features of zeolites with the chemical tunability of MOFs, offering exceptional thermal and chemical stability. This research culminated in a publication that became a defining milestone in my career, laying the foundation for my long-term engagement with porous crystalline materials.

Over the years, MOFs have evolved from a niche curiosity to one of the most active and influential areas of modern chemistry. Their impact now extends across chemistry, materials science, physics, and engineering. When Professor Yaghi was awarded the Nobel Prize in Chemistry in 2025 for his pioneering contributions to reticular chemistry and framework materials, it felt like a natural and inevitable recognition of a vision he had articulated decades earlier, long before it had gained widespread acceptance. Building on my training in MOFs, my current research focus is on Covalent Organic Frameworks (COFs), which represent a complementary and equally exciting class of porous crystalline materials. Unlike MOFs, COFs are constructed entirely from light elements (such as carbon, nitrogen, oxygen, and boron) connected through strong covalent bonds. This results in fully organic, metal-free frameworks with long-range order, permanent porosity, and exceptional structural precision.

COFs offer several advantages: low density, high thermal stability, tunable electronic properties, and designable π-conjugated backbones. These features make them particularly attractive for applications in photocatalysis, electrocatalysis, energy storage, optoelectronics, molecular separations, and heterogeneous catalysis. My current work focuses on designing robust COFs with tailored pore environments and functional architectures, extending the principles of reticular chemistry into purely covalent systems. In many ways, COFs represent a natural evolution of the ideas first crystallized in MOF chemistry, retaining the emphasis on structural predictability and modular design while opening up new possibilities in electronic and photophysical functionality. The conceptual foundation laid during my time with Professor Yaghi continues to guide this work.

In 2023, I had the privilege of participating as the only scientist from India in a Nobel Symposium in Sweden, where Professor Yaghi was also present. During an informal conversation, I reminded him that I was the first

FIG 3 : *At the Nobel Symposium at Björkborn Manor in Karlskoga. Björkborn Manor is now a museum where visitors can go back in time and imagine what it was like when Alfred Nobel spent his summers here.*

student to complete a PhD or postdoc?PhD entirely in India under his supervision. His response "the first and the best" was delivered with characteristic warmth and generosity.

Even today, his intellectual energy and passion for science remain undiminished. My interaction with Professor Omar Yaghi has not only shaped my scientific trajectory but also influenced my philosophy as a researcher and

mentor. The belief in bold ideas, perseverance in the face of skepticism, and commitment to intellectually honest, hard work continue to define my approach to science, which is now carried forward through my work on COFs and next-generation functional materials.

The Peacekeepers Within

A small group of calm "guardian" act as peacekeepers to prevent the immune system from turning against itself and maintain harmony among cells and organs.

The Peacekeepers Within

Pallavi Gupta,
Kushagra Bansal   *(Molecular Biology and Genetics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India)*

October 12, 2025

Regulatory T cells (Tregs) are a unique subset of CD4$^+$ T cells that play a crucial role in suppressing harmful autoreactive T cells in the body and maintaining peripheral immune tolerance. The 2025 Nobel Prize in Physiology or Medicine was awarded to Shimon Sakaguchi, Mary E. Brunkow, and Fred Ramsdell for their discoveries elucidating the cellular and genetic basis of regulatory T cells. The article **'The peacekeepers within'** delves deeper into the story behind their fantastic discovery that redefined immunology.

EDITED BY: Puppala Ainesh Kumar, Ishita Barddhan

Pallavi Gupta is a fourth-year Integrated PhD student at the Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR), Bangalore, where she is pursuing her doctoral research under the guidance of Prof. Kushagra Bansal. She has a strong interest in immunology, and her research focuses on understanding the regulatory role of Topoisomerase 1 in immune cell development.
Dr. Kushagra Bansal is an Associate Professor at Molecular Biology and Genetics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR), Bangalore. His research group focuses on the transcriptional regulation of immune cell development and function using murine models. He is a recipient of DBT-Wellcome Trust India Alliance Intermediate Fellowship.

Introduction: defense and restraint in the immune system

Multicellular organisms exist in constant exposure to a wide array of microbial threats. To counter these threats, vertebrates have evolved an immune system which is capable of pathogen detection and elimination by employing both innate and adaptive lines of defense. Innate immunity is the first line of defense to fight any pathogenic invasion immediately while adaptive immunity is the second line of defense that is slower and is characterized by 'memory' for faster future encounters. At the core of adaptive immunity are T lymphocytes that recognize antigens through T cell receptors (TCRs). These receptors exhibit an extraordinary diversity as they are generated via a gene rearrangement mechanism commonly known as somatic recombination. TCRs thus have the capability to recognize diverse set of antigenic peptides processed and presented by antigen presenting cells (APCs)[1]. However, the same random TCR rearrangement that allows T cells to recognize a wide range of antigens also creates an inherent risk: some TCRs can recognize self-antigens, potentially leading to autoimmunity - a condition where T cells start to attack our own tissues. However, under normal conditions, these autoreactive T cells are kept under check, and they are not allowed to cause damage to the self tissues - a process commonly known as immune tolerance. This raises an important question: what molecular mechanisms keep self-reactive T cells in check in the periphery? Are specific cell types responsible? Are soluble factors involved? Or is it achieved through a combination of cellular and molecular forces?

It took decades to find out the answer to this mystery. Through a series of experiments and combined efforts of three scientists– Shimon Sakaguchi, Mary E. Brunkow, and Fred Ramsdell– another regulatory layer to T cell function known as peripheral immune tolerance was uncovered. The 2025 Nobel Prize in Physiology or Medicine was awarded to Shimon Sakaguchi, Mary E. Brunkow, and Fred Ramsdell, who unwrapped the cellular and genetic architecture of 'peripheral immune tolerance'. Their discoveries reshaped immunology, providing a conceptual framework for understanding autoimmunity, allergy, transplantation, and cancer. The following sections trace the scientific journey that led to these transformative insights.

T cell development and tolerance mechanisms

T cells originate from common lymphoid progenitors (CLPs) in the bone marrow, but they complete their maturation in the thymus. Thymus is a small, bilobed organ located above the heart and contains two major compartments– the cortex and the medulla– whose organization is essential for proper T cell maturation
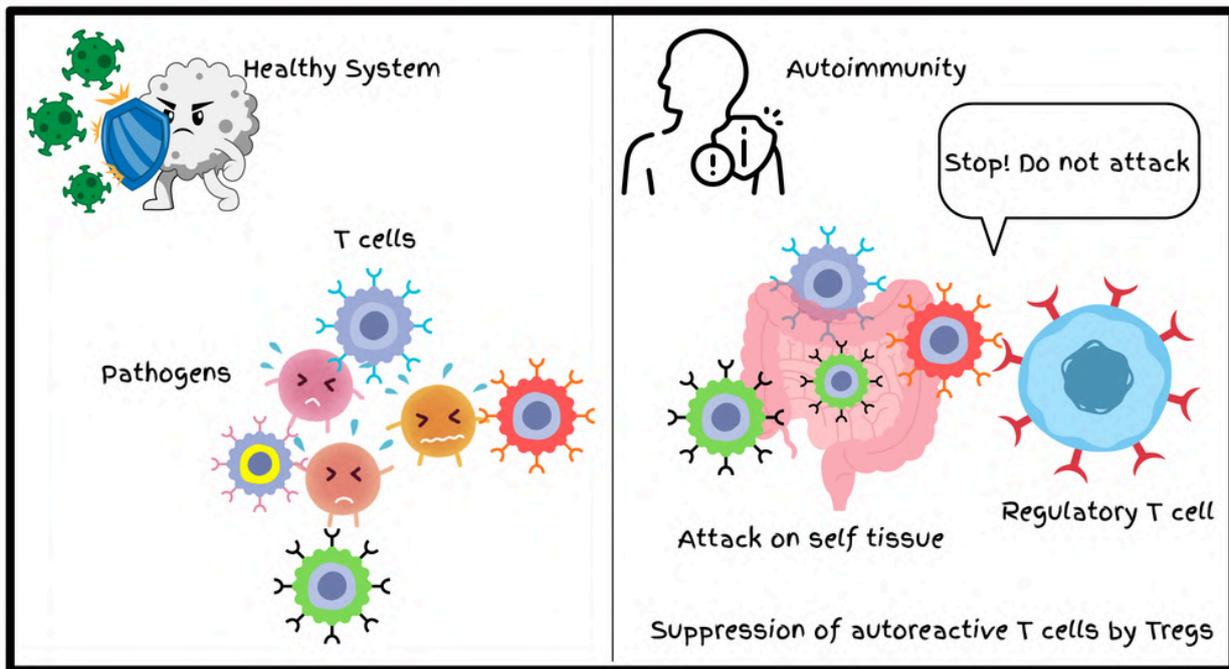
FIG 1 : ***T cells in immunity:*** *The schematic illustrates the dual nature of T cells. Left panel: In healthy system, T cells recognize the pathogens and eliminate infected cells. Right panel: In the absence of proper regulation, self-reactive T cells misidentify self-tissue antigens as foreign, initiating destructive inflammatory responses that are the characteristic of autoimmunity. Harmful autoreactive T cells are suppressed via peripheral immune tolerance.*

and TCR diversification. Once T cell progenitors enter the thymic environment, they progress through a series of orderly developmental stages that are defined largely by the expression patterns of surface proteins CD4 and CD8.

Early in their development, T cell precursors first lose the potential to differentiate into other lineages, such as B cells. They then enter the double-negative (DN) stage, characterized by the absence of both CD4 and CD8. As they mature, they transition into the double-positive (DP) stage, expressing both CD4 and CD8 simultaneously. Ultimately, they differentiate into single-positive (SP) thymocytes, expressing either CD4 or CD8, which determines their functional fate in the periphery[2].

During their developmental transitions, T cells have to cross 2 major checkpoints. First one is positive selection which ensures that only those T cells which are self-major histocompatibility complex (MHC) restricted survive; while second one is negative selection which facilitates the elimination of T cells that react too strongly to the molecules present on the body's own cells, commonly known as self-antigens[3]. This process helps in preventing autoreactive clones from entering the periphery and is also known as 'central immune tolerance'.

Once T cells maturation is complete, they exit the thymus and migrate to secondary lymphoid organs such as the spleen and lymph nodes. These are the sites where they encounter APCs, recognize antigenic peptides, and become activated.

Despite the stringent selection events in the thymus, some autoreactive T cells may escape central tolerance and enter the periphery. What happens then? How does the body prevent these cells from attacking self-tissues?

This is where peripheral tolerance comes into play. Peripheral tolerance ensures that autoreactive T cells that reach the peripheral tissues are either eliminated or functionally restrained so that they do not cause self-damage. Peripheral tolerance is accomplished through several mechanisms: promoting the activity of regulatory T cells, inducing anergy (a state in which T cells become unresponsive), or triggering clonal deletion, which removes these harmful T cells altogether.

But where do these regulatory T cells come from? Do they also develop in the thymus? Yes, a fraction of regulatory T cells (commonly known as Tregs) also develop in the thymus. During thymic selection, when the TCR on developing CD4[+] T cells engages with the cognate receptor (MHC-peptide complex) present on an APC, it triggers downstream TCR signalling. Surprisingly, the strength of this TCR signalling determines the fate of the CD4[+] T cell to either become a conventional CD4[+] T cell or a regulatory T cell (Treg). Low-affinity TCR signals favor development into conventional CD4[+] T cells, while intermediate-strength signals promote commitment to the Treg lineage. However, if a thymocyte receives very strong TCR signals, it undergoes negative selection and is deleted from the developing repertoire.

Controversial suppressor T cell hypothesis

For a long time, the thymus was seen as the sole guardian against autoimmunity. The occurrence of autoimmune diseases was largely interpreted as failures of thymic selection. However, the gradual accumulation of experimental evidence across the globe challenged this view. For example, people observed that in some animals despite having intact thymic architecture, they are prone to autoimmune symptoms.
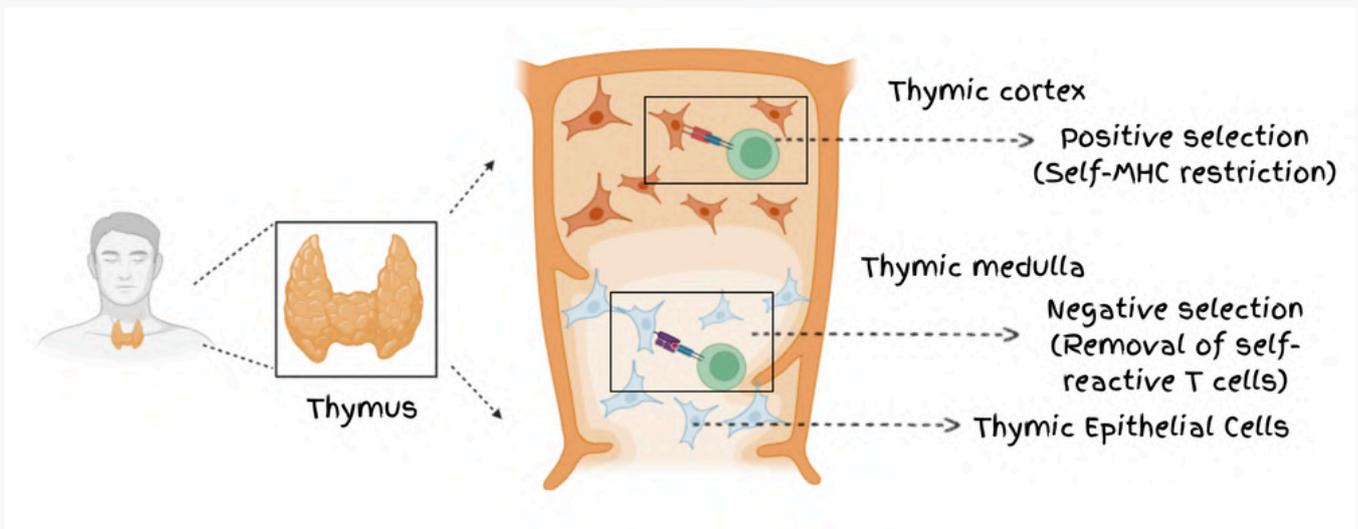
FIG 2 : ***Training of T cells in the thymus:*** *Thymic T cell development involves two major selection processes-positive and negative selection. Positive selection occurs in the thymic cortex and ensures that developing thymocytes are self-MHC restricted, while the negative selection occurs primarily in the thymic medulla and ensures the removal of self-reactive T cells.*

Not only this, reports from the 1970s hinted at a different layer of regulation which was the "suppressor T cells"[4]. These studies suggested that certain T cells could actively restrain immune responses, but reproducibility was poor, and no reliable molecular markers existed to prove their presence. As a result, the concept remained scientifically marginalized, and the observations were considered as misinterpreted data.

Amid this skepticism, Shimon Sakaguchi showed his determination and pursued the unconventional idea that a subset of peripheral T cells might actively suppress autoreactive clones. Building on the hints from earlier studies, he hypothesized that these cells could provide a critical layer of peripheral tolerance, setting the stage for experiments that would ultimately redefine our understanding of immune regulation.

The neonatal thymectomy paradox

According to the prevailing thoughts at that time, scientists predicted that thymectomy would lead to immunodeficiency, as the thymus was thought to be the sole source of functional T cells. Yet the results defied expectation. To their surprise, they observed that removal of thymus (thymectomy) in newborn mice at day 3 after birth did not lead to a weakened immune system. Instead, the mice developed severe multi-organ autoimmune disease, including thyroiditis[5,6], gastritis[7], and oophoritis[8,9]. This was very surprising! It suggested that thymus was not only responsible for the production of effector T cells but also a regulatory T cell population that restrains self-reactive T cells in the periphery.

So, to further test this hypothesis, Sakaguchi transferred a specific fraction of peripheral T cells from genetically identical, healthy mice into thymectomized mice recipients[10]. Remarkably, the autoimmune symptoms were reversed, providing the first functional evidence that peripheral T cells actively enforce self-tolerance.

This discovery shifted the conventional paradigm. The immune tolerance was not solely established through negative selection in the thymus but required active immune suppression in the periphery. It laid the foundation for identifying the T cell subset responsible for this critical regulatory role - the regulatory T cells (Tregs).

Sakaguchi: Discovery of CD25⁺ regulatory T cells

Motivated by the observations from thymectomy experiments, Sakaguchi desired to identify the cellular population responsible for maintaining peripheral tolerance. He focused on a rare subset of $CD4^+$ T cells expressing CD25 (a receptor present on $CD4^+$ T cells that binds to IL-2 - a critical growth factor for T cells), which had previously been overlooked and largely dismissed as merely activated effector cells. But how do we figure out the exact role of these CD25 expressing T cells? Sakaguchi took a clever approach. To test the function of this T cell subset, he selectively depleted $CD25^+$ T cells from healthy mice and transferred these cells in athymic 'nude' mice that are immunodeficient. The results were striking. These mice developed multi-organ autoimmunity, including tissues such as thyroid, pancreas, and stomach being attacked[11]. Interestingly, transfer of $CD4^+CD25^+$ T cells within a limited period after the transfer of $CD4^+CD25^-$ T cells prevented autoimmune responses. This provided definitive evidence that $CD25^+$ T cells were not ordinary effectors but instead a regulatory population critical for restraining self-reactive T cell clones. However, the story does not end here. The problem of not having a proper genetic marker for regulatory T cells was still there and was resolved only partially, as CD25 is expressed on activated effector T cells, though at a lower level. Thus, it limited its specificity and was not a perfect marker for the identification of this newly emerging regulatory T cells[11].

The field thus required a precise molecular identifier to define Tregs uniquely, setting the stage for the next critical discovery in the story of immune regulation.

Brunkow and Ramsdell: Solving the scurfy mouse mystery

While Sakaguchi was defining the cellular basis of immune regulation, Mary E. Brunkow and Fred Ramsdell were investigating a parallel line of puzzle of a peculiar mouse strain known as the scurfy mouse. These mice exhibited unique features like scaly skin, enlarged lymphoid organs, and severe multi-organ inflammation, with a pattern of male-specific lethality suggesting an X-linked mutation. The animals died early due to overwhelming autoimmunity, but the precise gene responsible remained unknown.

Brunkow and Ramsdell accepted the challenge of understanding the genetics behind scurfy phenotype. As limited molecular tools were available that time, they utilized the positional cloning techniques, genetic mapping, candidate gene sequencing and combinatorial breeding strategies to address this question. After years of meticulous work, they identified an insertion of two-nucleotides in a previously uncharacterized gene. It is worth noting that Brunkow and Ramsdell had to painstakingly screen multiple genes before reaching to the gene responsible for scurfy phenotype. This marks their dedication and efforts which they had put up into. Biologically, this mutation caused a frameshift and premature stop codon and thus abolishing the gene's function. Based on sequence homology with other forkhead box genes, they named it *FOXP3* (Forkhead box P3)[12].

The significance of this discovery became even more apparent when Brunkow and Ramsdell observed striking similarities between phenotype of scurfy mouse and human IPEX syndrome (Immune dysregulation, Polyendocrinopathy, Enteropathy, X-linked) and they started to suspect that IPEX is the human equivalent of scurfy mouse. Children with IPEX suffered from early-onset chronic diarrhoea, autoimmune enteropathy, and severe inflammatory skin rashes, often leading to death in infancy. Genetic analysis revealed mutations in *FOXP3*, paralleling the scurfy mouse defect[13,14].

This convergence of mouse genetics and human pathology established *FOXP3* as essential for immune tolerance. It further set the stage for linking two previously independent lines of investigation- Sakaguchi's cellular discoveries and Brunkow/Ramsdell's search for a definitive genetic regulator of autoimmune disease in mice and humans.

FOXP3: The master regulator of Tregs

For years, Sakaguchi's identification of CD25[+] regulatory T cells and Brunkow and Ramsdell's discovery of *FOXP3* had proceeded along parallel tracks. The unifying breakthrough came when Sakaguchi demonstrated that *Foxp3* is selectively expressed in CD4[+]CD25[+] Tregs[15]. Introducing *Foxp3* into conventional CD4[+]T cells further led to the conversion of conventional CD4[+] T cells into cells with suppressive function[15]. Not only this, the scurfy mice having mutations in *Foxp3* were found to be completely devoid of functional Tregs, whereas mice that overexpressed *Foxp3* carried excessive Treg cells[16].

This finding established FOXP3 as the master transcription factor defining the Treg lineage. What had once been a controversial concept of suppressor T cells

was now a well-defined biological entity. The concept of Tregs explained many long-standing questions including how the immune system maintains tolerance even after thymic selection.

Mechanisms of Treg-mediated immune suppression

The identification of cellular and molecular basis of peripheral immune tolerance was a key discovery, but then at functional level, how exactly Tregs provide protection against self-reactivity? The discovery of FOXP3 as the master regulator sparked a wave of curiosity about the biology of Tregs. With this molecular marker, researchers could isolate Tregs and investigate how these cells actively maintain immune tolerance. Tregs were found to employ multiple, complementary mechanisms to suppress immune activation.

Tregs secrete anti-inflammatory cytokines such as IL-10, TGF-β, and IL-35, which dampen immune responses in local tissues. They consume IL-2, a critical T cell growth factor, thereby limiting the proliferation of autoreactive effector T cells. Through CTLA-4 expression, Tregs modulate antigen-presenting cells, reducing their ability to activate effector T cells. Together, these mechanisms demonstrate that immune tolerance is actively maintained and is context-dependent, rather than a passive absence of immune response[17].

Clinical implications of regulatory T cells

The discovery of Tregs and *FOXP3* reshaped not only fundamental immunology but also the understanding of human diseases. Dysregulation of Tregs underlies a wide spectrum of pathological conditions, highlighting their central role in immune homeostasis.

Autoimmune diseases

Defective Tregs are now recognized as key contributors to disorders such as multiple sclerosis[18], rheumatoid arthritis[19], psoriasis[20], type 1 diabetes[21] and others. In these conditions, the failure of immune regulation allows autoreactive effector T cells to attack self-tissues, leading to chronic inflammation and progressive organ damage. The mechanistic insights into Treg biology have opened avenues for therapeutic intervention, including low-dose IL-2 therapy[22] to expand Treg population and engineered Treg infusions designed to restore tolerance[23].

Cancer

On the other hand, regulatory T cells play an opposite role in tumors by suppressing the anti-tumor immune responses. Cancer uses Tregs to create an immune suppressive shield and this helps tumor to grow and evade faster. Scientists are now trying to specifically target tumor-associated Tregs in order to alleviate their suppression of anti-tumor immune responses and to promote clearance of tumor[24].

Transplantation

One of the major challenges in organ transplantation is graft rejection due to alloreactive immune responses.
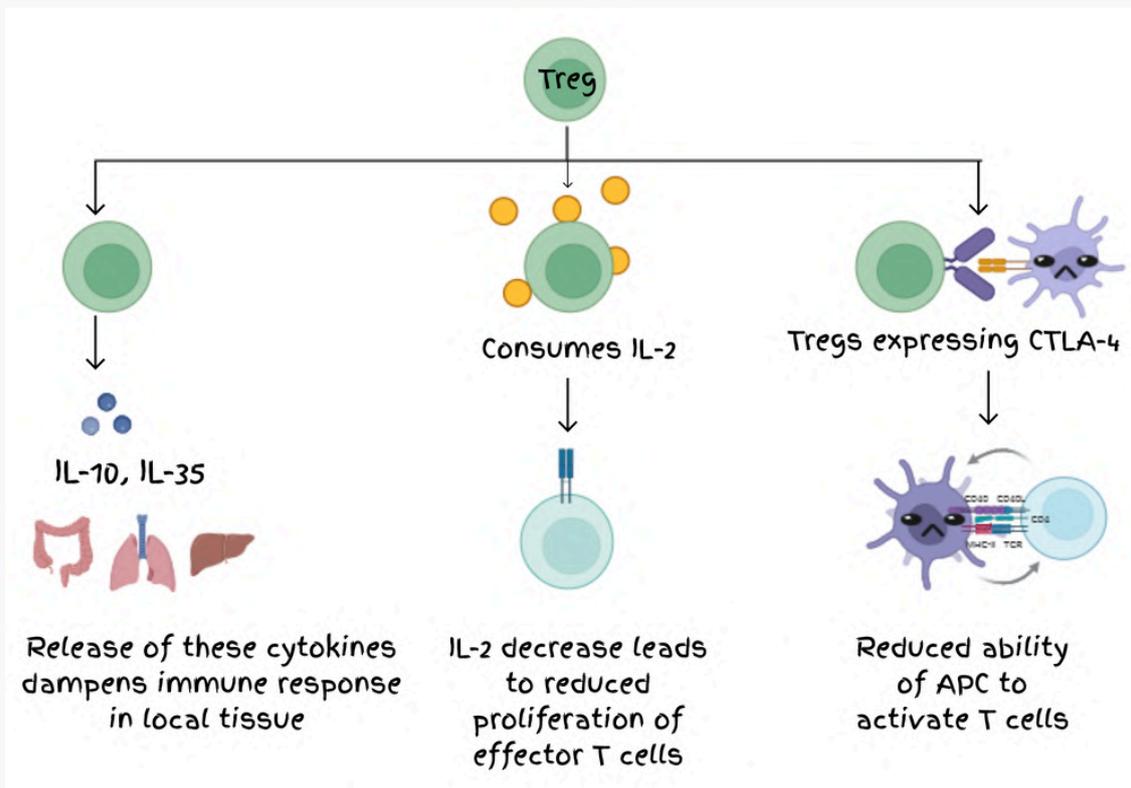
FIG 3 : ***Mechanisms of regulatory T cells:*** *Regulatory T cells carry out their suppressive functions through multiple complimentary mechanisms. These include the secretion of anti-inflammatory cytokines, the consumption of IL-2 which is vital for proliferation of effector T cells and the expression of CTLA-4 which competes and binds to co-stimulatory molecules on antigen-presenting cells (APCs). This leads to reduced ability of APCs to activate naïve T cells.*

FIG 4 : ***Nobel Prize in Physiology or Medicine 2025:*** *Mary E. Brunkow [Institute for Systems Biology, Seattle], Fred Ramsdell [Sonoma Biotherapeutics, San Francisco] and Shimon Sakaguchi [Osaka University, Osaka].*

Tregs can suppress these reactions and thus can promote long–term graft tolerance[25].

Taken together, by defining the molecular and cellular mechanisms governing Treg function, Sakaguchi, Brunkow, and Ramsdell provided a foundation for interventions across diverse medical fields.

Conclusion

The story of the 2025 Nobel Prize is not only about cells and genes, but it is also a testimony of scientific courage, continuous efforts, and curiosity. Sakaguchi pursued a hypothesis that many had long dismissed. Meanwhile,

Brunkow and Ramsdell devoted years to unravelling the genetic mystery of the scurfy mouse. Positional cloning at the time was a formidable endeavor, yet they pursued it despite the odds and technical limitations.

Their work ultimately bridged mouse and human immunology and illuminated the central role of *FOXP3* in peripheral immune tolerance. Their discovery has opened the door to innovative therapies ranging from Treg–based interventions in autoimmune diseases, allergy, and transplantation, to strategies that modulate Treg function in cancer immunotherapy. The immune system's peacekeepers "Tregs" stand as a testament

to nature's intricate design and the enduring power of scientific inquiry.

References

1. Charles A Janeway, J., Travers, P., Walport, M. & Shlomchik, M. J. Antigen recognition by T cells. in Immunobiology: The Immune System in Health and Disease. 5th edition (Garland Science, 2001).

2. Charles A Janeway, J., Travers, P., Walport, M. & Shlomchik, M. J. Generation of lymphocytes in bone marrow and thymus. in Immunobiology: The Immune System in Health and Disease. 5th edition (Garland Science, 2001).

3. Ashby, K. M. & Hogquist, K. A. A guide to thymic selection of T cells. Nat. Rev. Immunol. 24, 103–117 (2024).

4. Gershon, R. K. & Kondo, K. Cell interactions in the induction of tolerance: the role of thymic lymphocytes. Immunology 18, 723–737 (1970).

5. Kojima, A., Tanaka-Kojima, Y., Sakakura, T. & Nishizuka, Y. Spontaneous development of autoimmune thyroiditis in neonatally thymectomized mice. Lab. Investig. J. Tech. Methods Pathol. 34, 550–557 (1976).

6. Penhale, W. J., Farmer, A., McKenna, R. P. & Irvine, W. J. Spontaneous thyroiditis in thymectomized and irradiated Wistar rats. Clin. Exp. Immunol. 15, 225–236 (1973).

7. Tung, K. S., Smith, S., Teuscher, C., Cook, C. & Anderson, R. E. Murine autoimmune oophoritis, epididymoorchitis, and gastritis induced by day 3 thymectomy. Immunopathology. Am. J. Pathol. 126, 293–302 (1987).

8. Taguchi, O., Nishizuka, Y., Sakakura, T. & Kojima, A. Autoimmune oophoritis in thymectomized mice: detection of circulating antibodies against oocytes. Clin. Exp. Immunol. 40, 540–553 (1980).

9. Nishizuka, Y. & Sakakura, T. Thymus and reproduction: sex-linked dysgenesia of the gonad after neonatal thymectomy in mice. Science 166, 753–755 (1969).

10. Sakaguchi, S., Takahashi, T. & Nishizuka, Y. Study on cellular events in post-thymectomy autoimmune oophoritis in mice. II. Requirement of Lyt-1 cells in normal female mice for the prevention of oophoritis. J. Exp. Med. 156, 1577–1586 (1982).

11. Sakaguchi, S., Sakaguchi, N., Asano, M., Itoh, M. & Toda, M. Immunologic self-tolerance maintained by activated T cells expressing IL-2 receptor alpha-chains (CD25). Breakdown of a single mechanism of self-tolerance causes various autoimmune diseases. J. Immunol. Baltim. Md 1950 155, 1151–1164 (1995).

12. Brunkow, M. E. et al. Disruption of a new forkhead/winged-helix protein, scurfin, results in the fatal lymphoproliferative disorder of the scurfy mouse. Nat. Genet. 27, 68–73 (2001).

13. Bennett, C. L. et al. The immune dysregulation, polyendocrinopathy, enteropathy, X-linked syndrome (IPEX) is caused by mutations of FOXP3. Nat. Genet. 27, 20–21 (2001).

14. Wildin, R. S. et al. X-linked neonatal diabetes mellitus, enteropathy and endocrinopathy syndrome is the human equivalent of mouse scurfy. Nat. Genet. 27, 18–20 (2001).

15. Hori, S., Nomura, T. & Sakaguchi, S. Control of regulatory T cell development by the transcription factor Foxp3. Science 299, 1057–1061 (2003).

16. Khattri, R., Cox, T., Yasayko, S.-A. & Ramsdell, F. An essential role for Scurfin in CD4+CD25+ T regulatory cells. Nat. Immunol. 4, 337–342 (2003).

17. Vignali, D. A. A., Collison, L. W. & Workman, C. J. How regulatory T cells work. Nat. Rev. Immunol. 8, 523–532 (2008).

18. Viglietta, V., Baecher-Allan, C., Weiner, H. L. & Hafler, D. A. Loss of functional suppression by CD4+CD25+ regulatory T cells in patients with multiple sclerosis. J. Exp. Med. 199, 971–979 (2004).

19. Ehrenstein, M. R. et al. Compromised Function of Regulatory T Cells in Rheumatoid Arthritis and Reversal by Anti-TNFα Therapy. J. Exp. Med. 200, 277–285 (2004).

20. Nussbaum, L., Chen, Y. L. & Ogg, G. S. Role of regulatory T cells in psoriasis pathogenesis and treatment. Br. J. Dermatol. 184, 14–24 (2021).

21. Lindley, S. et al. Defective Suppressor Function in CD4+CD25+ T-Cells From Patients With Type 1 Diabetes. Diabetes 54, 92–99 (2005).

22. Klatzmann, D. & Abbas, A. K. The promise of low-dose interleukin-2 therapy for autoimmune and inflammatory diseases. Nat. Rev. Immunol. 15, 283–294 (2015).

23. Bender, C. et al. A phase 2 randomized trial with autologous polyclonal expanded regulatory T cells in children with new-onset type 1 diabetes. Sci. Transl. Med. 16, eadn2404 (2024).

24. Plitas, G. & Rudensky, A. Y. Regulatory T Cells in Cancer. Annu. Rev. Cancer Biol. 4, 459–477 (2020).

25. Walsh, P. T., Taylor, D. K. & Turka, L. A. Tregs and transplantation tolerance. J. Clin. Invest. 114, 1398–1403 (2004).
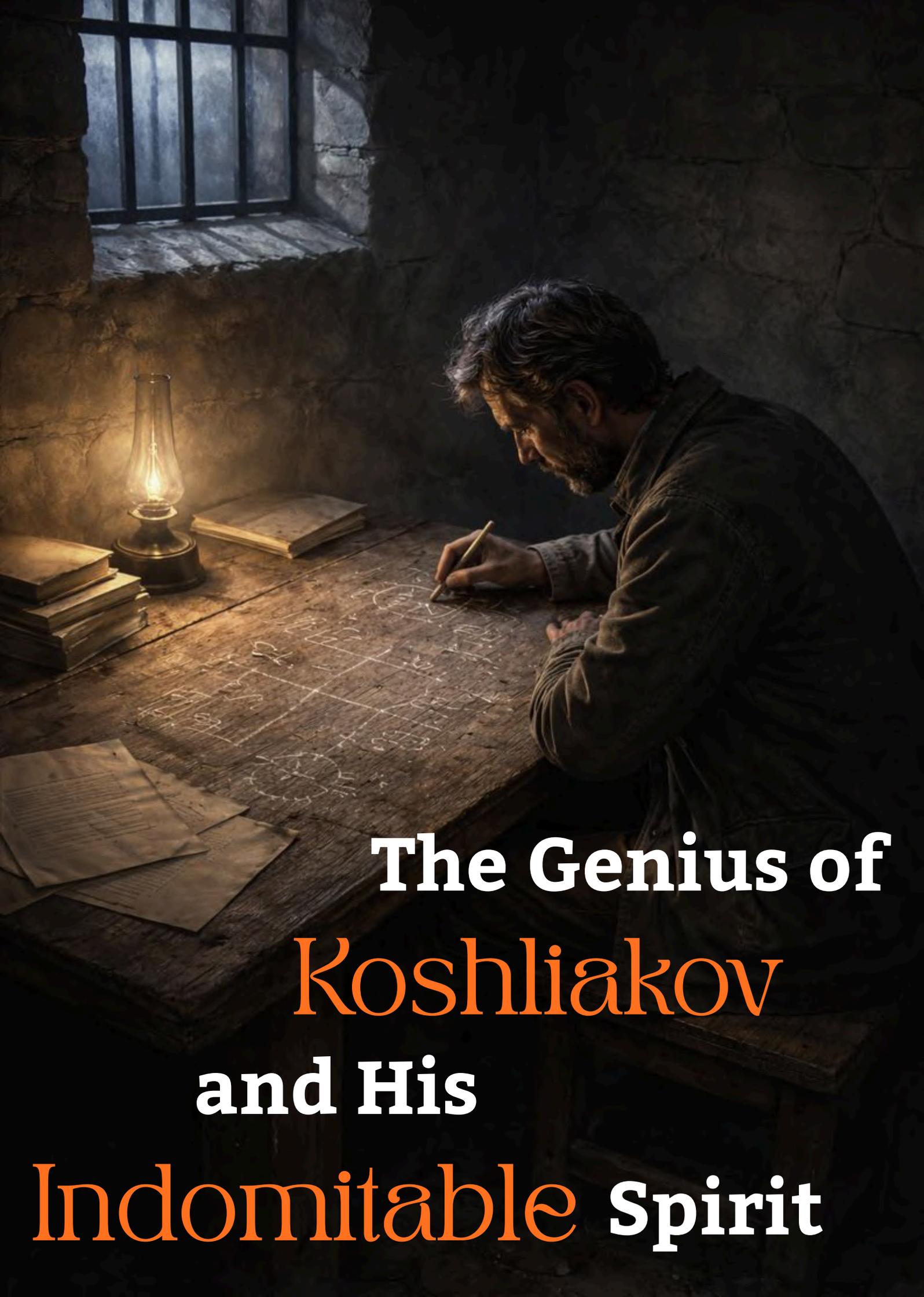
The Genius of
Koshliakov
and His
Indomitable Spirit

The Genius of Koshliakov and His Indomitable Spirit

Atul Dixit (*Department of Mathematics, Indian Institute of Technology Gandhinagar*)

December 22, 2025

From a Soviet labor camp, Nikolai Koshliakov developed a striking generalization of the Riemann zeta function under conditions that defy imagination. This article traces the origins of his work on transcendental functions arising from a generalized Riemann equation. Along the way, it reveals how profound mathematics can emerge even in the most hostile environments.

In memory of Nikolai Sergeevich Koshliakov

Introduction

The humankind has regularly witnessed the birth of influential personalities who have time and again guided people through difficult situations and have raised their spirits. Adversities or ordeals could not crush them, and instead, they emerged victorious through their heroic deeds.

One such role model that we will see in this article is the Russian mathematician Nikolai Sergeevich Koshliakov[1]. Koshliakov was born on 11th July 1891 in St. Petersburg, Russia. After finishing his gymnasium (high school and college), Koshliakov entered the Department of Physics and Mathematics at St. Petersburg University, graduating in 1914 with a diploma securing a first class. Before being accepted at St. Petersburg, Koshliakov had mastered differential and integral calculus on his own [1] (see also [10]). At the university, he was taught by excellent mathematicians such as Markov, Steklov, Uspenskii etc. He became interested in analytic number theory, heavily influenced by the work of G. F. Voronoï. After graduation, he remained at St. Petersburg for his Masters. Throughout his life, Koshliakov held several eminent positions at various universities. He was an excellent teacher, his lectures being precise and comprehensible. His textbook on differential equations [6] became greatly popular among students, the outgrowth of which is [9].

Koshliakov was a researcher par excellence who mainly worked in analytic number theory and differential equations. He published more than 65 papers ranging from summation formulas to problems in applied mathematics. His slick proof of the Voronoï summation formula [4] is, according to the present author, the shortest proof known so far and full of ingenuity!

The subject of this note is concerned with one manuscript written in Russian by Koshliakov [7], whose title translates to 'A study of a class of transcendental functions defined by the generalized Riemann equation'. But before we delve into it, it is important to understand the political situation in Soviet Union and the conditions in which Koshliakov wrote it.

Political situation in the Soviet Union during World War II, Koshliakov's arrest, and mathematics from jail

NKVD was the Soviet Union's secret police organization from 1934 to 1946. During the World War II, Axis powers undertook the "Siege of Leningrad", a military blockade, from 1941 to 1944. In [1], the authors write
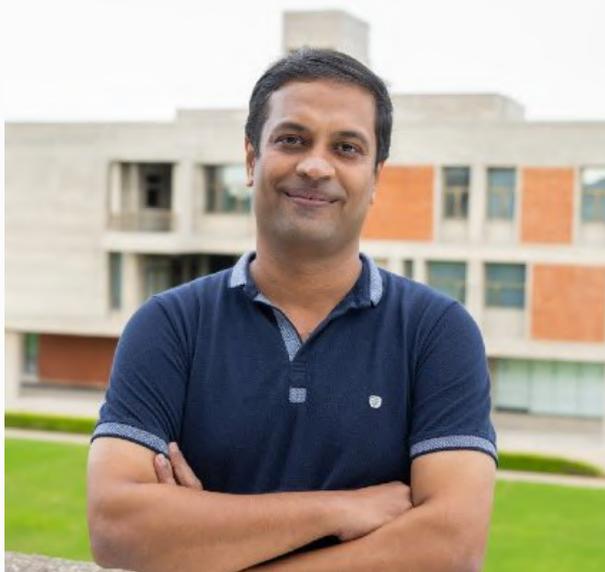
Atul Dixit is a Professor of Mathematics at IIT Gandhinagar and holds the N Rama Rao Chair position at the institute. Atul's research interests are in Analytic Number Theory, Special Functions, Theory of Partitions, q-series and Modular Forms. He also likes to work in the areas of mathematics developed by Srinivasa Ramanujan.

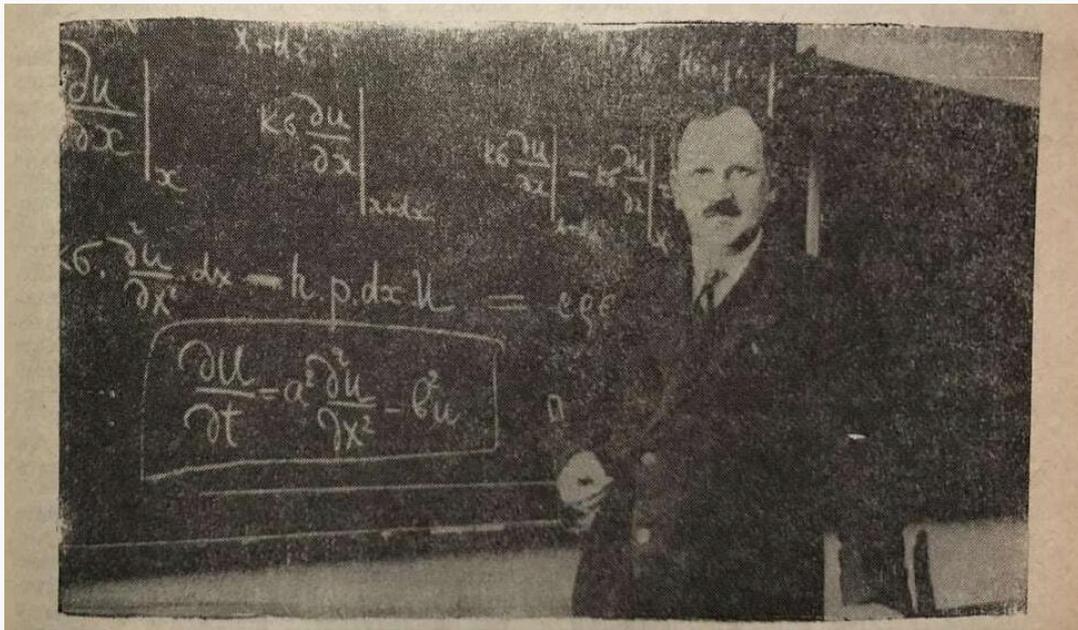[1]His last name is spelled in several different ways, the second most common being *Koshlyakov*.

FIG 1 : *Nikolai Sergeevich Koshliakov (Picture courtesy: V. N. Koshlyakov, O. A. Ladyzhenskaya and D. R. Merkin and M. M. Smirnov, Vestnik Leningrad Univ. Mat. Mekh. Astronom. (1991) No. 4, 81–83.)*

The repressions of the thirties which affected scholars in Leningrad continued even after the outbreak of the Second World War. In the winter of 1942 at the height of the blockade of Leningrad, Koshlyakov along with a group ... was arrested on fabricated ... dossiers and condemned to 10 years of correctional hard labour. After the verdict he was exiled to one of the camps in the Urals. ... On the grounds of complete exhaustion and complicated pellagra, Koshlyakov was classified in the camp as an invalid and was not sent to do any of the usual jobs. ... very serious shortage of paper. He was forced to carry out calculations on a piece of plywood, periodically scraping off what he had written with a piece of glass. Nevertheless, between 1943 and 1944 Koshlyakov wrote two long memoirs Issledovanie nekotorykh voprosov analyticheskoi teorii rational'nogo i kvadratichnogo polya *(A study of some questions in the analytic theory of rational and quadratic fields)*[2] *and* Issledovanie odnogo klassa transtsendentnykh funktsii, opredelyaemykh obobshchennym yravneniem Rimana *(A study of a class of transcendental functions defined by the generalized Riemann equation).*

Further details are given in [11, p 211−213], where the author says that the main purpose of the NKVD was to obtain signed "confessions"[3] from the accused as proofs of their guilt (of supporting the Axis powers). Koshliakov wasn't the only one. In fact, he was one among 13 scientists and mathematicians who were considered as the accused. Lorentz [11, p. 212] says, *'By April 1942, after seven hungry months, the accused were suffering from acute starvation; many of them had lost 1/3 of their normal weight and were close to death. It is humanly understandable that an offer of a bowl of soup was sufficient to force them to sign a "confession", an ultimate proof of guilt. Forced to stand for hours was another form of torture. No wonder that Koshlyakov signed:"I intended to establish relations with the German commandant... I would like to atone for my guilt, be it in a small measure, by participating in our working front.... In particular, would be very happy to complete my work on summation formulas which I have been conducting for 30 years....'.*

It is not our intention to delve a lot into politics but rather to put forth before everyone (especially students), how, in the extreme predicament that Koshliakov was going through, he could maintain his scientific fervour and produce magnificent research!

My "discovery" of Koshliakov's second manuscript written from jail: A personal story

The second manuscript which Koshliakov wrote from jail under his patronymic name N. S. Sergeev, namely, [7], although published, can still be considered as "lost" since nobody (except for a passing reference to it in the thesis of A. G. Kisunko [3, p. 4]) studied its contents for 70 years! I had the good fortune of "discovering" it in 2010 while I was a fourth year PhD student in Mathematics at the University of Illinois at Urbana-Champaign. Since this episode might be interesting to some of you, let me take some liberty to explain how it unfolded. I happened to come across the article [1] written on the centenary of Koshliakov's birth, where I first learned of his second manuscript [7]. The authors of [1] had discussed in it some of the contents of [7]. I was taken aback to see an integral involving the Riemann Ξ-function considered there which generalized an integral considered by Ramanujan in his published paper [12].

I was happy and anxious, at the same time, to see the manuscript [7], and decided to get hold of it, and also everything else that Koshliakov had published. This is because I, myself, was working on a generalization of

---

[2]It is believed that this manuscript was lost in transit from jail to Steklov Institute of Mathematics, however, Koshliakov resurrected it in the form of three papers[8] in 1954.

[3]This word in quotes, of course, means *fabricated confessions*.

FIG 2 : *Koshliakov graduated from the Faculty of Physics and Mathematics of the University of St. Petersburg in 1914.*

that very same integral of Ramanujan, and so I wanted to make sure that I am not replicating what Koshliakov did years ago! The manuscript was available at the Center for Research Libraries (CRL) in Chicago. I asked the UIUC Math librarian Tim Cole if we could get it for a few days via inter-library loan. I was pleasantly surprise to find that the CRL had scanned the 150-page manuscript and had emailed it to me! To my solace, Koshliakov had not done anything I was working on. I found the manuscript to be very interesting though.

However, it wasn't until ten more years that I decided to seriously study this manuscript along with my PhD student Rajat Gupta. It was then that we found out what a gem this manuscript is! We made two modest contributions to the theory [2].

Contents of the manuscript

To describe the main idea, let us first define the Riemann zeta function, one of the most important functions of mathematics. For Re(s) > 1, it is defined by the absolutely and uniformly convergent series $\zeta(s) := \sum_{n=1}^{\infty} n^{-s}$. It can be analytically continued to the entire complex plane except for a simple pole at $s = 1$. Koshliakov's theory stems from a problem in Physics on heat conduction. We refer the reader to [5] and [6, p. 488-489] for a description of this problem.

The equation $p \sin(\pi\lambda) + \lambda \cos(\pi\lambda) = 0$, where $p > 0$, forms the crux of Koshliakov's theory. It is the characteristic equation of the generalized heat equation encountered by Koshliakov in his problem on heat conduction. As $p \to \infty$, the positive roots λ_n of the characteristic equation are $\lambda_n = n$ (being the roots of the resulting equation $\sin(\pi\lambda) = 0$). Thus, in this special case, the series $\sum_{n=1}^{\infty} \lambda_n^{-s}$ is nothing but the Riemann zeta function $\zeta(s)$. Moreover, when $p \to 0$ and $\lambda \neq 0$, we get $\cos(\pi\lambda) = 0$, which implies that the positive roots are

$\lambda_n = n - \frac{1}{2}$. Thus, one can study the series $\sum_{n=1}^{\infty} \left(n - \frac{1}{2} \right)^{-s} = (2^s - 1)\zeta(s)$.

For any $p > 0$, one may then construct a general zeta function $\sum_{n=1}^{\infty} \lambda_n^{-s}$ for Re(s) > 1. In fact, Koshliakov considers a normalized version of this series, namely,

$$\zeta_{p(s)} := \sum_{j=1}^{\infty} \frac{p^2 + \lambda_j^2}{p(p + \frac{1}{\pi}) + \lambda_j^2} \cdot \frac{1}{\lambda_j^s}.$$

This is one of the two *Koshliakov zeta functions*. With this foundation, Koshliakov shows unmatched brilliance in developing the theory of these zeta functions and the functions associated to them. For details, the reader is referred to [7] and [2].

Concluding remarks

Koshliakov died of a brain haemorrhage on 23rd September 1958. While mortal remains put an end to a person's physical presence, the ideas he/she comes up with live forever. Koshliakov's legacy and his indomitable spirit will continue to inspire everyone as long his mathematics is studied!

Acknowledgements

References

1. N. N. Bogolyubov, L. D. Faddeev, A. Yu. Ishlinskii, V. N. Koshlyakov and Yu. A. Mitropol'skii, *Nikolai Sergeevich Koshlyakov (on the centenary of his birth)*, Uspekhi Mat. Nauk **45** (1990), No. 4, 173–176; English transl. in Russian Math. Surveys **45** (1990), No. 4, 197−202.

FIG 3 : ***Rajat Gupta*** *was then a PhD student under Atul Dixit when they studied Koshliakov's second manuscript in more detail and even managed to extend his theory.*

2. A. Dixit and R. Gupta, *Koshliakov zeta functions I: Modular relations*, Adv. Math. **393** (2021), 108093 (41 pages).

3. A. G. Kisunko, *Zeta functions and their application to solving some problems of mathematical physics* (Russian), Dissertation, Moscow State Institute, 1999.

4. N. S. Koshliakov, *On Voronoi's sum-formula*, Mess. Math. **58** (1929), 30−32.

5. N. S. Koshliakov, *An extension of Bernoulli's polynomials*, Mat. Sb. **42** no. 4 (1935), 425−434.

6. N. S. Koshlyakov, *Basic Differential Equations of Mathematical Physics*, ONTI, 4th edition, 1936.

7. N. S. Koshliakov (under the name N.S. Sergeev), *Issledovanie odnogo klassa transtsendentnykh funktsii, opredelyaemykh obobshcennym yravneniem Rimana* (A study of a class of transcendental functions defined by the generalized Riemann equation) (in Russian), Trudy Mat. Inst. Steklov, Moscow, 1949. (Available online at https://dds.crl.edu/crldelivery/14052)

8. N.S. Koshliakov, *Investigation of some questions of the analytic theory of rational and quadratic fields*, I, II and III (in Russian), Izv. Akad. Nauk SSSR Ser. Mat. **18** No. 2, 113−144; No. 3, 213−260; No. 4, 307−326 (1954).

9. N. S. Koshlyakov, M. M. Smirnov and E. B. Gliner, *Differential Equations of Mathematical Physics*, North-Holland Publishing Co., Amsterdam; Interscience Publishers John Wiley & Sons, Inc., New York, 1964. xvi+701 pp.

10. V. N. Koshlyakov, O. A. Ladyzhenskaya and D. R. Merkin and M. M. Smirnov, *Nikolai Sergeevich Koshlyakov (on the centenary of his birth)* (Russian), Vestnik Leningrad Univ. Mat. Mekh. Astronom. (1991) No. 4, 81−83.

11. G. G. Lorentz, *Mathematics and Politics in the Soviet Union from 1928 to 1953*, J. Approx. Theory **116** (2002), 169−223.

12. S. Ramanujan, *New expressions for Riemann's functions $\xi(s)$ and $\Xi(s)$*, Quart. J. Math. **46** (1915), 253−260.
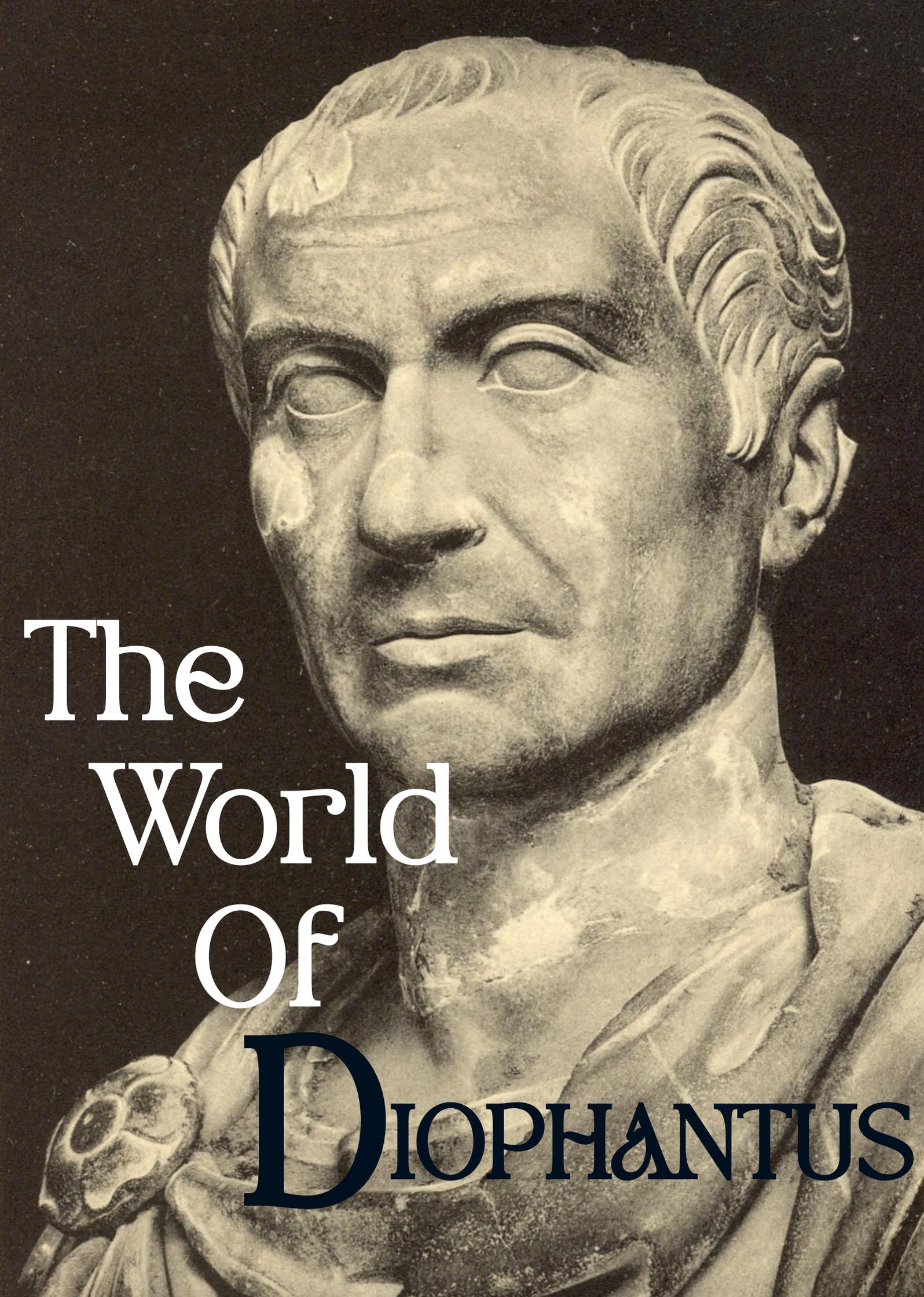
The
World
Of
DIOPHANTUS

The World of Diophantus

B Sury   (*Visiting Professor,International Centre for Theoretical Sciences Bangalore, India.*)

December 07, 2025

Sury offers a panoramic journey through Diophantine equations, weaving together history, puzzles, deep theorems, and modern breakthroughs—from Diophantus and Fermat to elliptic curves, Hilbert's 10th problem, and the ABC conjecture. Blending rigorous mathematics with anecdotes, poetry, and cultural context, it reveals how deceptively simple integer equations lead to some of the most profound ideas in mathematics.
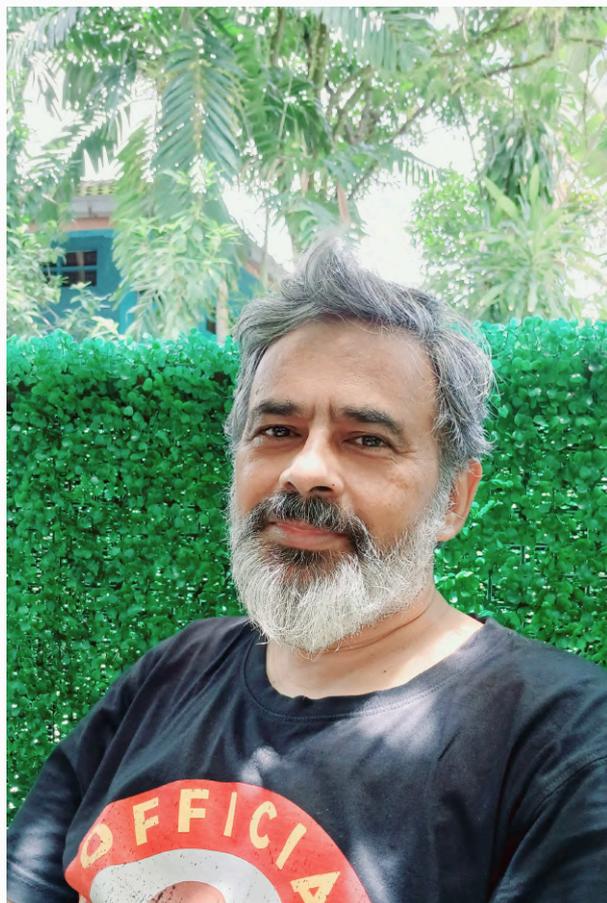
Sury was at the TIFR in Bombay during 1981-1999 and at the I.S.I. in Bangalore during 1999-2025. He is an elected Fellow of the National Academy of Sciences, India and was the National Co-ordinator for the Mathematics Olympiad Programme in India. His areas of research interest are Algebra and Number Theory. At present, he is a Visiting Professor at ICTS-TIFR, Bangalore.

No excuse is needed to celebrate mathematics, but there are two special days selected by the community. The National Mathematics Day is celebrated in India on Ramanujan's birthday, the 22nd of December but I defer the celebration of this to another time. The other one is Pi day, the 14th of March, which has been selected and universally accepted as the International Mathematics Day (there is actually a building in Hong Kong which is shaped like π). Regarding this choice of π-day, I would say:

> We had this year's Pi Day
> nine months back on a Friday.
> To Pi, 22 by 7 is closer
> but somehow 3.14 is kosher.
> But who cares? Let's celebrate anyway!

Diophantus of Alexandria

Metrodorus indicated the life span of Diophantus through a puzzle poetically as:

> 'Here lies Diophantus,' the wonder behold.
> Through art algebraic, the stone tells how old:
> 'God gave him his boyhood one-sixth of his life,
> One twelfth more as youth while whiskers grew rife;
> And then yet one-seventh ere marriage begun;
> In five years there came a bouncing new son.
> Alas, the dear child of master and sage
> after attaining half the measure of his father's life chill fate took him.
> After consoling his fate by the science of numbers for four years, he ended his life.'

FIG 1 : *Diophantus of Alexandria, Egypt lived during the 3rd century AD. Joseph-Louis Lagrange called Diophantus "the inventor of algebra".*

This puzzle implies that Diophantus's age x =?? is a solution of the equation:

$$x = \frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4.$$

For those who wish to verify the answer they got, here is a crossword type of hint: endless Hindustani Classical flautist!

Diophantus was interested in solving polynomial equations in many variables where he sought solutions in integers or, more generally, in rational numbers. He wrote a number of books titled 'Arithmetica' many of which have got lost. Thus, the nomenclature of Diophantine equations arose to describe the broad subject of finding integer solutions of polynomial equations. For instance, the Diophantine equation $x^2 + y^2 = z^2$ seeks integer triples (the so-called Pythagorean triples) describing the lengths of the sides of a right-angles triangle.

The amateur mathematician Pierre de Fermat had, in his copy of Bachet's translation of Diophantus's Arithmetica, made a famous marginal note which came to be known as Fermat's last theorem and which is now a theorem. It says that for any n > 2 there are no solutions of the equation $x^n + y^n = z^n$ in non-zero integers x, y, z; see the departure from the case $n = 2$ when we get infinitely many Pythagorean triples.

In regard to the history of Fermat's last theorem, many famous mathematicians like Cauchy had unsuccessfully attempted (and thought to have solved) it. The first seriously successful attempt was by Kummer when he proved Fermat's assertion for certain types of primes called regular primes. Sophie Germain (who had to go under an alias of a male name as women were not allowed to enrol at the university at that time) was a protege of Gauss and she proved the truth of Fermat's assertion when the exponent n is a prime p such that 1 + 2p is also a prime. Only in the 1980s, the whole picture changed with the advent of highly state-of-the-art methods into this topic by Gerd Faltings, Gerhard Frey, Kenneth Ribet and finally ending with the deep work of Andrew Wiles supported by Richard Taylor. One might say:

Solved first by the great Cauchy.
What a mistake made – Gosh, he!
Then it was Kummer's time.
He did every regular prime.
Germain came and shouted 'Whoopee!
Done, if we know about 1 + 2p'.
The uproar after Faltings, Gerd
went 'Mordell's done; spread the good word!'
We know who then jumped into the fray
whose elliptic curve could exist no way!
Ribet said 'Taniyama does
put a stop to all this fuss!'
And then Wiles was heard telling Taylor
now's the time; let's go and nail her!'
We conclude from this saga of Fermat
the importance, above all, of Karma!

The problem of finding integer solutions of polynomial equations is so basic and ubiquitous that it is not surprising Diophantine equations arise in diverse situations from the most abstract to the very concrete. In the following article, we take a peek into many Diophantine equations that arise in mathematics ranging from the recreational kind of questions to those that are more fundamental and deeper.

Sums of Powers

Fermat had conjectured that every positive integer is a sum of four squares of integers. Later in 1770, Waring conjectured that every positive integer N is a sum of at the most 9 cubes of positive integers. This was proved by Wieferich (1909) - with a correction carried out by Kempner (1912). In fact, if N is large enough, 7 cubes suffice (Linnik 1942); can 7 be reduced to 6 or 5 or 4? The answer is still unknown.

If we allow cubes of negative integers also, then 5 cubes suffice but, it is again unknown if 4 cubes suffice. Therefore, the problem as to which integers are sums of three integer cubes becomes very interesting.

Regarding this last question, by 2021, the only two elusive cases of 33 and 42 remained among the numbers up to 100. This was finally settled by Andrew Booker from
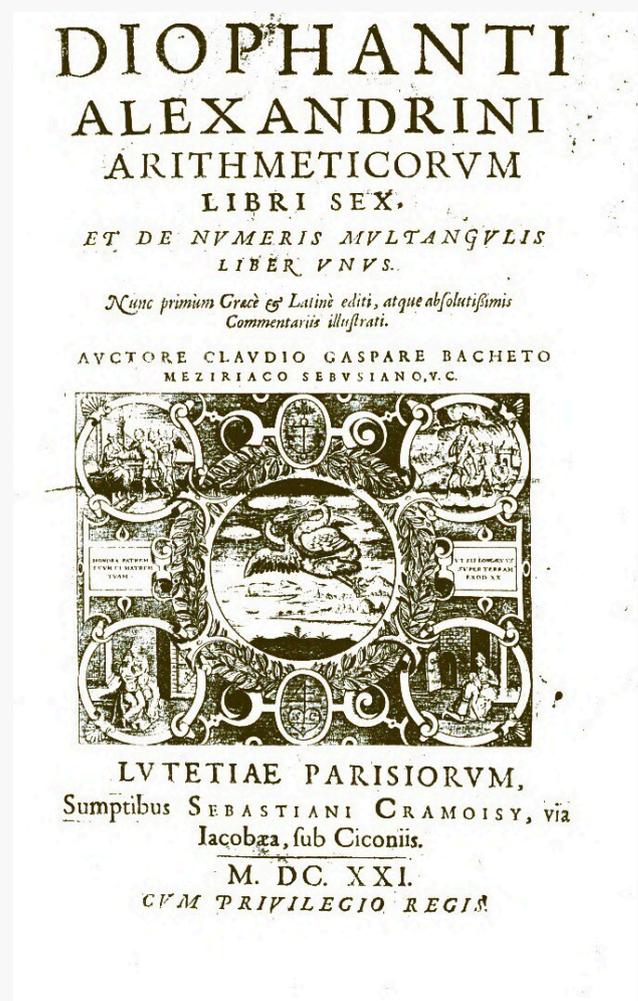

FIG 2 : *Diophantus's Arithmetica by Bachet. Arithmetica is the major work of Diophantus and the most prominent work on premodern algebra in Greek mathematics. It is a collection of 290 algebraic problems giving numerical solutions of determinate equations (those with a unique solution) and indeterminate equations.*

Bristol, and Andrew Sutherland from MIT - authorities on parallel computations. They used 'Charity Engine', a world-wide computer that harnessed idle, unused computing power from over 500000 home PCs to create a crowd-sourced platform.

The most difficult case of 42 resolved using a computing platform reminiscent of 'Deep Thought', the giant machine which gave the answer 42 in Douglas Adams's Hitchhiker's Guide to the Galaxy. For those who have not had the pleasure of reading this work, we recall from it: The Earth was actually a giant supercomputer, created by another supercomputer, Deep Thought. 'Deep Thought' built by its creators to give the answer to the "Ultimate Question of Life, the Universe, and Everything". After eons of calculations, the answer was given simply as "42". Deep Thought was then instructed to design the Earth supercomputer to determine what the Question actually is!

For the sake of those who are interested, here is the solution expressing 42:

$$42 = (-80538738812075974)^3 +$$
$$(80435758145817515)^3 +$$
$$(12602123297335631)^3.$$

In passing, we mention that it is easy to see the smallest solutions to expressing 3 as a sum of three cubes of integers are:

$$3 = 1^3 + 1^3 + 1^3 = 4^3 + 4^3 + (-5)^3.$$

For those who wonder how big the next solution could be, here it is:

$$3 = (569936821221962838 0720)^3 +$$
$$(-569936821113563493509)^3 +$$
$$(-472715493453327032)^3.$$

Concerning such questions, Terence Tao suggests as a challenge for the clairvoyant:

It occurs to me that these sorts of questions would be excellent challenge questions to pose to any psychics who claim to be in contact with superintelligent aliens, since the solutions are already expected to be produced by computer search in a few years but would be instantly verifiable evidence of some extraordinary computational or intellectual resource if produced sooner.

The Fab cab

Everyone must have heard of the famous taxicab number 1729 and Ramanujan's story thanks to Mahalanobis, who was a contemporary of Ramanujan at Cambridge (and who actually taught him for the first time how to get inside an English bed after realizing that for months Ramanujan had been freezing without being privy to this know-how). The Ramanujan taxicab number concerns the Diophantine equation $x^3 + y^3 = 1729$. When Hardy mentioned that the taxicab he travelled bore the number 1729 and this number appeared dull to Hardy, Ramanujan immediately responded that this was far from a dull

number as it was the smallest positive integer which is a sum of two cubes of integers in two different ways. The two integer solutions (12, 1) and (10, 9) are the only ones which is easy to prove, but a nice exercise it to check that each number < 1729 has at the most one expression as a sum of two cubes.

However, in this context, what may not be well-known is that $x^3 + y^3 = 1729$ has infinitely many rational solutions. For instance, if u, v is a solution, then so is $U = u(u^3 - 3458)/(1729 - 2u^3)$ and $V = v(u^3 + 1729)/(1729 - 2u^3)$.

Enter Sylvester

As alluded to earlier, Diophantine equations are polynomial equations for which one seeks integer or rational solutions. Existence of rational solutions is often an easier problem to study than that of determining if integer solutions exist. A heuristic reason is that the set of rational solutions may have a much better structure - for instance it often has the structure of a group where two rational points can be composed to give a new third rational point. For example, a Diophantine equation like $y^2 = x^3 + 54$ has only two integer solutions (x, y) = (3, 9), (3, -9) whereas if we consider a picture of the real locus of this equation, one may produce a new point with rational co-ordinates from two points with rational co-ordinates simply by drawing a chord between the two points and then reflecting about the x-axis the third point where this chord intersects the graph. To start with one can draw the tangent at (3, 9) and take the point where it cuts the graph and reflecting that point. In this manner, we can produce infinitely many solutions with rational co-ordinates! Of course, an equations like $y^2 = f(x)$ with $f(x)$ a cubic polynomial with distinct roots are special; they are called 'elliptic curves' and they have a group law as described above where two points give rise to a third point.

colorization © todayinsci.com

FIG 3 : *Ramanujan (1887-1920) from India was deeply interested in Diophantine equations, contributing explicit solutions to sums of two cubes and to Pell-type equations using continued fractions.*

Many classical problems lead to Diophantine equations which involve elliptic curves. One such is a problem due to Sylvester. Determining which integers n are sums of TWO rational cubes, has a rich history tracing back to Sylvester who predicted that prime numbers p which leave a remainder 2 or 5 when divided by 9 are not sums of two rational cubes, while primes which leave a remainder of 4, 7, or 8 are sums of two rational cubes. In contrast, primes p that leave a remainder of 1 or 8 when divided by 9 may or may not be sums of two rational cubes. The problem gets related to the elliptic curve $y^2 = x^3 - 432p^2$. Only very recently the last prediction mentioned above (considered particularly difficult) has been proved for infinitely many prime numbers.

In passing, we mention an amusing anecdote involving Sylvester. While teaching in the United States, he had an encounter with an unruly student and believed that the student had been fatally wounded by Sylvester during a skirmish. Sylvester returned to England still under the mistaken impression that he had killed a student! While in England, he had tutored many people in mathematics and statistics, one of whom was Florence Nightingale. Sylvester was also responsible for introducing several interesting names in mathematics - like syzygy.

Not as easy as Pi

Like the Diophantine equations, a related notion that arises in many situations is that of Diophantine approximation - we briefly mention one place where π occurs. Here is a routine-looking question? Is the infinite series $\sum \frac{1}{n^3 \sin^2(n)}$ convergent?

The convergence of the series depends on the behavior of the sequence $n|\sin(n)|$ as n gets infinitely large and, this behavior is mysterious. It depends on how well can π be approximated by rational numbers - something unknown as yet! To make it a bit more precise, we briefly describe something called the *irrationality measure $\mu(a)$* of an irrational number α. Consider the positive numbers a > 0 for which the distance $|a - p/q|$ of a from a rational number can be less than the quantity $1/q^a$ only for finitely many p, q. The smallest (or the infimum of) all such $a > 0$ is defined to be the irrationality measure of a. It quantifies how well-approximable the number a is by rational numbers.

The Dirichlet box principle implies that the irrationality measure $\mu(a) \geq 2$ and generically (that is, almost all) a have $\mu(a) = 2$. For a specific number, it is difficult to find μ; for instance, $\mu(e) = 2$ but, the constant $\mu(\pi)$ is still unknown! One knows $\mu(\pi) < 8$ but not much more is known Here is the shocker - the series

$$\sum \frac{1}{n^3 \sin^2(n)}$$

diverges if $\mu(\pi) > \frac{5}{2}$ and converges if, $\mu(\pi) < \frac{5}{2}$. So, take your pick! Before we move on to other things, we recall an interesting mnemonic to remember the first few digits of π:

> *Now I want a drink, alcoholic of course, after the heavy lectures involving quantum mechanics!"*

Hilbert's 10th problem

We already mentioned that many problems of mathematics can be formulated as seeking solutions of certain Diophantine equations. However, this statement goes much further. In a sense of formal mathematical logic, every mathematical problem can be so formulated! This is formally the so-called Hilbert's 10th problem - one of the 23 problems posed by david Hilbert during the 1900 International Congress of Mathematicians to lead the way for mathematics in the next several years.

Hilbert's famous 10th problem asserted:

> *Given a diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined by a finite number of operations whether the equation is solvable in rational integers.*

As it turned out, the answer was obtained in 1970 and unfortunately, it turned out to be negative. More precisely, the resolution due to Martin Davis, Putnam, Julia Robinson and Yuri Matiyasevich proves that there is no general algorithm that, given any Diophantine equation, decides whether it has solutions in positive integers or not. In more technical terms, the notions

FIG 4 : *Mahalanobis(1893- 1972) engaged with Diophantine problems and played a role in sustaining and promoting mathematical research traditions in India.*

of 'effectively enumerable' sets and 'Diophantine' sets coincide.

The ideas appearing in the resolution of the 10th problem yield interesting implications such as:

There exists a polynomial in 26 variables for which the set of positive values when the variables take positive integer values, equals the set of prime numbers!

One can write the polynomial down explicitly using the symbols a to z for the 26 variables.

Pell-Mell

We discuss a type of equation that arises in many contexts and we start with an elementary problem that leads to it.

Think of a fruit-seller arranging her fruits in a triangular pattern in the morning and in a square pattern in the evening. Can she do both of these with the same number of fruits? For instance, if he has 36 fruits, he can do this because $6^2 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$. Which other squares are so expressible as 'triangular numbers'?

If $n^2 = 1 + 2 + ... + k = k(k + 1)/2$, then $8n^2 = 4k(k + 1) = (2k + 1)^2 - 1$. Thus, $(2k + 1, 2n)$ is a solution of the equation $x^2 - 2y^2 = 1$. Being aestheticallyminded fruit-sellers, all of us of course want to know what the solutions of $x^2 - 2y^2 = 1$ are! Before addressing how to solve this equation, we jump to another instance when the same equation arises and it involves Ramanujan.

In the Strand magazine, Mahalanobis had seen the following problem which he mentioned to Ramanujan: Imagine that you are on a street with houses marked 1 through n. There is a house in between such that the sum of the house numbers to the left of it equals the sum of the house numbers to its right. If n is between 50 and 500, what are n and the house number? Ramanujan thought for a moment and replied "Take down the solution" and dictated something called a 'continued fraction' (to be explained below) saying that it contained the solution! Evidently, Ramanujan wanted to have some fun instead of directly giving the answer! So, what is behind this?

If the house number is r, then we have

$$1 + 2 + ... + (r - 1) = (r + 1) + ... + n$$

The LHS is $(r - 1)r/2$ and if we add $1 + 2 + ... + r = \frac{r(r+1)}{2}$ to both sides, we have $r^2 = n(n + 1)/2$. Multiplying by 8 and adding 1, we have $8r^2 + 1 = (2n + 1)^2$, the very same equation encountered by the fruit-seller! One can similarly look at $m^2 - dn^2 = 1$ for any square-free positive integer d. These equations are popularly (and erroneously!) known as *Pell equations.* Interestingly, it turns out that there are infinitely many pairs m, n of positive integer solutions of $m^2 - dn^2 = 1$ and essentially, they are all generated froma single pair. The ancient Indian mathematicians (especially Brahmagupta,Bhaskara II and Jayadeva) studied the equations $x^2 - dy^2 = \pm 1$ and solved them. What is more - they gave an algorithm (the so-called Chakravala or cyclic method) which produces all the solutions.

In 1657, Fermat, writing to his friend Frenicle, he posed "to the English mathematicians and all others" the problem of finding a solution of $x^2 - Ny^2 = 1$ "pour ne vous donner pas trop de peine" like N = 61, 109.

The 20th century great André Weil's comment on this was: "What would have been Fermat's astonishment if some missionary, just back from India, had told him that
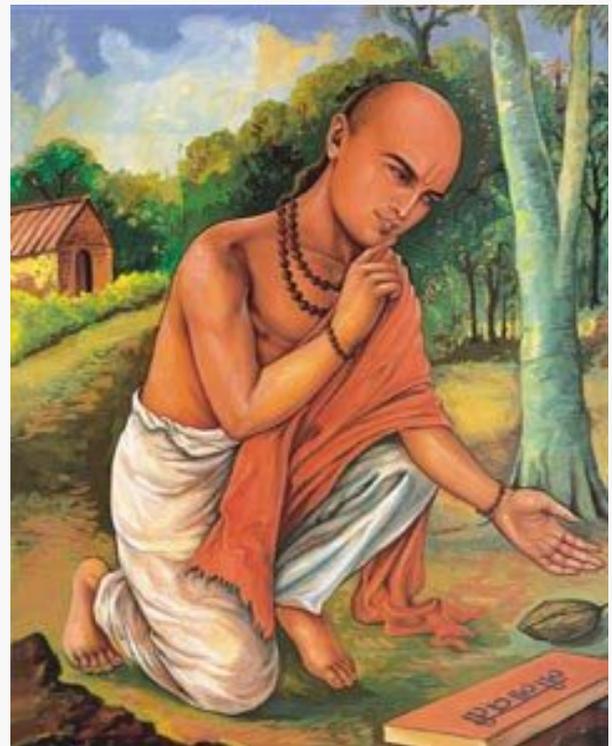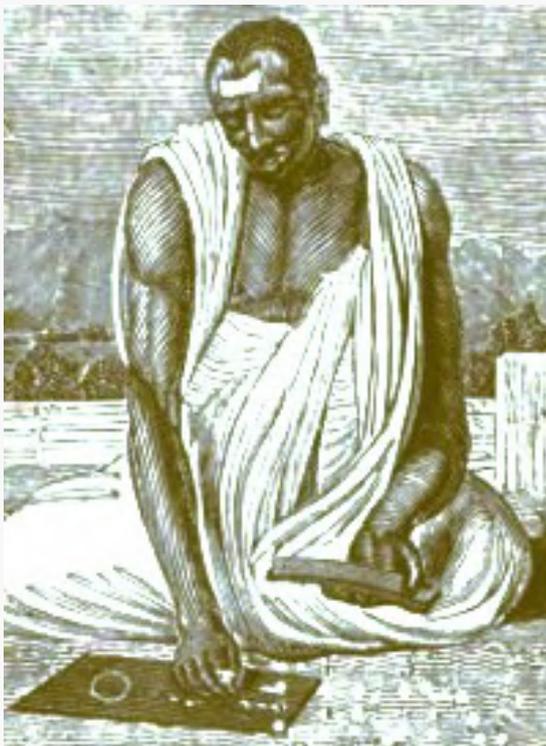

FIG 5 : *From left, Brahmagupta (598-670 AD) formulated general solutions for quadratic Diophantine equations and composition rules, while Bhaskaracharya (1114-1185 AD) systematized and perfected the Chakravāla method for solving equations of the form $x^2 - Ny^2 = 1$.*

his problem had been successfully tackled there by native mathematicians almost six centuries earlier ?"

Indeed, in 1150 A.D., Bhaskara II gave the explicit solutions

$$1766319049^2 - 61(226153980)^2 = 1$$

$$158070671986249^2 - 109(15140424455100)^2 = 1$$

Indeed, Brahmagupta (598-665) had already solved this equation in 628 A.D. for several values like N = 83 and N = 92.

Brahmagupta had remarked, "Kurvannaavatsaraad ganakah" – meaning (approximately), *"a person who is able to solve these within a year is truly a mathematician"'*!

The wrong attribution to Pell of these equations is due to the most prolific of mathematicians – Leonhard Euler, but the name has stuck. In view of the above understanding of mathematical history, now the equations are being increasingly referred to as the Brahmagupta-Pell equations. Perhaps the oldest Diophantine equation to be considered is one posed by Archimedes (287 –212 B.C.) to Eratosthenes known as the cattle problem. We avoid the long wording and just state the modern equivalent which is to find the smallest solution of the equation

$$X^2 - 410286423278424Y^2 = 1;$$

it turns out that the smallest solution has at least 206545 digits.

We make a brief interlude on continued fractions and indicate how they are used in solving the Brahmagupta–Pell equations.

A simple continued fraction (SCF) is a nested expression

$$a_0 + \frac{1}{a_1} + \frac{1}{a_2} + \ldots$$

where a_i are integers and $a_n > 0$ for all n > 0. That is, consider the sequence of fractions $a_0, a_0 + \frac{1}{\ell_1}$, where $\ell_1 = a_1 + \frac{1}{\ell_2}$, where $\ell_2 = a_2 + \frac{1}{\ell_3}$ etc. The above infinitely nested expression stands for the limiting value of the fractions $\ell_1, \ell_2, \ell_3, \ldots$ (which can be easily shown to exist). The main point is every real number t can not only be expanded as a decimal but also expanded as a simple continued fraction by the 'greedy' algorithm: Consider any real number t which is not already an integer. Then, take a_0 to be the greatest integer $[t]$ not exceeding t. Next consider the real number

$$t_1 := \frac{1}{t-a_0} < 1.$$

Take $a_1 = [t_1]$ and consider $\frac{1}{t_1-a_1}$ etc. In this manner, we get a continued fraction expansion

$$t = [a_0; a_1, a_2, a-3, \ldots].$$

It is easy to see that this is eventually periodic if and only if t is the root of a quadratic polynomial with rational coefficients. For any square-free positive integer N, the SCF of $\sqrt{N}$ looks like $\left[b_0; \overline{b_1, b_2, \ldots, b_{n-1}, 2b_0}\right]$.

For instance, using the greedy algorithm, let us find the SCF for the irrational number $\sqrt{7}$.

$$\sqrt{7} = 2 + \left(\sqrt{7} - 2\right) = 2 + \frac{3}{\sqrt{7}+2} = 2 + \frac{1}{(\sqrt{7}+2)/3}.$$

$$\frac{\sqrt{7}+2}{3} = 1 + \left(\frac{\sqrt{7}+2}{3} - 1\right) = 1 + \frac{\sqrt{7}-1}{3}$$

$$\sqrt{7} - 2 = \frac{1}{1+} \frac{\sqrt{7}-1}{3} = \frac{1}{1+} \frac{1}{(\sqrt{7}+1)/2} = \frac{1}{1+} \frac{1}{1+} \frac{\sqrt{7}-1}{2}$$

$$= \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{\sqrt{7}+2} = \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{4+\sqrt{7}-2}$$

Thus, we have a repetition and

$$\sqrt{7} - 2 = \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{4+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{4+} \cdots$$

So, $\sqrt{7} = \left[2; \overline{1, 1, 1, 4}\right]$.

Here is the interesting thing about SCFs of $\sqrt{N}$ and their relation to the Pell equation $x^2 - Ny^2 = 1$.

$$\sqrt{N} = \left[b_0; \overline{b_1, b_2, \ldots, b_{n-1}, 2b_0}\right],$$

the rational numbers $b_0 + 1/b_1, b_0 + 1/(b_1 + (1/b_2))$ etc. are called convergents of the SCF. When the SCF is periodic like that for $\sqrt{N}$, the penultimate convergent $p(n-1)/q(n-1)$ before the recurring period gives a solution of $x^2 - Ny^2 = (-1)^n$. This is so simple to prove that it is explained in the classical school level text by Hall and Knight. This is how Ramanujan could give the solution to the house number problem quickly.

For example, $\sqrt{7} = \left[2; \overline{1, 1, 1, 4}, \ldots\right]$ gives the penultimate convergent beforethe period to be [2;1,1,1] = 8/3.Then, (8, 3) is a solution of $x^2 - 7y^2 = 1$ (as the period is of even

FIG 6 : *Pierre de Fermat (August 17, 1601 to January 12, 1665). Fermat initiated the systematic study of Diophantine equations, introducing the method of infinite descent and posing problems that shaped number theory for centuries.*

length). Note that $x^2 - 7y^2 = -1$ has no solution as seen by looking at the remainders on dividing by 4.

Another example is $\sqrt{13} = \left[3; \overline{1, 1, 1, 1, 6, \ldots}\right]$ which has the penultimate convergent before the period to be [3; 1, 1, 1, 1] = 18/5. Then (18, 5) is a solution of $x^2 - 13y^2 = -1$ (as the period is of odd length). Think of this as the equality

$$\left(18 + 5\sqrt{13}\right)\left(18 - 5\sqrt{13}\right) = -1.$$

From this a solution for $x^2 - 13y^2 = 1$ can be obtained by considering $\left(18 + 5\sqrt{13}\right)^2 = 649 + 180\sqrt{13}$. Then (649, 180) is a solution of $x^2 - 13y^2 = 1$.

Congruent Number Problem

This is one of the oldest problems in Diophantine equations. A natural number d is said to be a congruent number if there is a right-angled triangle with rational sides and area d. For example, 5, 6, 7 are congruent numbers. Of course, 6 is congruent as seen from the usual 3, 4, 5 triangle. To see that 5 is a congruent number, we consider the right-angled triangle with sides 3/2, 20/3, 41/6. For 7, look at a right triangle with sides 35/12, 24/5, 337/60. How did we guess this? More importantly, how do we decide if a given number is a congruent number? This will be done by relating it to another problem! This is the following one.

Can we have an arithmetic progression of three terms which are all squares of rational numbers and the common difference d? That is, can $x^2 - d, x^2, x^2 + d$ be squares of rational numbers and x rational? The congruent number problem and the above question are equivalent!

Indeed, Let $u \le v < w$ be the sides of a right triangle with rational sides. Then $x = w/2$ is such that $\frac{(v-u)^2}{4}, \frac{w^2}{4}, \frac{(u+v)^2}{4}$ form an arithmetic progression. Conversely, if $x^2 - d = y^2, x^2, x^2 + d = z^2$ are three rational squares in arithmetic progression, then: $z - y, z + y$ are the legs of a right angled triangle with rational legs, area $(z^2 - y^2)/2 = d$ and rational hypotenuse 2x because $2(y^2 + z^2) = 4x^2$.

Now 1, 2, 3 are not congruent numbers. The fact that 1, 2 are not congruent numbers is essentially equivalent to Fermat's last theorem for the exponent 4(!) Here is the argument. If $a^2 + b^2 = c^2, \frac{1}{2}ab = 1$ for some rational numbers a, b, c then $x = \frac{c}{2}, y = |a^2 - b^2|\frac{1}{4}$ are rational numbers satisfying $y^2 = x^4 - 1$. In terms of integers, this is the Diophantine equation $u^4 - v^4 = w^2$. As this has no non-zero integer solutions, it follows that 1 is not a congruent number. Similarly, if $a^2 + b^2 = c^2, \frac{1}{2}ab = 2$ for rational numbers a, b, c, then $x = \frac{a}{2}, y = a\frac{c}{4}$ are rational numbers satisfying $y^2 = x^4 + 1$. Again, the nonexistence of solutions shows that 2 is not congruent.

Here is a (rather unusual!) way of using the above fact that 1 is not a congruent number to show that $\sqrt{2}$ is irrational. Consider the right-angled triangle with legs $\sqrt{2}, \sqrt{2}$ and hypotenuse 2. If $\sqrt{2}$ were rational, this triangle would exhibit 1 as a congruent number! Though it is an ancient problem to determine which natural numbers are congruent, it is only in late 20th century that substantial results were obtained and progress has been made which is likely to lead to its complete solution. The rephrasing in terms of arithmetic progressions of squares emphasizes

a connection of the problem with rational solutions of the equation $y^2 = x^3 - d^2x$; recall we referred to such equations as "elliptic curves". It is easy to show that: The positive integer d is a congruent number if, and only if, the elliptic curve $y^2 = x^3 - d^2x$ has a solution with $y \ne 0$.

In fact, $a^2 + b^2 = c^2, \frac{1}{2}ab = d$ implies $bd/(c - a), 2d^2/(c - a)$ is a rational solution of $y^2 = x^3 - d^2x$. Conversely, a rational solution of $y^2 = x^3 - d^2x$ with $y \ne 0$ gives the rational, right-angled triangle with sides $(x^2 - d^2)/y, 2xd/y, (x^2 + d^2)/y$ and area d.

In a nutshell, here is the reason we got this elliptic curve. The real solutions of the equation $a^2 + b^2 = c^2$ defines a surface in 3-space and so do the real solutions of $\frac{1}{2}ab = d$. The intersection of these two surfaces is a curve whose equation in suitable co-ordinates is the above elliptic curve! The connection with elliptic curves has been used, more generally, to show that numbers which are 1, 2 or 3 mod 8 are not congruent. This uses much deeper mathematics. Further, assuming the truth of a famous, deep, open conjecture known as the *weak Birch & Swinnerton-Dyer conjecture*, it has been shown that this is a complete characterization of congruent numbers.

Arithmetic Progressions

Here is a set of questions on arithmetic progressions of natural numbers which leads us to some very interesting
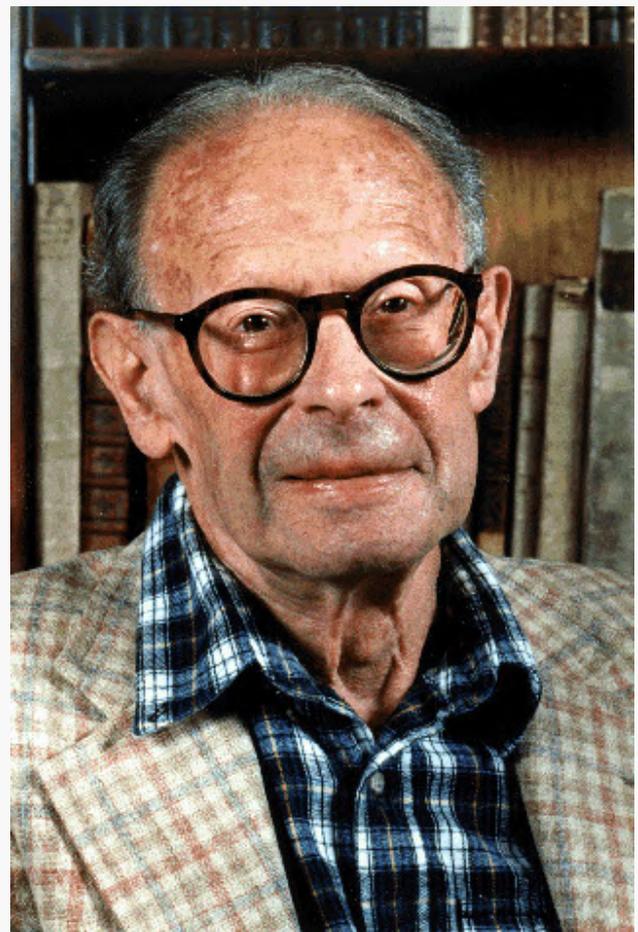

FIG 7 : *Andre Weil (May 6, 1906 to August 6, 1998) emphasized the structural depth of Diophantine equations, interpreting classical integer problems through algebraic curves and modern number-theoretic frameworks.*

Diophantine equations. Can we have a finite arithmetic progression

$$a, a + d, a + 2d, ..., a + nd$$

such that a first part $a, a + d, ..., a + (r - 1)d$ has the same sum as that of the second part $a + rd, a + (r + 1)d, ..., a + nd$? Note

$$20 + 25 + 30 = 35 + 40$$
$$14 + 21 + 28 + 35 + 42 + 49 = 56 + 63 + 70.$$

More generally, we can try to break an arithmetic progression into THREE parts with equal sums; here, we mean that each of the three parts consist of consecutive terms. The answer turns out to be 'No' which we leave as an exercise for the interested reader.

A related question is whether we can have four perfect squares of positive integers in arithmetic progression. Well, if a^2, b^2, c^2, d^2 are in arithmetic progression, then

$$b^2 - a^2 = c^2 - b^2 = d^2 - c^2,$$

and so

$$2a + 1, 2a + 3, ..., 2b - 1$$
$$2b + 1, 2b + 3, ..., 2c - 1$$
$$2c + 1, 2c + 3, ..., 2d - 1$$

would be 3 parts of an A.P. whose sums are all equal, which we mentioned above does not exist. Hence, we cannot have four perfect squares of positive integers in an arithmetic progression.

Erdös–Selfridge, Catalan

The fact that a product of (at least 2) consecutive positive integers can not be a perfect power was settled more than 50 years back by Erd¨os & Selfridge. In other words, the Diophantine equation

$$(x + 1)(x + 2)...(x + r) = y^2$$

does not solutions in positive integers $x, y, r > 1$.

Erdös–Selfridge theorem is so simple to state (and easy to prove for 2 or 3 consecutive numbers) that one may be tempted to think it could perhaps have an elementary proof. However, for r > 3, the proof needs deeper properties of prime numbers, such as a classical theorem due to Sylvester which asserts that any set of k consecutive numbers with the smallest one > k contains a multiple of a prime > k. The special case of this when the numbers are $k + 1, ..., 2k$ is known as Bertrand's postulate.

Yet another Diophantine equation arises from the natural question: *Which natural numbers have all their digits to be 1 with respect to two different bases?*

Equivalently, if the bases are x, y and he numbers have m and n digits respectively, then we wish to solve

$$\frac{x^m - 1}{x - 1} = \frac{y^n - 1}{y - 1}$$

in natural numbers $x, y > 1$; $m, n > 2$. For example 31 and 8191 have this property;

$$(11111)_2 = (111)_5, \quad (111)_{90} = 2^{13} - 1.$$

This was observed by Goormaghtigh nearly a century ago. However, it is still unknown whether there are only finitely many solutions in x, y, m, n. In fact, no other solutions are known. For any fixed bases x, y, it was proved only as recently as in 2002 that the number of solutions for m, n is at the most 2.

Another question is whether one can have different finite arithmetic progressions with the same product. For instance

$$2.6...(4n - 2) = (n + 1)(n + 2)...(2n)$$

for all natural numbers n. So, the question as to whether there are other solutions to the Diophantine equation

$$x(x + d_1)...(x + (m - 1)d_1) = y(y + d_2)...(y + (n - 1)d_2)$$

where d_1, d_2 are positive rational numbers and $d_1 \neq d_2$ if $m = n$ is quite open. It is only in 1999 that using deeper ideas from the subject of algebraic geometry, it was proved that if m, n, d_1, d_2 are fixed, then the equation has only finitely many solutions in integers apart from some exceptions which occur when $m = 2, n = 4$. A deep unsolved conjecture due to Erdös in 1975 asserts:

For each $c \in \mathbb{Q}$, the number of (x, y, m, n) satisfying

$$x(x + 1)...(x + m - 1) = c\, y(y + 1)...(y + n - 1)$$

with $y \geq x + m, \min(m, n) \geq 3$, is finite.

There are so-called transcendental methods that often prove by general ideas that certain classes of Diophantine equations can have only finitely many integer solutions. But, they are not 'effective' in the sense that one does not know any number which bounds the size of possible solutions. For instance, for the so-called Catalan equation $x^m - y^n = 1$, Robert Tijdeman proved (ineffectively) finiteness of the number of solutions. Later, although it was made effective by Langevin, he obtained an upper bound for x, y, m, n that was of the order of $\exp(\exp(\exp(\exp(730))))$ which is out of the range of any present-day computer. Of course, later in 2024, the Catalan equation was completely solved by Preda Mihailescu by totally different methods; he showed that the only perfect powers differing by 1 are 8 and 9. A more general conjecture due to S S Pillai is still open; it asserts that the gaps in the sequence of perfect powers tends to infinity.

Apollonian circle packing

Apollonius from 200 BC discovered something beautiful. If we have three circles touching each other, one may place another circle touching all three.

In the 17th century, Descartes discovered the remarkable fact that the four curvatures (which are taken to be the reciprocals of the radii) satisfy the Diophantine equation

$$\left((C_1 + C_2 + C_3 + C_4)^2\right) = 2\left(C_1^2 + C_2^2 + C_3^2 + C_4^2\right)$$

Here, the circles are supposed to have no common interior point which means by convention that the outermost circle's exterior is the interior and the interior

is the exterior and the radius is negative. Thus, if we are given 3 of the circles and they have integer curvatures, the fourth must also have integral curvature because of the equation! In the figure here, the curvatures are –1, 2, 2, 3.

ABC-conjecture

The ABC conjecture - formulated independently by Masser and Oesterl´e - supercedes many conjectures in Diophantine equations and implies many of them. It formalizes the intuitive observation that when two numbers A and B are divisible by large powers of small primes, then A + B tends to be divisible by small powers of large primes. More precisely:

> *For any $\epsilon > 0$, there are only finitely many triples A, B, C of relatively prime integers satisfying $A + B = C$, and $\max(A, B, C) > \text{Rad}(ABC)1 + \epsilon$, where $\text{Rad}(n)$ is the product of all distinct prime divisors of n.*

For instance, the ABC-conjecture implies Fermat's last theorem for sufficiently large exponents; in fact, it implies finiteness of the number of solutions of the generalized Fermat equation $Ax^r + By^s = Cz^t$. There is the as-yet-unsolved Beal's conjecture (also known as the Tijdeman- Zagier conjecture); until it is actually solved, one cannot predict what methods will work. The conjecture asserts that $x^a + y^b = z^c$ for a, b, c > 2 implies that x, y, z must have a common prime factor, and was formulated in 1993 by Andrew Beal, a banker and amateur mathematician.

FIG 8 : *In mathematics, Apollonian circle packing is a fractal generated by starting with a triple of circles, each tangent to the other two, and successively filling in more circles, each tangent to another three. Descartes discovered the remarkable fact that the four curvatures satisfy a Diophantine equation.*

One of Beal's goals is to inspire young people to think about the equation, think about winning the offered prize, and in the process become more interested in the field of mathematics. The prize money - now a million dollars - is being held by the AMS until it is awarded. The spendable income from investment of the prize money is used to fund the annual Erd¨os Memorial Lecture and other activities of the American Mathematical Society.

The conditions are necessary as shown by the examples

$$7^3 + 13^2 = 2^9,\ 27^4 + 1623 = 9^7.$$

Finally, we would like to mention that we have not discussed some other topics that involve the name of Diophantus. One is an unsolved problem that goes under the name of Diophantine tuples (both integer and rational) problem. Diophantus himself found a rational 4-tuple 1/16, 33/16, 17/4, 105/16. The question as to how large a set of positive rationals $a_1, a_1, ..., a_n$ can be found so that all $a_i a_j + 1$ are squares for $1 \le i < j \le n$ is still open? For integer tuples, 4 is the limit as was proved in 2016. Another fertile area of current research involves the so-called Frobenius Coin exchange problem: "Given n positive integers with no common factor > 1, what is the largest integer that cannot be represented as a nonnegative integral linear combination of the given integers?" Finally, we only alluded to briefly while describing a rational approximation to π

References

1. A. Granville, T.J. Tucker, "It's as easy as abc", Notices A.M.S. (2002), 1224-1231.
2. L.J. Mordell, "Diophantine equations", Academic Press 1969.
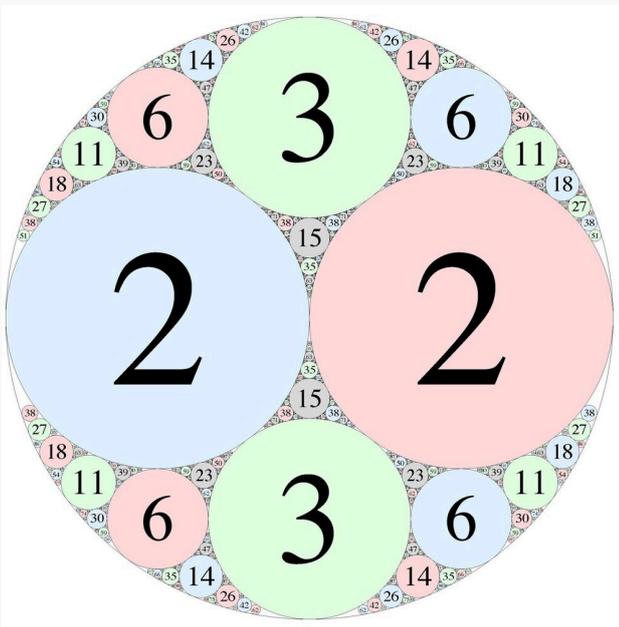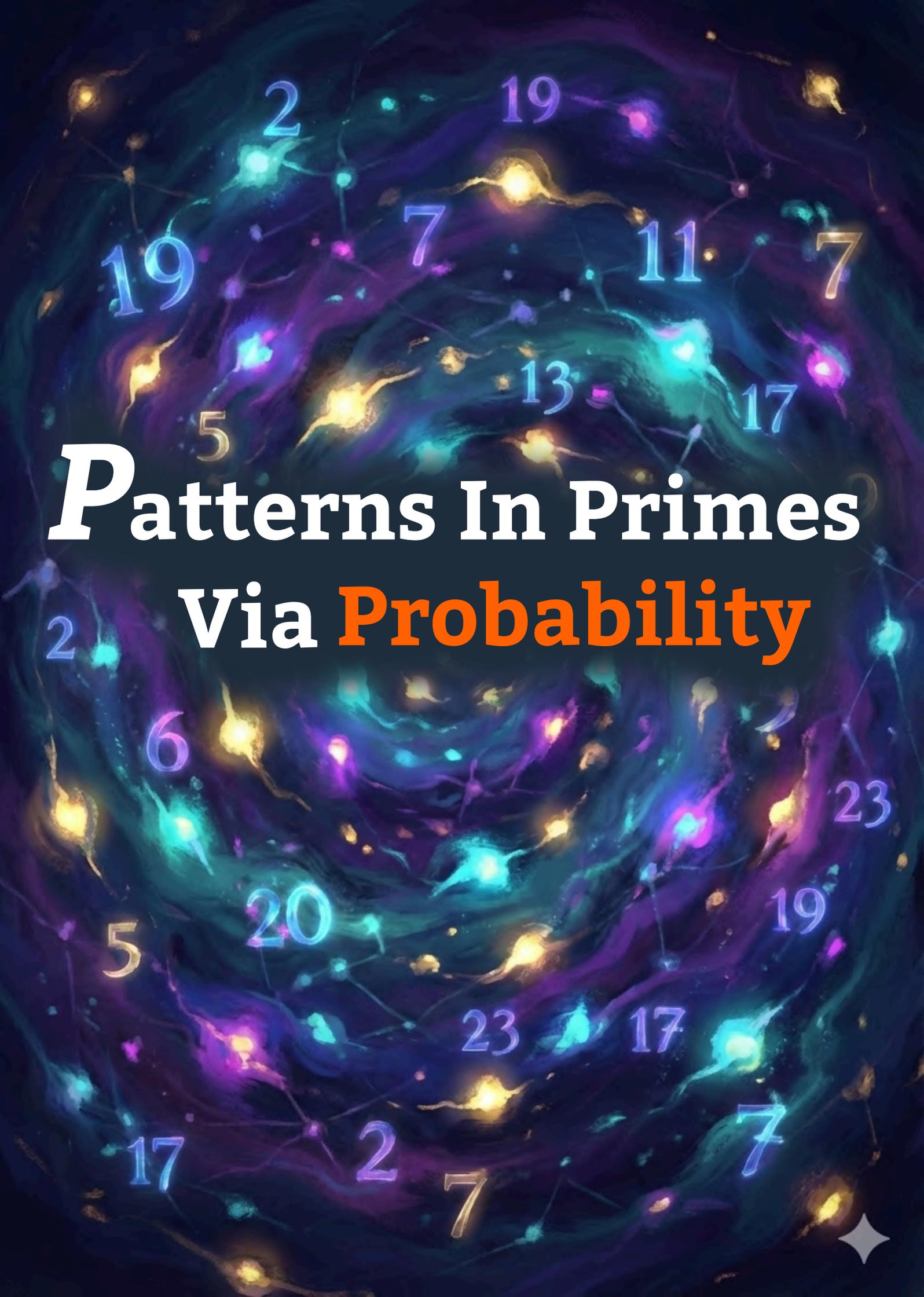3. R. C. Baker, "Diophantine inequalities", Clarendon Press, Oxford, 1986.

Patterns In Primes Via Probability

Patterns In Primes Via Probability

Anup Dixit   *(IMSc, India)*

Siddhi Pathak   *(CMI, India)*

December 22, 2025

Prime numbers appear to be scattered at random, yet their overall distribution follows striking statistical laws. This article explores how probabilistic thinking —especially Cramér's random model for primes—helps predict typical gaps between primes while revealing surprising limitations. Along the way, it shows how randomness and arithmetic structure coexist in one of mathematics' oldest problems.

Anup Dixit is an Assistant Professor at the Institute of Mathematical Science (IMSc) India. His research interest lies in number theory. Apart from various outreach programs for school students, he is also involved in training students for the Mathematics Olympiad.

Siddhi Pathak is an Assistant Professor at the Chennai Mathematical Institute (CMI). Her research focuses on topics in number theory. She is a corresponding editor for the mathematics magazine 'Bhavana'.

Introduction

The idea of a *number* lies at the very foundation of human civilization. Some of the earliest archaeological records of writing already reveal an awareness of natural numbers, marking a crucial step in the development of abstract thought. Reflecting on this, R. Dedekind famously wrote that "numbers are free creations of the human intellect; they serve as a means of grasping more easily and more sharply the diversity of things". Since numbers provide a lens through which we interpret the world, their study has fascinated mathematicians for centuries.

One of the earliest and most profound discoveries in mathematics, dating back to Euclid, is that every natural number can be written as a product of prime numbers. In this sense, primes serve as the fundamental building blocks of arithmetic. A *prime number* is a natural number $n > 1$ that has no divisors other than 1 and itself. For instance, 2, 3, 5, 7, and 11 are prime, whereas $9 = 3^2$ is not. The study of prime numbers has attracted many of the greatest mathematicians, including Euler, Gauss, Dirichlet, and Riemann.

One of the first striking facts about primes is Euclid's theorem asserting that there are infinitely many of them. Yet even a brief inspection of the primes among the first 100 natural numbers reveals an apparent lack of regularity. Aside from the trivial observation that all primes except 2 are odd, no simple pattern emerges. It is therefore unsurprising that there is no fast and fully deterministic procedure for locating very large primes. Indeed, the largest known prime as of December 2025 is
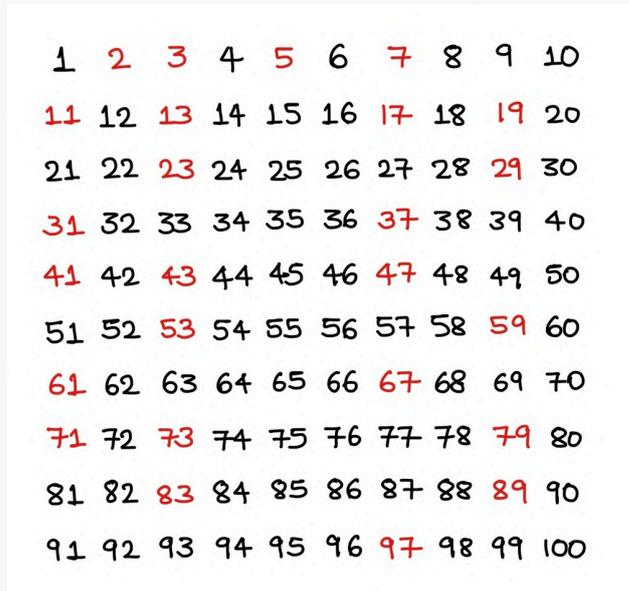

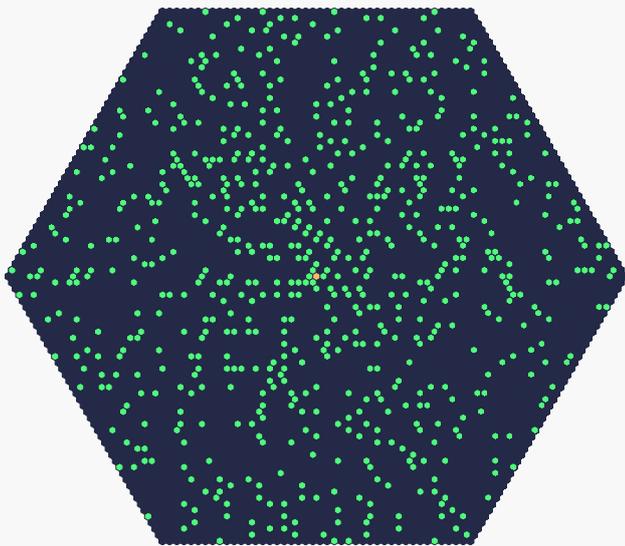**FIG 1** : *Apparent lack of patterns among the primes.*

FIG 2 : *The above construction represents numbers in a hexagonal pattern (the innermost hexagon shows 1 through 6, and then the numbers keep spiralling outward). The green mini hexagons represent prime numbers, and their frequency decays as the numbers get bigger (from inside to outside). This relationship is consistent with the prime number theorem which says that the frequency of prime numbers goes down as* $1/\log n$. *[Idea for the hexagonal representation - Quanta Magazine]*

$$2^{136,279,841} - 1,$$

a number discovered through extensive computation rather than a simple formula.

Despite this apparent irregularity, prime numbers exhibit remarkable *statistical* regularities. In the late eighteenth century, Adrien-Marie Legendre and Carl Friedrich Gauss independently observed, based on extensive numerical evidence, that the number of primes up to a large number N satisfies

$$\pi(N) := \#\{p \leq N : p \text{ is prime}\} \sim \frac{N}{\log N} \text{ as } N \to \infty.$$

Gauss further suggested that primes occur near a large number n with density approximately $1/\log n$, leading to the refined approximation

$$\pi(N) \sim \int_2^N \frac{dt}{\log t} =: \text{li}(N),$$

known as the *logarithmic integral*. This approximation is surprisingly accurate: for example, there are exactly 78498 primes below 10^6, while $\text{li}(10^6) \approx 78627$, an error of only about 0.16%.

This observation, now known as the *Prime Number Theorem*, was proved independently by Jacques Hadamard and Charles Jean de la Vall'ee Poussin in 1896, building on ideas introduced earlier by Riemann. Thus, although primes may appear erratic, their overall distribution follows a precise and elegant law.

Still, the intuition that primes behave in many ways like an unpredictable sequence remains useful. In this article, we explore how this perspective can lead to meaningful predictions, focusing in particular on the behavior of gaps between consecutive primes. The smallest possible

gap is 2, as seen in twin primes such as (3, 5) or (11, 13). On the other hand, by considering the numbers

$$n! + 2, n! + 3, \ldots, n! + n,$$

one can construct arbitrarily long stretches of composite numbers, and hence arbitrarily large gaps between primes. This naturally raises the question: near a large number N, what should a *typical* gap between consecutive primes look like?

To investigate this, consider an interval $[N, N + H]$, where N is large and H is much smaller than N, but still large enough to contain many primes. The Prime Number Theorem predicts that the number of primes in this interval is approximately

$$\pi(N + H) - \pi(N) \approx \frac{H}{\log N}.$$

Dividing the length of the interval by the number of primes it contains suggests that the *average* gap between consecutive primes near N is about $\log N$. This reasoning also reinforces Gauss's original insight that roughly one out of every $\log N$ numbers near N is prime.

This line of thought inspired the Swedish mathematician Harald Cramér in 1936 to introduce a probabilistic model for primes. Imagine an infinite sequence of urns $(U_n)_{n\in\mathbb{N}}$. The urn U_1 contains only blue balls, U_2 only red balls, and for $n \geq 3$, the urn U_n contains both colors. Suppose that when drawing a ball from U_n, the probability of obtaining a red ball is $1/\log n$. Drawing independently from each urn produces an infinite sequence of red and blue outcomes. Let P_n denote the index of the urn from which the n-th red ball is drawn. The resulting sequence $(P_n)_{n\in\mathbb{N}}$ is increasing and serves as a model for the sequence of prime numbers.

If we define

$$\Pi(N) := \#\{n \in \mathbb{N} : P_n \leq N\},$$

then

$$\mathbb{E}[\Pi(N)] = \sum_{n=1}^{N} \frac{1}{\log n} \approx \text{li}(N) \approx \frac{N}{\log N}.$$

This mirrors the behavior of $\pi(N)$, reinforcing the analogy between (P_n) and the primes. In particular, one
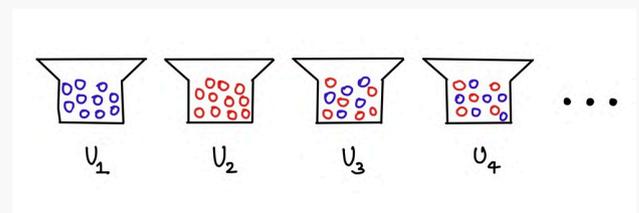
FIG 3 : *Cramér's conjecture is a prediction about how large the gaps between consecutive prime numbers can get. Using a probabilistic model in which an integer near n is "randomly prime" with probability $1/\log n$, Cramér argued that unusually large gaps should be rare and that the largest prime gaps near n should grow no faster than about $(\log n)^2$. In simple terms, the conjecture says that although prime gaps do grow without bound, they grow much more slowly than the primes themselves, and their size is tightly constrained by logarithmic factors.*
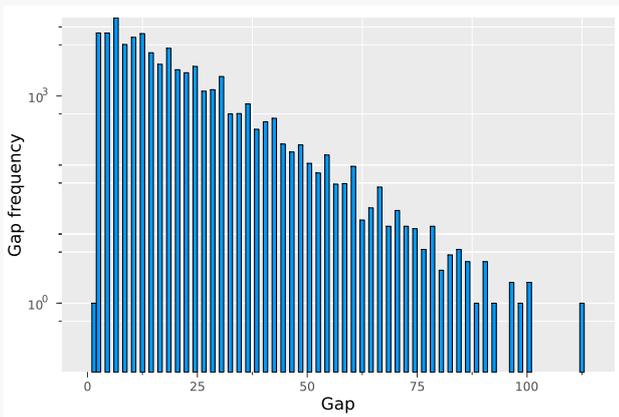
FIG 4 : *A histogram of gaps between consecutive prime numbers in a large numerical range. While individual prime gaps fluctuate irregularly, the distribution as a whole reflects the Prime Number Theorem: most gaps cluster around a size comparable to $\log N$ for primes near N. This visualization supports the heuristic argument that, despite their apparent randomness, primes obey predictable average behavior—an idea that motivates probabilistic models such as Cramér's and underlies modern conjectures on extreme prime gaps. [Source: InScight]*

FIG 5 : *Gauss's (left) empirical observations on the density of prime numbers led to the Prime Number Theorem and the logarithmic integral. Cramér's (right) probabilistic model builds on this viewpoint to predict the typical and maximal size of gaps between consecutive primes, highlighting both the power and the limitations of treating primes as a random sequence.*

may expect the largest gaps between primes to resemble the largest gaps between successive P_n.

Using results from probability theory, Cramér showed that with probability one,

$$\limsup_{n\to\infty} \frac{P_{n+1}-P_n}{(\log P_n)^2} = 1.$$

Motivated by this, he conjectured that prime gaps satisfy a similar bound.

Conjecture 1 (Cramér's Conjecture)
Let $2 = p_1 < p_2 < p_3 < \ldots$ be the sequence of prime numbers.
1. (Strong form) $\limsup_{n\to\infty} \frac{p_{n+1}-p_n}{(\log p_n)^2} = 1$
2. (Weak form) $p_{n+1} - p_n \ll (\log p_n)^2$

Although powerful, Cramér's model is undeniably simplistic. It ignores basic arithmetic constraints, such as the fact that all primes except 2 are odd, or that no number divisible by 3 can be prime. Consequently, relying on the model without modification can lead to misleading predictions.

A striking example concerns twin primes—pairs of primes $(p, p + 2)$. While it is widely believed that infinitely many twin primes exist, this remains unproven. Cramér's model predicts that the number of twin primes up to N should be approximately

$$\sum_{n=2}^{N} \frac{1}{\log(n)\log(n+2)} \approx \frac{N}{(\log(N))^2}$$

However, the same reasoning would predict a similar number of *consecutive* primes, which is clearly impossible beyond the pair (2, 3). This illustrates the limitations of the model.

Nevertheless, when refined to account for divisibility by small primes, Cramér's framework yields more accurate predictions. Granville showed that such corrections replace the constant 1 in the strong conjecture with

$2e^{-\gamma} \approx 1.229$, where γ is Euler's constant. Similarly, the refined model predicts that the number of twin primes up to N should be

$$\sim 2C_2 \frac{N}{(\log N)^2}, \quad C_2 = \prod_{\substack{p \text{ is prime} \\ p\geq3}}\left(1 - \frac{1}{(p-1)^2}\right) \approx 0.66016$$

Cramér's model also sheds light on the distribution of primes in short intervals. Under this framework, intervals $[N, N + H)$ with $(\log N)^{2+\delta} < H < N$ typically contain about $H/\log N$ primes. In 1943, assuming the Riemann Hypothesis, A. Selberg showed that this holds for *almost all* such intervals. For many years, it was believed that this behavior should hold uniformly. However, in a groundbreaking result, H. Maier demonstrated in 1985 that there are infinitely many short intervals where the number of primes deviates significantly from this expectation.

Seen in this light, Cramér's model reveals both the power and the limitations of probabilistic reasoning in number theory. It captures the correct scale of prime gaps while falling short of fully encoding arithmetic structure. Its successes and failures together show why such models are best viewed as guiding principles rather than final answers. By exploring where the model works, where it breaks down, and how it can be refined, mathematicians continue to uncover subtle patterns beneath the surface, ensuring that the study of prime numbers remains a vibrant and evolving field.

Integer Partitions in Three Acts: Combinatorics, Number Theory, and Analysis

Koustav Banerjee   (*Postdoctoral Researcher, University of Cologne*)

December 18, 2025

This article explores the mathematics of integer partitions, a simple counting problem with deep consequences. Beginning with Euler's ideas, we trace how the subject evolved through the work of Hardy, Ramanujan, and Rademacher. The story shows how combinatorics, number theory, and analysis come together to reveal the hidden structure behind counting numbers.

Koustav Banerjee *has obtained his doctoral degree from Research Institute of Symbolic Computation (RISC), Johannes Kepler University, Austria under supervision of Prof. Peter Paule. After spending one year and nine months as a postdoctoral fellow under mentorship of Prof. Paule, he has joined Prof. Kathrin Bringmann's group. Since November, 2024, he is working as a post-doc at University of Cologne with Prof. Bringmann.*

The Partition Function: A Combinatorial Genesis

*P*rimarily, there are two ways of decompose a natural number. One way is multiplicative, precisely, factoring a number into primes and other way is additive. In this section, we discuss the additive decomposition in brevity. This additive decomposition is well-known as "integer partitions". In the history of the literature on partitions, Leibniz seems to be the first person who defined integer partitions. In a 1674 letter [12, page 37], he asked J. Bernoulli about the number of "divulsions" of an integer. In modern terminology, "divulsion" is rephrased as the number of partitions of a positive integer. Leibniz observed that there are three partitions of 3 counted by 3, 2 + 1, 1 + 1 + 1, five partitions of 4 counted by 4, 3 + 1, 2 + 2, 2 + 1 + 1, 1 + 1 + 1 + 1, seven partitions of 5 and eleven partitions of 6. This examples lead to a problem which is still open:

Are there infinitely many integers n for which the total number of partitions of n is a prime?

Keeping this question aside, let us move on define the partition function.

A *partition* of a positive integer n is a finite non-increasing sequence of positive integers $\lambda_1, ..., \lambda_r$ such that $\sum_{j=1}^{r} \lambda_j = n$. The λ_j are called the parts of the partition. The partition $(\lambda_1, \lambda_2, ..., \lambda_r)$ is denoted by λ, and we write $\lambda \vdash n$ to denote that λ is a partition of n. The partition function $p(n)$ is the number of partitions of n. The set of all partitions of n is denoted by $P(n)$. Euler undertook a rigorous and systematic investigation of the theory of partitions. Ph. Naudé [6] wrote a letter to Euler asking about the number of partitions of n with the total number of parts in each partition being m. Precisely the question of Naudé was: what is the total number of partitions of 50 into seven distinct parts? It is quite unlikely to get the total number by writing down all the partitions of 50 into seven distinct parts. To avoid this, Euler introduced the concept of generating functions. Let $p_{m(n)}$ denote the number of partitions of n into m parts. Then following Euler's observation, we get

$$\sum_{m,n \geq 0} p_m(n) z^m q^n = \prod_{k=1}^{\infty} \left(1 + z q^k\right)$$

$$= (1 + zq) \prod_{k \geq 1} \left(1 + (zq) q^k\right)$$

$$= (1 + zq) \sum_{m,n \geq 0} p_m(n) z^m q^{m+n}.$$

Comparing the coefficients of $z^m q^n$ on both sides of the above identity, we find the following recursive formula

FIG 1 : *Leonhard Euler (1707–1783), who introduced generating functions and laid the foundational framework for the theory of integer partitions.*

$$p_m(n) = p_m(n - m) + p_{m-1}(n - m),$$

which gives $p_7(50) = 522$. Euler proceeded further to obtain a generating function for $p(n)$. Euler's calculation can be put in the following way:

$$
\begin{aligned}
P(q) &:= \sum_{n \geq 0} p(n) q^n \\
&= \left(1 + q^1 + q^{1+1} + \ldots\right)\left(1 + q^2 + q^{2+2} + \ldots\right) \\
&\quad \left(1 + q^3 + q^{3+3} + \ldots\right)\ldots \\
&= \prod_{n \geq 1}\left(1 + q^n + q^{n+n} + \ldots\right) = \prod_{n \geq 1} \frac{1}{1 - q^n}.
\end{aligned}
$$

Note that, rather than working with sequences, we are now in the regime of function, a switch between discrete to continuous. In order to simplifying (and doing fast) calculations for $p(n)$, Euler realized that a power series expansion for $\prod_{n=1}^{\infty}(1 - q^n)$ is essential. His empirical discovery leads to the following identity which is known as Euler's Pentagonal Number Theorem.

$$\prod_{n \geq 1}(1 - q^n) = \sum_{n \in \mathbb{Z}} (-1)^n q^{\frac{3n^2 - n}{2}}.$$

This identity was proved by Euler himself many years after the discovery. For a modern exposition of Euler's proof, we refer to Andrews' [1]. Putting (1.1) and (1.2) together, we see that

$$\sum_{n \geq 0} p(n) q^n \sum_{n \in \mathbb{Z}} (-1)^n q^{\frac{3n^2 - n}{2}} = 1,$$

and comparing the coefficients of q^n on both sides of the last identity, Euler found the following recurrence for $p(n)$: $p(0) = 1$, and for all $n \geq 1$,

$$p(n) - p(n - 1) - p(n - 2) + p(n - 5) + p(n - 7) - \ldots = 0.$$

After Euler, the theory of partition propagates through the works of Sylvester, Cayley, Jacobi, MacMahon, Hardy,

Ramanujan, Rademacher, Gordon, and Andrews among many others. The reader can consult Andrews' magnum opus [2]. In addition to that, the entire history of partitions up to 1918 is documented in [5], and for a survey article, we refer to [7].

The counting problem for $p(n)$ (for large values of n) has been one of the most predominant themes in the literature on integer partitions. First of all, we point out a simple fact: $(p(n))_{n \geq 1}$ is a strictly increasing sequence. For a partition $\pi \vdash n - 1$, define a map $\phi : P(n - 1) \to P(n)$ by $\phi(\pi) = (\pi, 1)$; i.e., insertion of 1 as part in π that yields a partitions of n and it is clear that ϕ is an injective map and $P(n) \setminus \phi(P(n - 1))$ is the set of all partitions of n where 1 is not a part (also known as non-unitary partitions of n). Let us try to formulate the problem of counting $p(n)$ in terms of counting partitions of n subject to the condition that each partition has at most k parts. Let $p_{\leq k}(n)$ denotes total number of such partitions of n. Observe that for $k = n$, $p_{\leq k}(n) = p(n)$. Cayley [4] and Sylvester [18] gave a number of formulas for $p_{\leq k}(n)$ with small values of k, which was anticipated by Herschel [10]. For example, $p_{\leq 2}(n) = \lfloor \frac{n+1}{2} \rfloor$. Now the question we may ask how far we can compute $p(n)$. Is there any simple formula for counting $p(n)$ apart from (1.3)? Using (1.3), MacMahon computed $p(200)$ by hand and

$$p(200) = 3972999029388.$$

This was a remarkable achievement and of great importance[4]. Now the question is

Problem 1.1. *How fast does $p(n)$ grows?*

To understand the problem and in order to get an answer, we need some artillery from analysis and number theory, precisely, the notion of asymptotics and modular forms.

Preliminaries

Approximations and asymptotics: According to Russell,

All exact science is dominated by the idea of approximation."

Since antiquity, the notions of approximations have played a crucial role in major disciplines of science and philosophy. The formula for approximating the square root of a number is often attributed to the Babylonians. Since then, mathematical formulae were developed to assist in approximating transcendental functions. Probably the first application of theory of approximations was due to Euler who tried to solve a problem of drawing a map of the Russian empire with exact latitudes. After Euler[5], Gauß, Laplace, Fourier, Cauchy, Chebyshev, Lagrange, Poisson, Fejér, Weierstraß, Runge among many others expanded the theory of approximations while working on several problems in different domains of mathematics and physics. Asymptotic analysis is a branch of mathematical analysis that provides a rigorous foundation to understand the language of approximation. Let us start with a well-known asymptotic result, so-called Stirling's formula: $\lim_{n \to \infty} \frac{n!}{\sqrt{2\pi n}\, n^n e^{-n}} = 1$. In the language of

---

[4]We will discuss the importance in the final section.

[5]Among many others, the most celebrated one is perhaps Euler–Maclaurin summation formula.

asymptotics, we say $n! \sim \sqrt{2\pi n}\, n^n e^{-n}$, as $n \to \infty$. Now, the question naturally arises what is the significance of this limit formula when one can easily compute $n!$ with a computer? The point is, as n became larger, we do not know how the function $n!$ really behaves. Thanks to Stirling's approximation, we have now the information that $n!$ has exponential growth, i.e., we perceive the "unknown" function $n!$ in terms of well-known elementary functions. Let us conclude this section with another deep and famous asymptotic formula. Let $x \in \mathbb{R}_{>0}$ and $\pi(x)$ denotes the number of primes not exceeding x. Based on the tables by Felkel and Vega, Legendre conjectured in 1797-1798 that $\lim_{x \to \infty} \frac{\pi(x)}{\left(\frac{x}{A\log x + B}\right)} = 1$, and later in 1808, he proposed that $A = 1$ and $B \approx -1.08366$. The prime number theorem, originally conjectured by Gauß, and independently proved by Hadamard [8] and de la Vallée Poussin [14], states that

$$\pi(x) \sim \frac{x}{\log(x)} \text{ as } x \to \infty.$$

For an elementary proof of the prime number theorem, we refer the reader to Selberg's proof [16].

Modular Forms: We begin with elementary trigonometric functions. Euler discovered that $e^{ix} = \cos x + i \sin x$, where i is an imaginary number (termed by Descartes) satisfying $i^2 = -1$. In 1807, Fourier introduced a series, what is well-known as "Fourier series", for the purpose of solving the heat equation in a metal plate. Roughly we can say that a Fourier series is an infinite sum that represents a periodic function as a sum of sine and cosine functions. Both sine and cosine function are periodic with period 2π, i.e., $\sin(x + 2\pi) = \sin x$ and trivially, $\sin(x + 2\pi k) = \sin x$ for all $k \in \mathbb{Z}$. In group theoretic language, it is equivalent to say that $\sin 2\pi x$ is invariant under the abelian group $(\mathbb{Z}, +)$. Therefore, the question arises which class of functions are invariant under the action of a non-abelian group? Before giving an example of such a class of functions, let us introduce a few preliminary definitions.

Define $\mathbb{H} := \{\tau \in \mathbb{C} : \operatorname{Im} \tau > 0\}$. Let $\operatorname{GL}_2(\mathbb{Z})$ be the set of 2×2 matrices with integer entries and non-zero determinant. The special linear group[6] $\operatorname{SL}_2(\mathbb{Z})$ be a subgroup of $\operatorname{GL}_2(\mathbb{Z})$ with determinant one. The non-abelian group $\operatorname{SL}_2(\mathbb{Z})$ acts on $\mathbb{H}$ in the following way: for $\gamma \in \operatorname{SL}_2(\mathbb{Z})$ and $\tau \in \mathbb{H}$, $\gamma\tau := \frac{a\tau + b}{c\tau + d}$. Let $k \in \frac{1}{2}\mathbb{Z}$ and a holomorphic function $f : \mathbb{H} \to \mathbb{C}$ is called a *modular form* of weight k over $\operatorname{SL}_2(\mathbb{Z})$ if for $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \operatorname{SL}_2(\mathbb{Z})$ and $\tau \in \mathbb{H}$, it satisfies[7] $f(\gamma\tau) := (c\tau + d)^k f(\tau)$ (up to an automorphy factor) along with f is bounded as $v \to \infty$ (with $\tau = u + iv$). Note that if $k = 0$, then f is invariant under the non-abelian group $\operatorname{SL}_2(\mathbb{Z})$.

Note that for $\gamma = T$, we have $f(\tau + 1) = f(\tau)$. From the theory of complex analysis, we know that such a periodic function admits a Fourier expansion and thus $f(\tau) = \sum_{n \geq -m} a_f(n) q^n$, where $q = e^{2\pi i \tau}$ and $a_f(n)$ are called Fourier coefficients. The partition function $p(n)$ are Fourier coefficients of $q^{-\frac{1}{24}}\eta(q)$, where $\eta(\tau)$ is the Dedekind eta function[8], defined by $\eta(\tau) := q^{\frac{1}{24}} \prod_{n \geq 1}(1 - q^n)$.

FIG 2 : *G. H. Hardy (1877–1947), whose analytic approach and development of the Circle Method led to the first asymptotic formula for the partition function $p(n)$.*

For a more brief overview on modular forms, we refer to [3].

In the next section, we will see how these two topics, seemingly different, comes together to address Problem 1.1.

Asymptotic Formula for $p(n)$

In 1918, using the modularity of P, Hardy and Ramanujan [9] developed the Circle Method and found an asymptotic series for $p(n)$. The simplest form of their result reads

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}} \text{ as } n \to \infty.$$

A few years later, Uspensky [19] independently discovered this. In [9, equation (2.11)], Hardy and Ramanujan first proved that there exist $H, K > 0$ such that for $n \in \mathbb{N}$, $\frac{H}{n}e^{2\sqrt{n}} < p(n) < \frac{K}{n}e^{2\sqrt{2n}}$. Thus, the next step is to determine C, where $C = \lim_{n \to \infty} \frac{\log p(n)}{\sqrt{n}}$, which is documented in [9, Section 3]. Next, applying the Cauchy integral formula, we have

$$p(n) = \frac{1}{2\pi i} \int_{\Gamma} \frac{P(q)}{q^{n+1}} dq,$$

where the path Γ encloses the origin and lies entirely inside the unit circle. Truncating (1.1), we observe that $P_N(q) := \prod_{n=1}^{N} \frac{1}{1-q^n}$ has a pole at $q = 1$ of order N, a pole at $q = -1$ of order $\lfloor \frac{N}{2} \rfloor$, poles at $q = e^{\frac{2\pi i}{3}}$ and $q = e^{\frac{4\pi i}{3}}$ of order $\lfloor \frac{N}{3} \rfloor$, and so on. Hardy and Ramanujan defined the following auxiliary function $F(q) := \frac{1}{\pi\sqrt{2}} \sum_{n \geq 1} \Psi(n) q^n$, where $\Psi(n) := \frac{d}{dn}\left(\frac{\cosh C\lambda_n - 1}{\lambda_n}\right)$, $C = \pi\sqrt{\frac{2}{3}}$, and $\lambda_n = \sqrt{n - \frac{1}{24}}$. Now the behaviour of P and F is similar inside the unit circle and in the neighbourhood of $q = 1$. Applying Cauchy's integral formula for $P - F$, they obtain the first term of the asymptotic series

$$p(n) = \frac{1}{2\pi\sqrt{2}} \frac{d}{dn}\left(\frac{e^{C\lambda_n}}{\lambda_n}\right) + O\left(e^{D\sqrt{n}}\right)$$

---

[6]This discrete subgroup is also called the *full modular group.*

[7]This condition is called *Modularity.*

[8]This is a modular form of weight $\frac{1}{2}$

where $D > \frac{c}{2}$. Taking $n \to \infty$, (3.3) gives (3.1). But how close the formula (3.3) with real values of $p(n)$? For example, taking $n \in \{61, 62, 63\}$, (3.3) gives 1121538.672, 1300121.359, 1505535.606, whereas the exact values are 1121505, 1300156, 1505499. So the errors alternate in sign. To explain this factor, the same principle is applied near the point –1 on the unit circle which contributes to the second term in the series for $p(n)$; i.e.,

$$p(n) = \frac{1}{2\pi\sqrt{2}} \frac{d}{dn}\left(\frac{e^{c\lambda_n}}{\lambda_n}\right) + \frac{(-1)^n}{2\pi} \frac{d}{dn}\left(\frac{e^{\frac{c\lambda_n}{2}}}{\lambda_n}\right) + O(e^{D\sqrt{n}}),$$

where $D > \frac{c}{3}$. This process can be continued further by taking into consideration the points on the unit circle where P has singularities. For example, the singularities which are important after $q = -1$ are $q = e^{\frac{2\pi i}{3}}$ and $q = e^{\frac{4\pi i}{3}}$, and so on. The major obstacle to proceeding systematically is to construct the auxiliary functions associated with the points $q = e^{\frac{2\pi i h}{k}}$ of singularity lying on the unit circle. To illustrate this, define $F_{h,k}(q) := \omega_{h,k} \frac{\sqrt{k}}{\pi\sqrt{2}} F_{\frac{c}{k}}(q_{h,k})$, where $\omega_{h,k}$ is a 24th root of unity, $q_{h,k} = qe^{\frac{-2\pi i h}{k}}$, and for a being positive and independent of n, define

$$\Phi(q) := P(q) - \sum_{k=1}^{a\sqrt{n}} \sum_{\substack{1 \le h \le k \\ (h,k)=1}} F_{h,k}(q)$$

If then $F_{h,k}(q) = \sum c_{h,k,n} q^n$, we obtain from Cauchy integral formula,

$$p(n) - \sum_{k=1}^{a\sqrt{n}} \sum_{\substack{1 \le h \le k-1, \\ (h,k)=1}} c_{h,k,n} = \frac{1}{2\pi i} \int_\Gamma dq (\Phi(q))/(q^{n+1}),$$

where Γ is a circle of radius $R < 1$ and its center is the origin. By dissecting the circle Γ by means of Farey series and computing the bounds of the integral on the right–hand side of (3.5), Hardy and Ramanujan finally proved that the error term is of order $O\left(\frac{1}{n^4}\right)$. The final form of their formula for $p(n)$ can be stated as follows.

Theorem 3.1. *There exists an $a \in \mathbb{R}_{>0}$ such that $n \gg 1$,*

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k=1}^{a\sqrt{N}} \sqrt{k} A_k(n) \frac{d}{dn}\left(\frac{e^{\frac{c\lambda_n}{k}}}{\lambda_n}\right) + O\left(n^{-\frac{1}{4}}\right),$$

where

$$A_k(n) := \sum_{\substack{1 \le h < k \\ (h,k)=1}} e^{-\frac{2\pi i n h}{k} + \pi i s(h,k)} \quad \text{and}$$

$$s(h,k) := \sum_{\mu=1}^{k-1}\left(\frac{\mu}{k} - \lfloor\frac{\mu}{k}\rfloor - \frac{1}{2}\right)\left(\frac{h\mu}{k} - \lfloor\frac{h\mu}{k}\rfloor - \frac{1}{2}\right).$$

To know in detail about this collaboration, we refer the reader to [13, 17]. We end this discussion by quoting further two instances for verifying (3.6) with the actual values of $p(n)$. MacMahon[9] computed values of $p(n)$ for $1 \le n \le 200$. The actual values for

$$p(100) = 190569292 \text{ and } p(200) = 3972999029388,$$

whereas if taking the first six terms of (3.6) for $n = 100$ and first eight terms of (3.6) for $n = 200$ gives

$$p(100) \approx 190569291.996 \text{ and } p(200) \approx 3972999029388.004.$$

This proves the accuracy of the formula (3.6). In [9, Section 6, 6.22], they remarked that it remains unanswered whether the infinite series (by extending $n \to \infty$ in (3.6)) is convergent or divergent and if it is convergent, then whether it represents $p(n)$. Lehmer [11] proved that (3.6) is divergent when $N \to \infty$. In the fall of 1936, Rademacher[10] [15] perfected the Hardy–Ramanujan circle method and derived a convergent infinite series for $p(n)$ stated below

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k \ge 1} \sqrt{k} A_k(n) \left[\frac{d}{dx}\left(\frac{\sinh(\frac{\pi}{k}(\frac{2}{3}(x-\frac{1}{24}))^{\frac{1}{2}})}{(x-\frac{1}{24})^{\frac{1}{2}}}\right)\right]_{x=n}.$$

He [15] also proved that if the series (3.7) is truncated after N terms, the absolute value of the error is bounded by

$$\frac{2\pi^2}{9\sqrt{3}N} e^{\frac{\pi}{N+1}\frac{\sqrt{2n}}{3}},$$

which tends to 0 as $N \to \infty$. If we truncate the series (3.7) at N and compare it with (3.6), it clearly shows two significant differences between them:

1. In (3.6), the parameters n and N are entangled whereas in (3.7), we have the complete freedom over n and N.
2. The exponential function in (3.6) is replaced by hyperbolic trigonometric function (precisely $\sinh$) in (3.7) which made the series convergent.

FIG 3 : ***Hans Rademacher*** *(1892–1969), who perfected the Hardy–Ramanujan circle method and obtained a convergent exact series for the partition function $p(n)$.*

---

[9]See the table [9, page 377-378]
[10]Selberg [17, page 705] came up with the same formula Equation (3.7) for $p(n)$ around the same time but never published his result when he came to know that Rademacher already had it.

FIG 4 : *Srinivasa Ramanujan (1887–1920), whose deep insights into partitions and modular forms were central to the asymptotic theory of the partition function p(n).*

So, summarizing the topics covered above, we see that how through a simple counting function $p(n)$, three different domains (in fact many more which is beyond the scope to present in this short exposition) in mathematics come together so that (3.7) comes into existence, like a piece of music where different notes and tones sequentially placed make a harmony to give a birth of ragas or symphonies.

Acknowledgements

References

1. G. Andrews, Euler's pentagonal number theorem, Math. Mag., 56 (1983), 279-284.
2. G. Andrews, The Theory of Partitions, Addison-Wesley, Reading, Mass, 1976; reprinted, Cambridge University Press., 1998.
3. J. Bruinier, G. Harder, G. van der Geer, and D. Zagier, The 1-2-3 of modular forms: lectures at a summer school in Nordfjordeid, Norway (ed. K. Ranestad), Universitext, Springer-Verlag, Berlin-Heidelberg-New York (2008).
4. A. Cayley, Researches on the partition of numbers, Philos. Trans. Roy. Soc. A 146 (1856), 127-140.
5. L. Dickson, History of the theory of numbers, Vol. 2, Carnegie Institution, Washington 1920; reprinted: Chelsea, New York, 1952.
6. L. Euler, Observationes analyticae variae de combinationibus, Comm. Acad. Petrop., 13 (1741-1743, 1751), 64-93.
7. H. Gupta, Partitions: a survey, J. Res. Nat. Bur. Standards, 74B (1970), 1-29.
8. J. Hadamard, Sur la distribution des zéros de la fonction S et ses conséquences arithmétiques, Bulletin de la Societé mathematique de France, 24 (1896), 199-220.
9. G. Hardy and S. Ramanujan, Asymptotic formulæ in combinatory analysis, Proc. Lond. Math. Soc. (3) 2 (1918), 75-115.
10. J. Herschel, On circulating functions, and on the integration of a class of equations of finite differences into which they enter as coefficients, Philos. Trans. Roy. Soc. A 108 (1818), 144-168.
11. D. Lehmer, On the Hardy-Ramanujan series for the partition function, J. Lond. Math. Soc. 1 (1937), 171-176.
12. G. Leibniz, Math. Schriften, Vol. IV (2), Specimen de divulsionibus ae- quationum. (see D. Mahnke, Leibniz auf der Suche nach einer allgemeinen Primzahlgleichung, Bibliotheca Math., 13 (1912-1913)), Letter 3 dated Sept. 2, 1674, 29-61.
13. J. Littlewood, Littlewood's miscellany, Cambridge University Press, 1986.
14. C. de la Vallée Poussin, Recherches analytiques sur la théorie des nombres premiers, Annales de la Société scientifique de Bruxelles, Imprimeur de l'Académie royale de Belgique, (1897).
15. H. Rademacher, On the partition function p(n), Proc. Lond. Math. Soc. (3) 2 (1938), 241-254.
16. A. Selberg, An elementary proof of the prime-number theorem, Ann. Math. 50 (1949), 305-313.
17. A. Selberg, Reflections around the Ramanujan centenary, Resonance 1 (1996), 81-91.
18. J. Sylvester, On subvariants, that is, semi-invariants to binary quantics of an unlimited order, Amer. J. Math. 5 (1882), 79-136.
19. J. Uspensky, Asymptotic formulae for numerical functions which occur in the theory of partitions, Bull. Russ. Acad. Sci. 14 (1920), 199-218.

Guiding Tomorrow's Minds

With
Nayan Bhunia
and
Dibyendu Biswas

interview by
Subham Sarkar

Guiding Tomorrow's Minds: Conversation with Nayan Bhunia and Dibyendu Biswas

Subham Sarkar　　(Department of Biotechnology, St. Xavier's College, Kolkata)

December 13, 2025

In their June 20, 2025 interview, Mr. Nayan Bhunia and Mr. Dibyendu Biswas highlighted the realities of Ph.D. life in India, stressing strong academic foundations, collaboration, multidisciplinary thinking, and resilience. They emphasized **patience**, financial independence, work–life balance, and awareness of narrowing research novelty as key considerations for aspiring researchers.

EDITED BY: Medha Roy Choudhury, Budhaditya Banerjee

Subham Sarkar is a final year Masters student at St. Xavier's College (Kolkata). His research interests converge at the intersection of biomaterials, nanotechnology, and molecular mycology. He is interested in exploring the design and engineering of nanomaterials towards creating innovative platforms to mitigate spread or aggravation of infectious diseases, among other things.

Mr. Nayan Bhunia and Mr. Dibyendu Biswas, research scholars from University of Calcutta, India, in an interview on June 20, 2025, discussed the importance of research and the key considerations students should keep in mind before entering this field. Their expertise revolves around photophysical characterization of lanthanide-doped inorganic nanoparticles and the exploration of their applications in various fields. They, along with their principal investigator, **Dr. Prasun Mukherjee**, have published several research articles communicating their ideas and findings in reputed international journals. They spoke about the things young students should keep in mind before embarking on the long-term commitment of a Ph.D. programme. They candidly discussed the importance of academic knowledge, collaboration, meaningful discussions, proper training, constructive criticism, multidisciplinary applied research, **hard work**, work–life balance, and **time management**. They also highlighted that many research themes are becoming saturated and the scope for novelty is narrowing. They emphasized the importance of **patience** and financial independence during the research journey. Overall, their interview provides a comprehensive insight into Ph.D. life in India.

Nayan Bhunia on Educating for the Future of Research

In an insightful interview, **Mr. Nayan Bhunia**, Senior Research Fellow at the Centre for Research in Nanoscience and Nanotechnology, University of Calcutta, shared his profound perspectives on nurturing a strong scientific ecosystem. Bhunia emphasized the critical shift towards applied and translational research, highlighting its importance in bridging the gap between scientific discovery and real-world solutions. This means moving beyond theoretical exploration to actively developing practical applications and translating laboratory findings into tangible benefits for society, such as new medicines, technologies, or environmental solutions. His message resonated with optimism, offering a clear roadmap for aspiring scientists and the broader research community. Bhunia stressed the fundamental need for researchers to develop the ability to imagine and correlate ideas. This capacity, he explained, isn't just about creativity; it is about making intellectual leaps, connecting seemingly disparate pieces of information, and envisioning novel approaches to problems. This ability to see patterns and relationships where others don't is crucial for fostering innovative thinking and leading to groundbreaking discoveries. He further elaborated on the significance of well-designed projects that deepen understanding and advance knowledge, underscoring that well-designed research initiatives with clear objectives and robust

methodologies are the bedrock of meaningful scientific progress. These aren't just experiments; they're thoughtfully constructed investigations that expand our knowledge base.

A key theme in *Bhunia's* discussion was the undeniable power of **collaboration**. He advocated for the importance of **collaboration** with leading labs, communication with experts, and networking. This isn't just about being friendly; it's about actively seeking out partnerships with leading research groups, engaging in regular dialogues with established experts in one's field, and building a wide network of contacts. These interactions, he believes, are vital for knowledge exchange, accessing diverse perspectives, sharing resources, and significantly accelerating scientific advancements. Complementing this, he highlighted the importance of training and internships, providing invaluable practical experience and exposure to real-world research environments. These hands-on opportunities are crucial for developing essential lab skills and understanding the day-to-day realities of scientific work. *Bhunia* also underscored the non-negotiable habit of reading quality scientific literature, asserting that a strong, up-to-date foundation in existing scientific knowledge, gleaned from reputable journals and publications, is paramount for any researcher to build upon. A poignant observation from Bhunia was the perceived scarcity of genuinely passionate and knowledgeable people within
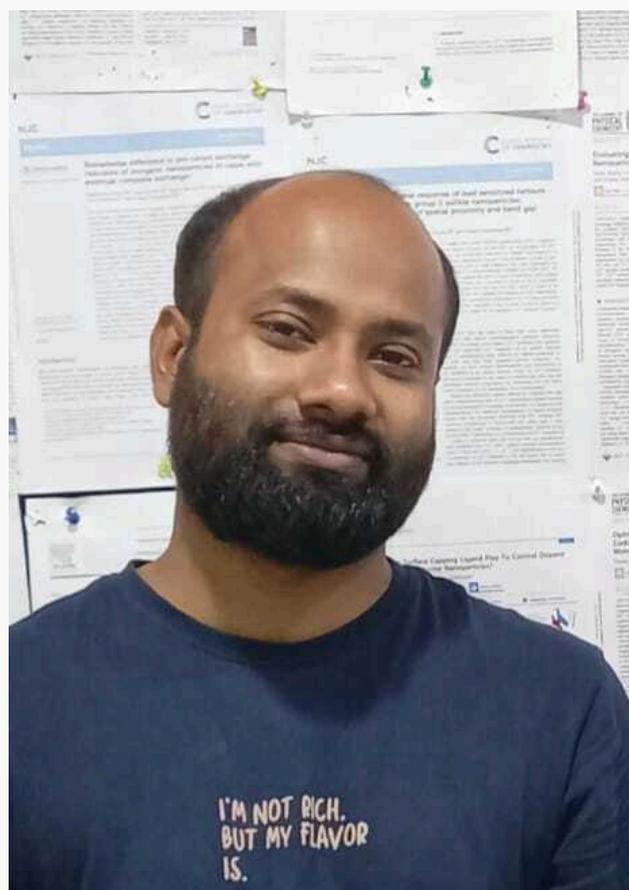

FIG 1 : *Mr. Nayan Bhunia. His research focuses on the electronic spectroscopic investigation of lanthanide containing co-doped inorganic nanoparticles for multiplex assays. He also investigates inter-dopant electronic interactions as a function of dopant's relative concentration and excitation energy.*

FIG 2 : *These seven points outline the key factors students should carefully evaluate before pursuing a Ph.D.. They reflect the academic, personal, and practical realities of research life. The insights are drawn from the experiences shared by Nayan Bhunia.*

the research and scientific community. He lamented this deficit, emphasizing that true passion, a deep intrinsic drive to understand and discover, and combined with a profound understanding of one's subject matter, is indispensable for driving innovation and maintaining the integrity and quality of scientific inquiry. Without this core enthusiasm and expertise, research can become stagnant or lack true impact.

For students embarking on their scientific journey, Bhunia offered practical and profound advice. He urged them to prioritize academic merit, meaning striving for excellence in their studies and developing a strong theoretical foundation. Equally important, he stressed the need to exchange knowledge with their peers and mentors, fostering an environment of mutual learning and intellectual growth. Crucially, he advised aspiring scientists to stay away from negative and discouraging people, recognizing the detrimental impact of negativity, skepticism, or a lack of understanding on intellectual curiosity, perseverance, and overall well-being. Surrounding oneself with supportive and intellectually stimulating individuals is key. In a forward-looking statement, Bhunia asserted that modern chemical research should not proceed alone, instead should combine with biological, mathematical, and physical understanding. This **interdisciplinary** approach, he believes, is essential for tackling complex scientific challenges that often transcend traditional disciplinary boundaries. For example, understanding a new drug's effect requires not only chemistry but also biological insights into its interaction with living systems, mathematical modelling of its kinetics, and physical principles governing its structure. This holistic perspective leads to more comprehensive and effective solutions. Prospective Ph.D. candidates, according to Bhunia, must ensure their basic knowledge is clear and rationalize their thinking. This means having a firm grasp of fundamental scientific principles and being able to apply logical reasoning to complex problems, rather than relying on guesswork or intuition. He passionately advocated for the importance of critical investigation, good observation skills, and professional and polite conversation to communicate ideas and results, instead of a confrontational attitude. This emphasizes the need for researchers to meticulously analyze data, notice

subtle details, and present their findings in a respectful, clear, and constructive manner. The goal is to **collaborate** and build upon ideas, not to confront or undermine.

Finally, Bhunia touched upon the practical realities of research funding. He stated that research funding agencies want productive outcomes, and impressing them is an important responsibility. This highlights the need for researchers to not only conduct excellent science but also effectively articulate its potential impact, demonstrate its feasibility, and clearly communicate its value to secure the necessary financial support. It's about demonstrating both scientific rigor and strategic vision. Beyond the technicalities of research, Bhunia provided invaluable insights into the less discussed but equally crucial aspects of the scientific profession: lab culture and workplace culture. He strongly suggested that individuals aspiring to join a lab should do a thorough background check – talking to current and former lab members, observing the lab environment, and assessing the principal investigator's mentorship style – to ensure the environment aligns with their values and professional goals. This foresight can significantly impact a researcher's productivity, mental wellbeing, and overall career trajectory.

Bhunia said, *"As a researcher, I prefer the students who are willing to pursue a Ph.D., they should be extremely passionate, have a good academic record, and most importantly, should be hardworking, focused and collaborative."* *"You should never forget your roots and reading is absolutely necessary,"* he added. Mr. Bhunia's comprehensive and optimistic message serves as a beacon for the scientific community, urging a collective effort towards fostering a culture of excellence, collaboration, and genuine passion. His insights are invaluable for shaping the next generation of scientists and driving impactful research that benefits society.

Dibyendu Biswas on Navigating Research Life in India

In an era defined by rapid scientific advancement, the allure of a career in research beckons to many, yet the pathway remains a mystery to most. To demystify this intricate world, an exclusive and in-depth interview was conducted with Mr. Dibyendu Biswas, a distinguished Senior Research Fellow at the Centre for Research in Nanoscience and Nanotechnology, University of Calcutta. Mr. Biswas, a researcher in the realm of groundbreaking scientific inquiry, offered an insightful discussion on various facets of research, providing valuable guidance for those contemplating a journey into the uncharted territories of discovery.

The interview, held amidst the stimulating environment of the Centre, began with a thorough exploration of the research environment itself. Mr. Biswas described it not merely as a physical space, but as a dynamic ecosystem demanding intellectual curiosity, collaboration, and an inherent drive to solve complex problems. *"A vibrant research environment is one where ideas are challenged, hypotheses are rigorously tested, and failures are seen not as setbacks, but as crucial learning opportunities,"* he explained, emphasizing the importance of a supportive and intellectually stimulating atmosphere for fostering innovation. He highlighted that a truly productive research setting encourages open

discussion, peer review, and a constant thirst for new knowledge, transforming it into a fertile environment for breakthroughs.

Moving on to the essential qualities of budding scientists, Mr. Biswas meticulously outlined the prerequisites of research students. Beyond strong academic foundations, he stressed the paramount importance of an inquisitive mind, an innate sense of curiosity, and an insatiable desire to understand "why" and "how." "A good researcher," he asserted, "is not just someone who knows facts, but someone who relentlessly questions them." He elaborated on the need for critical thinking, problem-solving skills, and a high degree of perseverance. *"The path of research is often fraught with dead ends and unexpected challenges. It's the ability to pick yourself up, learn from the experience, and try again, that truly defines a successful research student."* This resilience, he noted, is often coupled with meticulous attention to detail and an ethical compass to guide their investigations.

The conversation then navigated the vast scope of research, with Mr. Biswas emphasizing its profound impact across every sector of modern life. From healthcare and energy to environmental sustainability and advanced materials, research serves as the engine of progress. He passionately conveyed that research isn't just an academic pursuit but a powerful tool for societal transformation. To succeed in this expansive field, he underscored the critical significance of focus, **hard work**, and the determination required of aspirants. *"Talent is certainly a gift,"* he acknowledged, *"but it's the relentless grind, the countless hours spent in the lab, and the unwavering dedication to a problem that ultimately yields results."* He encouraged aspiring researchers to cultivate a deep sense of purpose, to be disciplined in their approach, and to embrace the often-long and arduous journey towards scientific discovery.

A core theme woven throughout the discussion was the profound gratification derived from being a part of the scientific community and contributing through meaningful and impactful research. Mr. Biswas spoke

FIG 3 : *Mr. Dibyendu Biswas. His research encompasses surface ligand engineering, photophysical characterization and applications of quantum dots, and metal and lanthanide doped semiconductor / perovskite nanocrystals. He also investigates surface chemistry, exciton dynamics, and optoelectronics of doped nanomaterials.*

with palpable passion about the shared mission of researchers to advance human knowledge and address pressing global challenges. He emphasized the ethical responsibility that comes with scientific inquiry – the duty to ensure that research serves the greater good and contributes to a more sustainable and equitable world. "*Our work should not exist in an ivory tower,*" he declared. "*It should be a bridge between fundamental understanding and tangible solutions that improve lives.*" This sense of collective endeavor and the potential for real-world impact, he believes, is one of the most rewarding aspects of a research career.

Mr. Biswas offered a perspective on scientific inquiry. While echoing the sentiment for an **interdisciplinary** approach in research, a crucial theme also highlighted by his colleague Mr. Nayan Bhunia, Biswas delved deeper into the personal journey of a researcher, emphasizing strategic choices and individual development. Biswas began by reinforcing the vital role of **interdisciplinarity**, explaining that the most impactful research today seldom confines itself to a single field. He stressed that properly defining the scope and research question for such cross-cutting projects is paramount. This initial clarity helps in effectively integrating diverse knowledge sets, whether combining chemistry with biology, or physics with materials science, to address complex, real-world problems. He then shifted focus to the crucial personal choices that shape a scientific career. Biswas underscored the choice of the right

FIG 5 : *Research is a structured yet evolving journey that begins with curiosity and develops through academic grounding, practical training, and continuous critical evaluation. Progress emerges through persistence, ultimately leading to contributions that extend beyond the laboratory and into society as suggested by Mr. Biswas*

guide as a foundational decision, recognizing that a good mentor is instrumental in shaping a researcher's intellectual journey and practical skills. He also urged aspiring scientists to prioritize the identification of unexplored areas rather than simply chasing saturated fields. This advice encourages originality and the pursuit of truly novel contributions, moving beyond incremental advancements in crowded research landscapes

For success in these often-uncharted territories, Biswas highlighted the importance of learning and optimizing. This isn't just about acquiring new knowledge, but about refining experimental techniques, computational skills, and analytical methods to achieve the best possible results. He stressed that **hard work** remains an indispensable ingredient for scientific progress, coupled with astute **time management** to navigate the demanding nature of research. Beyond the technical skills, Biswas emphasized understanding one's own scope and capabilities as an individual. This self-awareness helps researchers align their aspirations with their strengths, leading to greater efficiency and, ultimately, more profound research satisfaction. The joy of discovery, he implied, is deeply tied to working within one's capabilities.

To students embarking on their research careers, Biswas offered a trove of practical wisdom. He suggested they be flexible in terms of research aspirations, understanding that initial interests might evolve as they gain more experience and encounter new opportunities. He strongly advised **reducing distractions** and working wholeheartedly to achieve their aims, recognizing that focus and dedication are key to overcoming challenges inherent in scientific exploration. Perhaps most importantly, Biswas encouraged students to take criticisms as a means to improve oneself and become successful. He framed critique not as an attack, but as invaluable feedback essential for growth and refinement, turning potential setbacks into stepping stones toward achievement.

FIG 4 : *The University of Calcutta, where Nayan Bhunia and Dibyendu Biswas are carrying out their doctoral research at the Centre for Research in Nanoscience and Nanotechnology. Their work spans photophysical characterization, surface chemistry, and the design of advanced inorganic and lanthanide-doped nanomaterials with emerging technological applications.*

The practical aspects of scientific training were then brought to the fore, with Mr. Biswas ardently championing the importance of practical knowledge

and training. He argued that theoretical understanding, while foundational, must be complemented by extensive hands-on experience. "*Science is not just about equations and theories; it's about experimentation, observation, and the meticulous execution of protocols,*" he explained. He stressed the value of internships, laboratory work, and collaborative projects in developing the essential skills needed for real-world research. This practical grounding, he added, is intimately linked to the crucial decision of the wise selection of a specialization. He urged aspiring researchers to engage in self-reflection and thoroughly explore various sub-disciplines before committing to a specific area. "*Don't just follow the crowd or the perceived 'hot' topic,*" he advised. "*Choose a field that genuinely excites you, one where your curiosity is boundless, as that passion will sustain you through the inevitable challenges.*"

While the demands of research are undeniably high, Mr. Biswas provided a refreshing and pragmatic perspective on the necessity of maintaining work-life balance. He acknowledged the long hours and intense focus often required but cautioned against the dangers of burnout. "*A sustainable research career is one that allows for personal well-being, hobbies, and time with loved ones,*" he asserted. He highlighted that stepping away from the lab periodically can often lead to new perspectives and a renewed vigor for research. This emphasis on balance flowed seamlessly into his counsel to wholeheartedly embrace one's passion. He firmly believes that genuine enthusiasm for a subject is the most potent fuel for innovation and resilience. "*If you are truly passionate about what you do, the challenges become less daunting, and the pursuit of knowledge becomes an exhilarating adventure,*" he opined.

Mr. Biswas then offered a candid and balanced assessment of the pros and cons of being a research fellow. On the positive side, he reiterated the intellectual stimulation, the thrill of discovery, the opportunity to contribute to human knowledge, and the potential for significant personal and professional growth. The intellectual freedom to explore intriguing questions and the satisfaction of uncovering new insights were among the key benefits he listed. However, he was equally forthright about the drawbacks. These can include demanding work schedules, the pressure to publish frequently, the competitive nature of funding, and the often-protracted timelines required to achieve significant breakthroughs. "It's not always attractive," he conceded, "*there are long hours, failed experiments, and*

moments of frustration. But the highs, when they come, are rewarding".

Concluding the insightful discussion, Mr. Biswas provided a captivating glimpse into the future prospects of nanotechnology research, with a particular emphasis on inorganic nanoparticles and photoluminescence. He articulated the immense potential of these areas to revolutionize various sectors. From developing highly efficient solar cells and advanced biomedical imaging techniques to creating novel drug delivery systems and environmental sensors, the applications are seemingly endless. He spoke enthusiastically about the capacity of inorganic nanoparticles to exhibit unique optical properties through photoluminescence, paving the way for innovations in quantum dots, biosensing, and next-generation display technologies. He stressed the global impact of such advancements, predicting that breakthroughs in nanotechnology will undoubtedly lead to transformative solutions for some of humanity's most pressing challenges, from sustainable energy to disease eradication, fostering a healthier and more technologically advanced future worldwide.

Overall, Mr. Dibyendu Biswas's interview was far more than a mere report on scientific advancements; it was a profound invitation to introspection. His words served as an invaluable guide for anyone contemplating a research career, prompting readers to deeply identify and reflect on their research interests. More fundamentally, he challenged them to honestly assess whether they possess the innate drive, the relentless curiosity, and the enduring dedication required to truly be a part of the research community. His comprehensive insights underscored that while the path of research is demanding, for those who embrace its challenges with passion, determination, and perseverance, it offers an unparalleled journey of discovery, contribution, and ultimately, a chance to leave a lasting mark on the tapestry of human knowledge.

Acknowledgement

FIG 6: *This illustration shows the key elements that support scientific research. Academic knowledge, collaboration, training, and perseverance work together. Research progress comes from their combined effort, not from a single factor.*

How Magnetism Defies Classical Mechanics
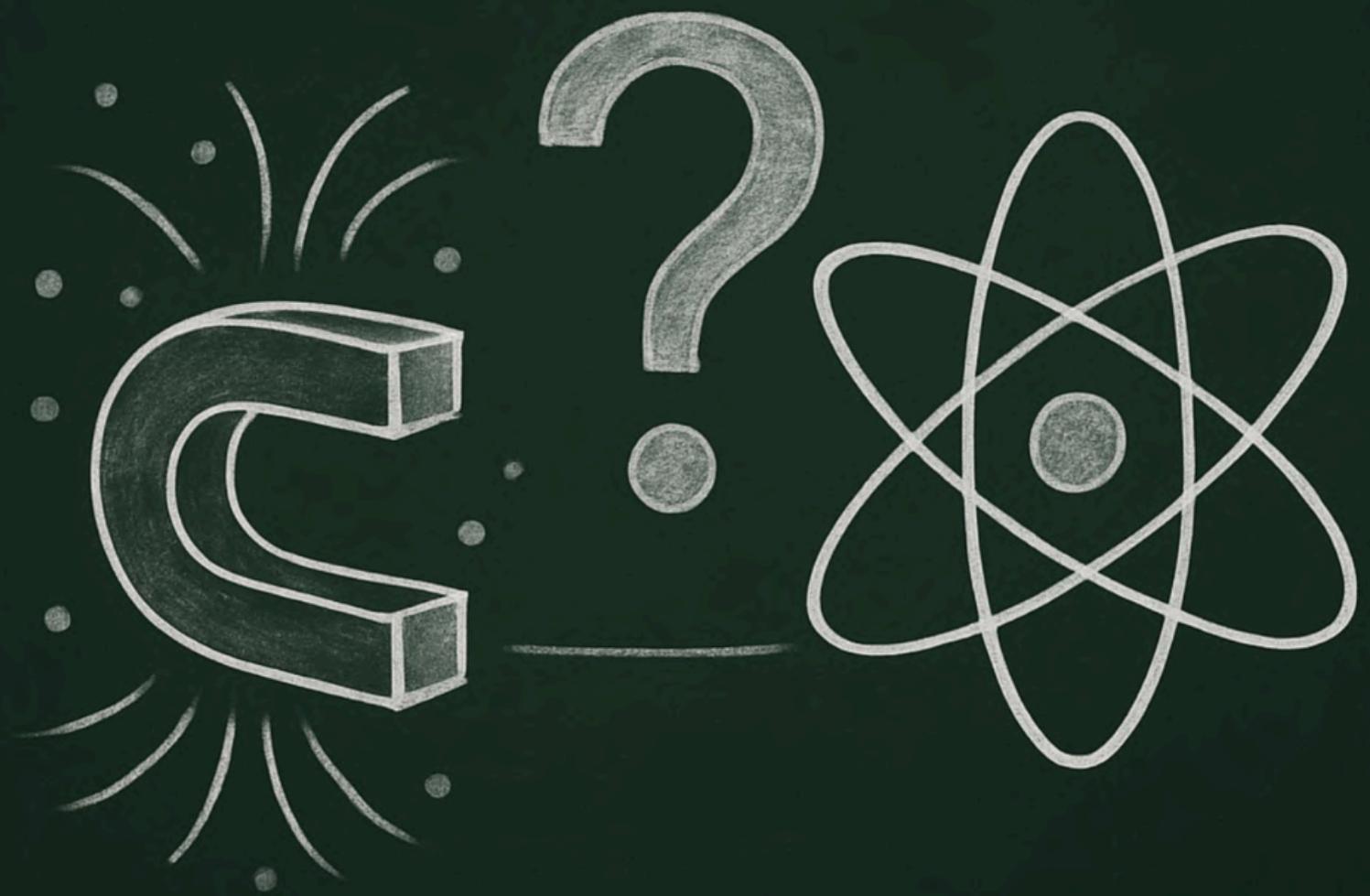
__ Raj Gaurav Tripathi
(IISER Kolkata)

How Magnetism Defies Classical Mechanics

Raj Gaurav Tripathi   (*Department of Physical Sciences, IISER Kolkata*)

June 10, 2025

While the ultraviolet catastrophe is famously credited for the birth of quantum mechanics, the failure of classical physics to explain simple magnetism provided an equally vital, yet often overlooked, confirmation. This article explores the Bohr-Van Leeuwen theorem, which mathematically demonstrated that in a purely classical universe, magnets simply cannot exist. Discover how this fundamental paradox drove Niels Bohr to cement the foundations of the quantum revolution.

EDITED BY: Debanuj Chatterjee

Hey! I am Raj, a 4th year physics major at IISER Kolkata. I have worked in spintronics and optics, and am currently working in theoretical cosmology. My interests range from massless photons to supermassive black holes, particularly everything:)

Introduction

The early 20th century was a major turning point in physics, often considered the birth of modern physics. It marked a fundamental shift away from the classical worldview that had dominated since Newton. Classical physics (Newtonian mechanics, Maxwell's electromagnetism, and thermodynamics) worked well for everyday phenomena, but it failed at extreme conditions: "Couldn't explain the behavior of light and matter at atomic scales"! One such roadblock was faced by none other than Niels Bohr in 1911. Bohr found that classical electron theory was fundamentally inadequate to explain observed magnetic properties, such as the Hall effect(The development of voltage across a current-carrying conductor placed in a perpendicular magnetic field). Through his rigorous application of classical principles, he demonstrated that, in thermal equilibrium, the net magnetization of a collection of electrons would always vanish. This implied that phenomena like paramagnetism, diamagnetism, and ferromagnetism could not be explained by classical physics alone(10)! Isn't it fascinating that a seemingly classical, age-old phenomenon like magnetism could actually have a fundamentally nonclassical explanation (Fig. 1)?

Eight years later, Hendrika Johanna van Leeuwen independently derived the same theorem in her 1919 doctoral thesis, thus the result is now formally known as the Bohr-Van Leeuwen theorem (4). Notably, she was unaware of Bohr's prior discovery. Her research similarly concluded that classical physics and statistical mechanics could not account for the existence of magnetism, emphasizing its quantum mechanical nature.

This independent convergence on a critical result powerfully validates the idea, thereby compelling the scientific community to seriously consider and develop alternative, non-classical frameworks.

The Langevin Function: Accidental Quantization

When classical physics fell short, Niels Bohr sparked a revolution in 1913 by proposing that electrons don't whirl randomly around the nucleus — they jump between fixed orbits like steps on an invisible ladder. These quantized orbits gave atoms their stability and hinted at the origin of magnetic moments. Arnold Sommerfeld later in 1916 added elegance to Bohr's sketch, allowing electrons to follow elliptical orbits and even move relativistically!! (11)

Given that classical physics forbids spontaneous magnetization in equilibrium, one might wonder how magnetization was nevertheless described successfully for decades? Where did the ignorance sweep in? Let's see an example. Paul Langevin's 1905 theory

FIG 1 : *Isn't this magnet big enough to obey classical mechanics?*

of paramagnetism(weak attraction of materials to magnetic field due to unpaired electrons aligning with it), embodied by the Langevin function, is historically regarded as a classical approach. It was developed before the widespread acceptance of quantum physics and relied solely on classical concepts like Boltzmann distribution for alignment in an external magnetic field. But how did he get paramagnetism classically?

The "borderland" nature of the Langevin function arises from Langevin's implicit "quantization" of the system. As J. H. Van Vleck famously observed, "When Langevin assumed that the magnetic moment of the atom or molecule had a fixed value, he was quantizing the system without realizing it (1)". This assumption of a fixed, discrete magnetic dipole moment, rather than a continuously variable one, aligns with a quantum mechanical perspective where certain physical properties are quantized. This "accidental quantization" in Langevin's work is a fascinating historical and conceptual point. It suggests that the empirical observations of magnetism were already subtly hinting at the discrete, quantized nature of physical properties, even before the formal development of quantum mechanics. Interestingly, in that same year (1905), Einstein introduced the concept of the photon through his paper on the photoelectric effect, marking the true beginning of quantum theory.

Formal Statement

Theorem (Bohr-Van Leeuwen). At any finite temperature, and in all finite applied electric or magnetic fields, the net magnetization of a collection of electrons in thermal equilibrium vanishes identically.

$$\langle \mu_{\text{tot}} \rangle = \frac{e}{2c} \Sigma_i \langle r_i \times v_i \rangle = 0,$$

where
- $\langle \mu_{\text{tot}} \rangle$ – The classical statistical average magnetic moment.
- e – Electron charge (magnitude $\approx 1.6 \times 10^{-19}$ C)
- c – Speed of light (appears in Gaussian/CGS units).
- I – Index for different particles.
- r – Position vector of the electron relative to the nucleus.
- v – Velocity vector of the electron.

The Bohr−van Leeuwen theorem (Bohr, van Leeuwen) shows that if one applies classical mechanics together with classical statistical mechanics to an ensemble of charged point particles (using the usual Hamiltonian with kinetic energy and Coulomb/Lorentz forces and Maxwell−Boltzmann statistics), the thermal-average magnetization vanishes. In other words, classical point-particle statistical mechanics — without any additional structure — cannot produce the orbital contributions to diamagnetism or paramagnetism, that is, the magnetic effects arising from the orbital motion of electrons around nuclei, which in quantum mechanics manifest as diamagnetic and paramagnetic responses. By contrast, phenomenological classical models that assume pre-existing magnetic moments (for example, fixed magnetic dipoles or rigid extended current loops, or models built from a continuous/extended charge distribution) are not within the
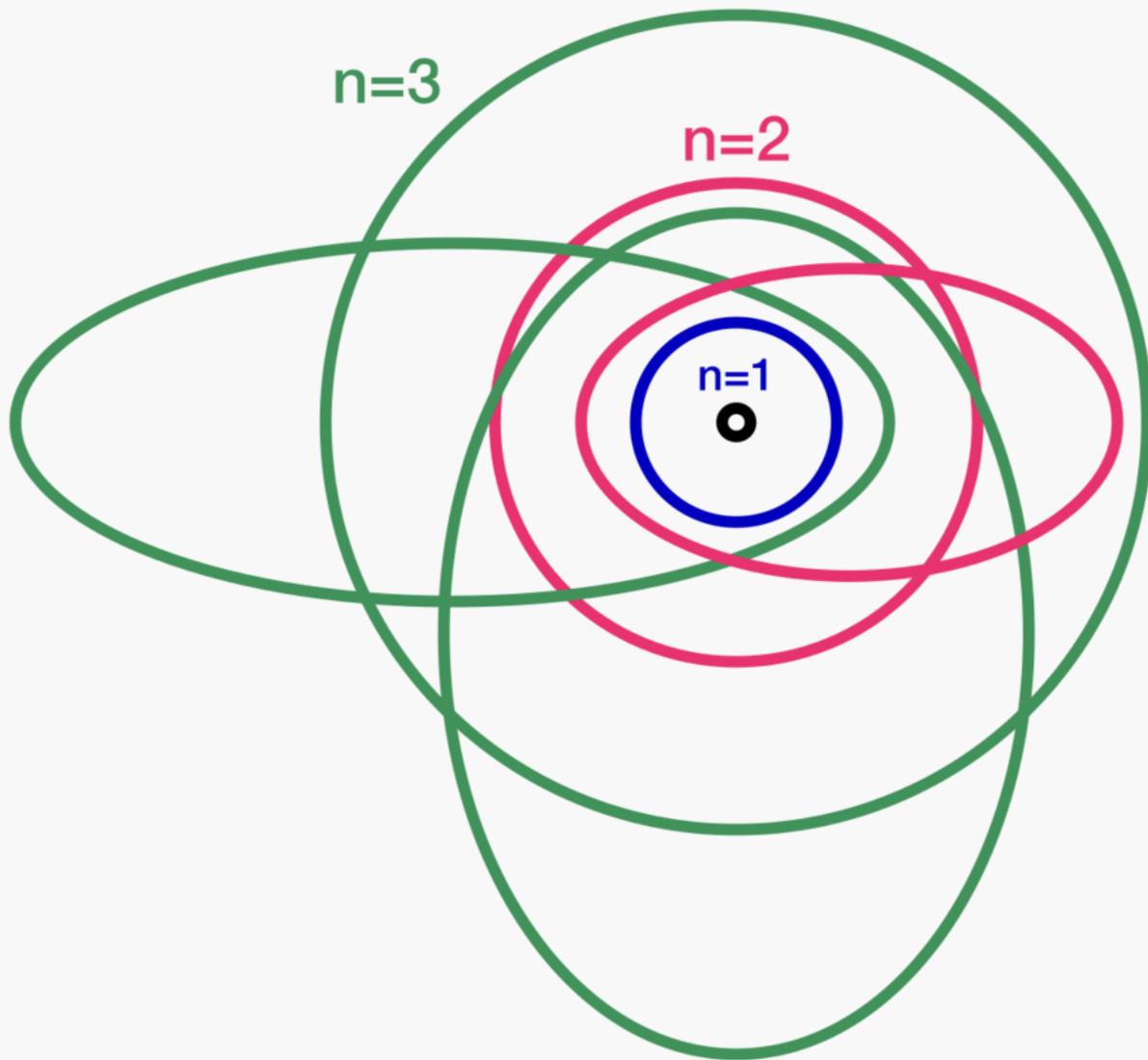
FIG 2 : *The Sommerfeld extensions of the 1913 solar system Bohr model of the hydrogen atom showing the addition of elliptical orbits to explain spectral fine structure.*

strict assumptions of the theorem: they start by postulating a magnetic moment or current distribution and therefore can yield nonzero magnetization and reproduce results such as the Langevin susceptibility (Curie's law) for an ensemble of permanent dipoles. (27) The crucial distinction is therefore one of assumptions. The Bohr–van Leeuwen result does not say "classical physics cannot describe any magnetic phenomena"; rather it shows that classical point-particle Hamiltonian mechanics + classical statistical mechanics, taken literally, cannot derive magnetization — you must either introduce extra (and effectively quantum) ingredients (intrinsic magnetic moments, quantized angular momentum, exchange interactions) or relax the theorem's assumptions (e.g., allow rigid extended currents, constraints, or non-equilibrium/rotating systems). (10)

Hendrika Johanna van Leeuwen

3 July 1887, The Hague (Netherlands) – 26 February 1974, Delft (Nether lands) Daughter of Professors Pieter Eliza van Leeuwen and Maria Wilhelmina Schepman, studied secondary education in her hometown, and con -tinued her training in physics at the University of Leiden, under the supervision of Hendrik Antoon Lorentz. In 1919.(2) She obtained her PhD, her thesis entitled "Problems of the electronic theory of magnetism" focused on explaining why magnetism is essentially a quantum mechanical effect (a result now known as the Bohr-van Leeuwen theorem). In this respect, Van Leeuwen was unaware of the results presented 8 years earlier by Niels Bohr also in his doctoral thesis (4). Fig 3. Hendrika Johanna van Leeuwen (1887-1974). (3)

Intuitive Explanation

We will see a non-formal explanation here, it begins by considering the system's energy distribution as predicted by Maxwell-Boltzmann statistics, which is proportional to $\exp\left(-\frac{U}{k_BT}\right)$, where U is the total energy (kinetic plus potential). A crucial step is recognizing that the magnetic field (B) does not contribute to the system's potential energy. This is because the Lorentz force ($F = q(E + v \times B)$), while dependent on the magnetic field, does no work on charged particles ($F \cdot v = qE \cdot v$, which is independent of B).

Consequently, the total energy of the system remains independent of the magnetic field, which in turn implies that the statistical distribution of particle motions is also unaffected by the magnetic field. In the absence of an external magnetic field, and given the constraint that the system cannot rotate, there is no net ordered motion of charged particles, resulting in an average magnetic moment of zero. Since the distribution of motions and thus the energy are independent of the magnetic field, the average magnetic moment, being zero in a zero field, must remain zero in any applied magnetic field.

Assumptions

1. **Isolated System**: The system must be isolated and explicitly constrained from rotation. If the system is allowed to rotate in response to an external magnetic field, the theorem's conclusion of zero magnetization does not necessarily hold.
2. **Thermal Equilibrium**: The system must be in a single state of thermal equilibrium at a given temperature and magnetic field, and sufficient time must be allowed for it to reach this equilibrium after a field is applied.
3. **Position-Dependent Interactions**: A critical assumption in the standard formulation is that charged point particles interact solely via position-dependent potential energy. This is a key point of contention in contemporary analyses.
4. **Maxwell-Boltzmann Statistics**: The proof fundamentally relies on the principles of Maxwell-Boltzmann statistics to describe the probability distribution of states within the classical ensemble.
5. **Infinite Phase Volume**: The theorem's reliance on idealizations, such as infinite phase volume(volume in the multi-dimensional phase space, which is a space where each axis represents a coordinate and a momentum for a degree of freedom of a system) for the momenta, is crucial. This defines the exact boundaries of the theorem's applicability.

Critiques of the Proof

While the theorem's summary as "no classical magnetic susceptibility, in particular no diamagnetism(magnetism in which a material is repelled by an external magnetic field)" is widely circulated, some researchers consider this statement "seriously misleading". A primary critique centers on the theorem's assumption of only position-dependent interactions between charged particles, arguing that this is not a universal requirement of classical physics. Since Charles Galton Darwin's work in 1920, it has been understood that the accurate description of magnetism arising from classical charged point particles necessitates the inclusion of velocity-dependent interactions within the Lagrangian. When this crucial assumption is relaxed, classical diamagnetism can indeed be derived. Recent experimental observations, such as perfect conductors exhibiting classical perfect diamagnetism (akin to the Meissner effect in superconductors (7)), appear to contradict the strict Bohr-Van Leeuwen conclusion (5). However, these phenomena can be consistently explained within the Darwin formalism, which accounts for velocity-dependent interactions and is applicable to dissipationless systems. Another significant nuance concerns the proof's reliance on infinite limits in the phase space integrals. For systems with finite phase volume, a non-zero classical diamagnetic orbital moment can be derived. This suggests that the theorem's strict zero-magnetization conclusion is an idealization applicable to infinitely extended classical systems.

Beyond Magnetism: Connection to Triboelectricity

Classical Inability

Just as the Bohr–van Leeuwen theorem demonstrates that classical physics cannot account for net magnetization in thermal equilibrium, it likewise fails to explain the persistent surface charges observed in triboelectric systems. Thus, the implications of the theorem extend beyond magnetism, revealing a deeper limitation of classical physics—its inability to describe stable charge separation. Although triboelectric charging is often classically attributed to electron transfer between materials with different electron affinities during contact or friction, the theorem underscores a broader principle: within classical statistical mechanics, equilibrium systems of charged particles cannot sustain either net charge separation or persistent current distributions. (13)

The key feature of triboelectricity is not only the transfer of charge but also the persistence of that charge on surfaces—charges remain localized for long periods instead of randomizing. According to classical theory, such stable charge separation should vanish, just as the Bohr–Van Leeuwen theorem predicts zero net magnetization in thermal equilibrium.

Fig.4.

This indirect connection of the Bohr-Van Leeuwen theorem to the classical inability to explain triboelectricity suggests that its implications are broader than just magnetic moments. It implies a more general

FIG 3 : *Hendrika Johanna van Leeuwen (1887-1974)*

classical failing: that classical statistical mechanics, when applied to microscopic charged systems in thermal equilibrium, cannot spontaneously generate or maintain any net macroscopic effect arising from ordered charge motion (magnetism) or stable charge separation/retention (triboelectricity). Both phenomena require non-random, stable configurations of charge that classical thermal equilibrium, by its nature, tends to randomize and dissipate. Therefore, the theorem is not solely about magnetism but underscores the fundamental inadequacy of classical equilibrium thermodynamics to explain any persistent, ordered microscopic electrical or magnetic phenomena without the explicit inclusion of quantum effects.

Application in Plasma Physics

The Bohr-Van Leeuwen theorem finds significant utility in plasma physics (6), particularly in understanding diamagnetism within plasma elements (10). Discussions of the theorem in this context often reference Niels Bohr's classical model of a gas of charged particles enclosed within a perfectly reflecting container. In this model, the walls reflect particles elastically, ensuring that every orbital current induced by the magnetic field is exactly balanced by an opposing current upon reflection. This symmetry leads to complete cancellation of net magnetization from the interior of the plasma element, resulting in zero net diamagnetism under equilibrium conditions. However, it is crucial to distinguish that diamagnetism of a purely classical nature can indeed occur in plasmas, but this is typically a consequence of thermal disequilibrium, such as a gradient in plasma density (9). This highlights a key boundary condition for the theorem, as it strictly applies to systems in thermal equilibrium. In classical kinetic equilibrium, the theorem
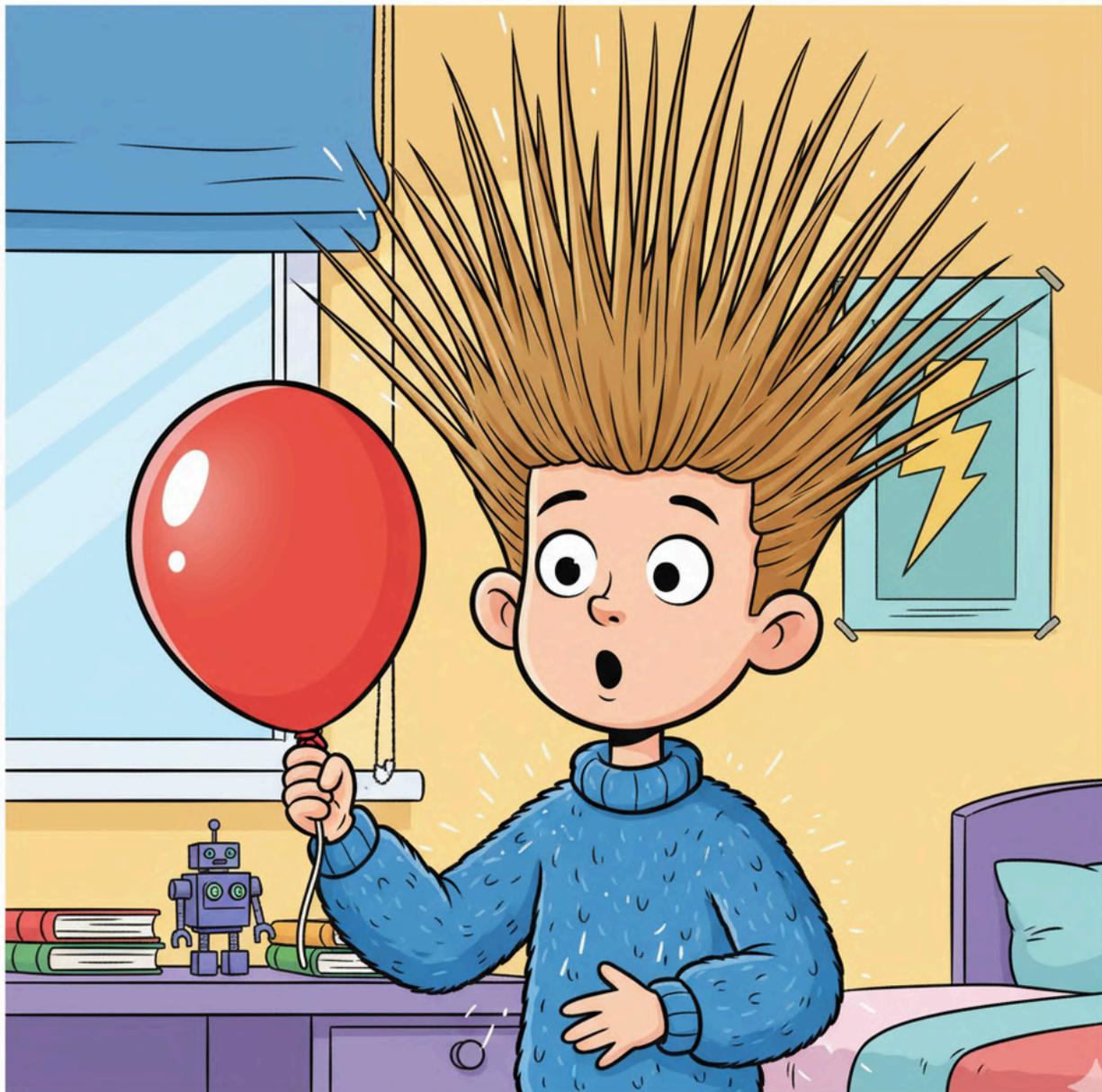

FIG 4 : *The triboelectric effect causes hair to become charged when rubbed against another material.*

implies that diamagnetic currents within the interior of a plasma element are identically zero.

Conclusion

The Bohr-Van Leeuwen theorem's "no-go" conclusion for classical magnetism profoundly impacted the trajectory of physics, propelling the development and widespread acceptance of quantum theory. This doesn't just state a limitation; it effectively functions as a "proof by contradiction" . If the established, well-tested framework rigorously leads to a result that contradicts observation (zero magnetization), then a new, fundamentally different framework must be required to explain the observed phenomena. Classical theories also couldn't account for why blackbodies didn't radiate infinite energy (the ultraviolet catastrophe), why light could eject electrons only above a certain frequency (the photoelectric effect), or why atoms emitted light at discrete wavelengths instead of continuous spectra. Models like Bohr's quantized orbits and Sommerfeld's refinements hinted that energy and motion inside atoms were not continuous but quantized. As more puzzles appeared — from atomic stability to specific heats and magnetism — scientists realized that a completely new framework was needed, leading to the birth of quantum mechanics: a theory where nature operates not in smooth curves, but in tiny, indivisible steps.

References

1. Vleck, J.H. Van. QUANTUM MECHANICS THE KEY TO UNDERSTANDING MAGNETISM. 1997
2. 3 of our "Women of Quantum" series: Theoretical Physicist Professor Hendrika Johanna van Leeuwen
3. Hendrika van Leeuwen, buitenstaander in de natuurkunde - NRC
4. Blaauboer, Miriam. H. Johanna van Leeuwen, the Other Scientist behind the Bohr–Van Leeuwen Theorem. Cambridge, Cambridge University Press, 2025
5. Fiolhais, Miguel C.N. Electrodynamics of Perfect Conductors. 2013. Springer Nature
6. Roth, J. Reece. "PLASMA STABILITY AND THE BOHR - VAN LEEUWEN THEOREM." NASA TECHNICAL NOTE, NASA, 1967, p. 56.
7. Squires, Gordon Leslie. "Meissner effect | Superconductivity, Magnetic Fields & Temperature." Britannica, 12 September 2025
8. Essen, Hanno. "Classical diamagnetism, magnetic interaction energies, and repulsive forces in magnetized plasmas." EPL (Europhysics Letters) 94 (2011) 47003-p1-p5, vol. 3, no. 1, 2011, p. 5.
9. Engelhardt, Wolfgang. "Is a Plasma Diamagnetic? ,Cornell University, 2005
10. Singh, Navinder. "1903.07031v1 [physics.pop-ph] 17 Mar 2019 A Brief history of mangnetism." arXiv, 17 March 2019
11. Cardoso, Wilder R. "Revisiting Sommerfeld's atomic model using Euler–Lagrange dynamic." American Journal of Physics, 2024
12. By CielProfond - Own work, CC BY-SA 4.0
13. Robert Alicki1,and Alejandro Jenkins." Quantum Theory of Triboelectricity"
14. "Theory of Magnetism" (lecture notes) by Carsten Timm, TU Dresden (2019) — section 1.1 : Theory of Magnetism

Interview of
Prof. Ram Murty
and
Prof. Kumar Murty

Interview of Prof. Ram Murty and Prof. Kumar Murty

interview by **Abhisruta Maity** (IISER Kolkata), **Shirshendu Chowdhury** (IISER Kolkata)
December 12, 2025

Two distinguished mathematicians reflect on the origins of their curiosity, the mentors who shaped their paths, and the philosophy that sustained their research journeys. From self-learning calculus in high school to mastering seminal papers and embracing collaboration without ego, they reveal the human side of mathematical discovery. This conversation offers rare insights into creativity, perseverance, and the mindset required to thrive in mathematics.

Profs. M. Ram Murty and V. Kumar Murty *are fellows of the Royal Society of Canada. The former is the A. V. Douglas Distinguished University Professor & Queen's Research Chair at Queen's University in Canada. The latter is the Director of the Lodha Mathematical Sciences Institute in Mumbai while also a Professor of Mathematics at University of Toronto in Canada. Prof. Ram Murty was an adjunct professor at IISER Kolkata.*

Q: First let us talk about your journey to this day. You both have had great and successful mathematical careers. When and what sparked your mathematical interest?

RM: Well, I think, we began in high school and we wanted to learn calculus. We bought a book, "Teach Yourself Calculus", and we learned from that book – teaching ourselves mainly. We also had some good teachers, but essentially, we taught ourselves from books. So, I think books are important from that perspective.

KM: Books are important, but also, we're lucky to have each other. Siblings are important. Family is important. Learning together is important. Books are a good support, but basically the human touch is the most important. And we're very, very fortunate to have that.

Q: When did you know that it was mathematics?

RM: It's a good question. I think throughout high school, we were more or less clear that it was math; it was one of our strengths. When we went to university, we were taking courses in other topics as well: physics, computer science, even philosophy. But it was clear that math is going to be the core thing. And, from then onwards, I think after reaching university, we focused on math and math alone.

KM: But, I remember even before high school, it was elementary school, and we had just written the final exam in mathematics. And I remember telling my best friend (Neil Kumar) at that time, "I'm going to be a mathematician." And the interesting thing as I reflect on that, two things come to my mind. Firstly, that day and that moment and that conversation are very clear in my mind. I still remember where we were standing and what I said. And second thing is, I had no idea what a mathematician is. So why did I say that? I don't know. But clearly the idea was already there and then it grew.

Q: During your undergraduate education, was there anyone or anything that influenced you significantly?

RM: Yes, absolutely. We had an assortment of professors in the undergraduate period. Some of them were good. Some of them were not so good. But certainly, there was one professor, who was encouraging. And there were a few other professors who were also encouraging, but this professor was so encouraging that he would keep tabs on what we were doing. There was also some encouragement to go to conferences and present papers. We had, as undergraduates, written a paper or two, and he was very supportive in going to the Canadian Math Society Conference and presenting these works. But the most important role that he had was to tell us that,

"For graduate school, you shouldn't stay here". And for us, it was nice to stay home. You can do your PhD. The University near our home has a PhD program, just do it. You don't have to set up house, you don't have to learn cooking. You don't have to do any of these things. So, it was attractive to stay where we were, for us. But then, he was very adamant. And he was saying, "No, that would be a mistake." But he didn't leave it at that. He told us to apply to the big schools. And he pestered us every day and asked "Did you send the applications for MIT, Harvard, Princeton? That's where you people belong." And we said, "No, we didn't do that." And he kept on pestering. So, we got the forms. And then next day he would say, "Did you fill those forms out?" We said, "No, we didn't fill those forms out." "Why not?" Well, because a section on financial information was there and we didn't want to give that. He said, "Leave that blank, fill the rest, and just send it in." So, you see, clearly there was some sort of divine intervention in the form- of this advice, which is out of the ordinary. If I were in his place and I told you, let's say, to apply to these big schools, and you said you didn't, I would have said, "Okay." I wouldn't be pestering you every day and then telling you what you should do. Perhaps I'd rather say, if you don't want, you don't need to fill that out. But he used to say, "Get the forms." This was for Harvard and Harvard had more forms. I didn't feel like doing it. He said, "No, get the form just for me. You don't need to do it. I won't see them." It's unusual. That's why I'm saying that sometimes, you know, when I see these episodes, which are very unusual, there is some divine intelligence moving around. I don't know what it

is. We tend to think in terms of humanoid personalities, but there may not be. There may be some transcendental intelligence, as I would like to call it, divine mother. What you would like to call if you'd like to speak in those terms, that seem to have these extraordinary events. I'm sure there are such events in your life too. I think there are such events, they are there in everybody's life – turning points and things that happened that changed your life; thanks to some person who went out of the way to encourage you to do this or do that. So, I think these are important things when looking back. You can see the presence of the divine mother.

Q: During your undergraduate education did you face any significant difficulty? And if you had, then how did you resolve it? Could you share?

KM: Not really difficulty. No, I don't think we had any difficulty, but there were a few professors who perhaps were not totally encouraging. Let's put it that way. We didn't know why. For example, if you wanted to take an advanced course they said, "No, you're in second year. Why do you want to take a fourth-year course?" But overall, I would say it was a positive experience. And you were asking just before about people who had an influence or impact and we were talking about this one professor, but there was another person who had a big influence on our thinking at that time. And he was not at the university at all. He was Paul Erdős. We used to read a lot of his papers, and we thought that's the way to write papers too, which is not a good thing to do. So, our first papers were based on that kind of style. And our second paper, especially, the second published paper – well, he has a whole bunch of papers, like "Some Topics in Graph Theory", "some problems in number theory" .... So, our paper was called "Some Results in Number Theory". If I was a referee nowadays- I'd probably say, "What is this nonsensical title Put a proper title." But we got away with that... and probably because we later learned Erdős was the referee! So, I think, mathematically, at that time when we were undergraduates, Erdős was a big influence. We used to read; we used to spend a lot of time in the library. There was a whole floor for math, and sciences was on the bottom floor of the library. And we used to spend hours and hours there, pouring over journals. All the spare time was spent in the library. And therefore, we were aware of many things in mathematics that maybe even the faculty did not know because we were studying them.

KM: Exactly. I remember once: you went to class and I was in the library. I picked a book on topology. I didn't know what topology was. I was a first-year undergraduate. And I read it, then after you came, we were going to lunch, I said, "There's this thing called topology." So, it was a golden time in the sense that apart from a few hiccups, we had a lot of opportunities to read, to study beyond classrooms. And, the biggest influences were of course that professor we mentioned, but also Erdős. And so, we were very happy when in our final year, I think we were able to get Erdős to visit. We met him in person. And we even started a joint paper with him. And so now we both have Erdős number one.

Q: Erdős is considered as a great problem solver. And so, if you start navigating his papers,

you will see lots of ideas coming in from different kinds of mathematics. But in that case, because he used to get many ideas from other domains, at an early age, what strategy would you take to understand those papers? For example, for understanding some specific constructions, specific mathematical constructions.

RM: When we say we studied Erdős' papers, we did not do a systematic study starting from paper one. We didn't do that. What we did was we came across a paper or two of Erdős' and we were baffled by, "How does one prove this?" For example, one of the first papers that we came across was a paper on the number of integers less than X such that N and phi of N are relatively prime. And so, the question is, how do you even begin to solve this? And what the paper entails is, an application of Brun's Sieve. Now Brun's Sieve is one of the hardest of the sieves, actually. I mean, I wrote a book on Sieve Theory afterwards, so I know that probably that's the dumbest sieve to start with. You're better off doing large sieve or Selberg sieve or something. But anyway, Erdős somehow mastered Brun's Sieve. After Viggo Brun, probably Erdős was the expert on Brun's Sieve. And so, this paper was a very clever application of analytic number theory. How to subdivide, chop, chop, chop- and estimate and see where the main term is coming from, where the other terms or error terms were coming from, how do you estimate all these things? And so, we mastered that paper. It was a paper published in the Journal of the Indian Math Society in 1948. It must have been submitted during independence time. And it was not a long paper,

FIG 2 : *Kumar Murty delivering a public lecture on **"The Art of Research"** at IISER Kolkata.*

maybe six or seven pages long. We mastered it, and then we could generalize the question. And generalization is how often how many prime factors does N and phi of N actually have? And so when we worked it out, let's say it has K prime factors. So, the case that Erdős did is a case where it has no prime factor, since N and phi of N are relatively prime. But if you do the K prime factor case, we discovered it had a Poisson distribution. So, it was kind of something new, and that was the paper Erdős refereed. He told us later.

KM: But in that paper something interesting happened. That's not the theorem we were trying to prove. We thought, okay, he did N and phi of N co-prime, but how often do N and phi of N have GCD K, let's say? Can you count that? And we tried and we tried and we tried, and we couldn't get it. And we were stuck on that, but then Ram was the one who observed, "Just a minute now, we didn't solve the problem we wanted to solve. But you see that all these lemmas we put together actually solve this other problem on the number of prime divisors. That was an important observation. And so, we wrote that paper. Yes. So, this means that you set out to answer a question, and you think hard and deep about it, and you, you make a lot of progress, and still you think, "I didn't solve the problem." But maybe, you solved some other problem. You have to realize that.

RM: Yes that is important...

KM: You must keep your mind open.

RM: Absolutely. I think if there's some mature advice that I can give anybody is that if you work on something, let's say for a month, you cannot possibly tell me that you didn't solve anything. You may not have solved that; you may have solved something else. Find it!

KM: In fact, even now, this is the advice I give to PhD students. You don't need to know everything. Just one paper, become the world's expert on it. Just one paper, even if it's a short paper, guaranteed you'll get a new theorem. Guaranteed. Your thesis is done. Just become the master of one paper. We proved it from the beginning, but I've seen it play out many times with students. Know one fundamental paper inside out. And you'll be surprised. The deeper the paper is, the deeper the result will be. But any paper can produce this.

RM: Any paper. And the maturity in research has to do with, understanding what it is that you're doing. So, if you are investigating something, you're trying to solve, let's say, problem A. In the context of trying to solve problem A, you may not solve problem A. But you may give rise to problem B or problem C. You must have respect for these problems also. And then have the courage to sit down and say, "Yes, I did solve problem C in the context of this question. I was after problem A, but I didn't solve that. I solved problem C." You write it up. You write them. That's the way it works. Sometimes we get obsessed with some problem that you're trying to solve. While trying to solve it, you probably have been solving some other problem. Find that problem that you've been solving, or you may have now the tools to solve something else.

KM: It turned out many years later, both of us were in Chennai, visiting the Institute of Mathematical Sciences. And there was a conference proceedings volume. I was supposed to give a paper, but I usually like to write a paper slowly. So, they were telling me, "The deadline

is soon, you need to give a paper." So, I said, "Okay, fine." I went into my office and thought for some reason, started thinking about this old problem again that we had tried when we were undergrads and that we couldn't solve. Then I just saw everything fell into place. Just one afternoon, I got it, and then I went and showed it to you (Ram Murty) and Balasubramaniam. And thinking that I made a mistake. You guys checked it, and you said it was right. So, it was solved, but many years later. The point is that the whole process of discovery is quite wondrous. You know? It may not follow the plan that you thought, but it will follow some plan. And you must keep your mind open for that journey. On this one, one more point I want to make about discovering new things. We really mastered that paper of Erdos, and it gave rise to this theorem. And I'm telling you that now master any paper; you'll get a new theorem. And then the deeper the paper, the bigger the theorem. So many years later, there was a paper of Serre. It's published in the publications of the IHES. And we found it fascinating. In fact, you had a course on it at Harvard, right? And we found it fascinating. And I remember I was in Princeton, soon after it appeared, I used to walk around with it everywhere. I always had it in my hand. And I'd always believe in it. And the result was, within those three months, we had improved Serre's results. I'm telling you; it is a principle, you can take it almost as a theorem: you master a paper, you'll get a new theorem!

RM: And mastering a paper means you should be able to present the paper with your bare hands. No notes! That is the test of real understanding.

KM: If I wake you up in the middle of the night and I say, "Explain this." You immediately know. So, you need to know it that well.
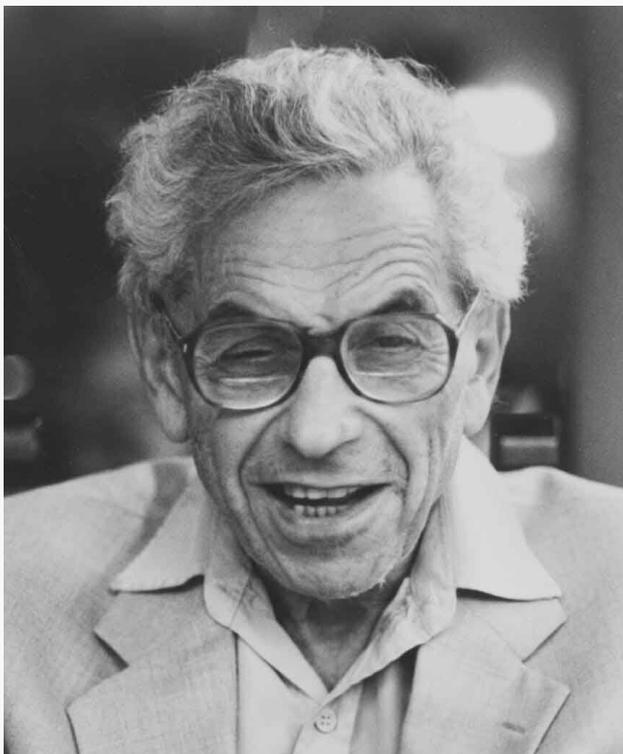

FIG 3 : *Paul Erdős (1913–1996), A towering figure of twentieth-century mathematics, was a formative influence on Ram Murty and Kumar Murty.*

RM: And that's why seminars are important. See, seminars for especially youngsters. Have a seminar every week, you know, like a weekly seminar. Encourage younger people to read a paper. Okay, if they need to use the notes, fine. But try to lecture without notes. I mean, I'm not saying that you must memorize numbers and know technical calculations. But you must have the conceptual idea, understanding of the conceptual idea, so clearly in your head that you can explain it. And, you know, to at least one group of other fellow graduate students or fellow colleagues. And when you understand something deeply, you'll even have the conviction, "I can explain this to a child."

KM: And we've seen this play out many times with my own students. For example, every Thursday, I have a seminar. It's called the Ganita Seminar. And the format of it is a student lecture. My students and my postdocs too. And many of them have written their theses based on the talks they gave in the seminar. "Here's a paper, read this paper and lecture on it." So, they read, they start lecturing. They don't understand everything. We ask questions, many questions. We interrupt a lot, ask questions. And then they say some things they understand, then fine. Go think about it some more. Next week, speak again. They speak again. That happens several times. Before you know it, they've got that paper. And then before you know it a little bit more, they've got a new theorem. And they've got another new theorem, until their thesis appears. So, it's a formula that works. And, you know I keep telling them also that it is hard not to prove a theorem. Mathematics is so rich. You must try hard not to prove a theorem. All you need to do is... the algorithm is right there for everybody. Just take one paper that you find interesting, master it. You'll get a new theorem automatically.

Q: Thank you very much for your advice. Let us now come to your doctoral studies. You were supervised by one of the greatest architects of modern mathematics: Professor John Tate. How was your personal experience: what could you learn from him?

KM: I think it was an interesting experience because every teacher is different. His approach was treating the students like colleagues. So that has its advantages and disadvantages. The advantage, of course, is you get to talk to, you know, a famous mathematician. But the disadvantage is he's not going to give you any allowance because you're a student. If you say something nonsensical, he'll tell you: it's nonsense. There are many, many stories of students who have been sort of shocked a little bit, about how rough he could be. Rough meaning, not physically. I mean verbally. He could tell them this, "Your idea is stupid." You know? He's not trying to insult anybody. He's just saying, he's talking like he would banter with a colleague. If you're expecting some understanding and kid gloves, no. He's not going to give that. So that was one thing. Second thing is, he expected us to work on hard problems. I remember telling him once an idea and he said, "Okay, it looks interesting. Do it." I said, "No, I don't think it'll work because of the implication of this... it'll solve a famous problem." "Yeah? So, what are you waiting for?" I was afraid to do it. But I was thinking, "It can't possibly be right, because it's consequence would be that." And he's saying, "That's

why you should work on it." So those two things I remember. That is, he expected high standards. That is, he treats you like a colleague. Don't come and try to tell him nonsense or things you didn't think hard about. And second thing is, attack hard problems. Attack, big, open problems.

Q: And your doctoral education was at MIT with two mathematicians, Harold Stark and Dorian Goldfeld. So, what would you say about that time?

RM: Good point. Yes. Both Stark and Goldfeld were firstly, classical number theory people. And I also consider Serre as one of my teachers. In fact, for my PhD thesis, I solved one of Serre's conjectures. And Serre even read my preliminary draft, because he used to come to Harvard regularly, and he read my preliminary draft. So, I consider Serre one of my mentors. And the first year I was there, I was exposed to a course by Serre, in which he very cleverly was telling the interconnections between analytic number theory and algebraic number theory. Okay, so, now Serre started as an algebraist. And in this course, he was learning analytic number theory. And on the flip side, I had two very strong analytic number theory people. And both Goldfeld and Stark were very inspiring, I would say. Very easy to talk to. And, you know, one of the things that I think, the disease of many students – and there were many at MIT: students in number theory, is what I would call the allure of fashion. Okay, so fashion being big words, big programs, you know. You kind of rally as if it's some sort of religion, you rally around a particular person and you use big words, let's say, Langland's program or whatever. You know... I'm not saying some specific thing, but there's this tendency to be attracted by fashion.

It's important for the student to pursue what they actually love and not change what they wanted to learn to something else because it was fashionable. I'm not making any sweeping statements against other subjects, but I... for example, when I was a fresh PhD student, there were two or three other PhD students who were always talking about representation theory, Langlands program, Lie theory; that you had to learn Lie theory in order to do number theory. That was another big thing. Of course, they're right. But if you came to work on analytic number theory and then thought that Lie theory was more fashionable and tried to learn that you may shortchange yourself, because it's hard to make a big discovery in something that you're a fresh, novice into. Whereas if you already have a certain tendency for a particular subject and you follow your heart – So I think that's what I learned from all three. What I found with Stark and Goldfeld was this classical school of number theory that they were both part of. Then with Serre, I could see that there's no need to abandon that tradition. There's a way of moving into the algebraic setting and applying that. So, there was a connection, but there was a kind of a symbiosis, which I really liked. And Serre also talked about elliptic curves at that time. And I was learning a little bit about elliptic curves. And my thesis was something about cyclicity of the group of points mod p of elliptic curves. So, I did solve that unconditionally. Serre had solved it using the generalized Riemann hypothesis, and I managed to remove the Riemann hypothesis. So, I thought that was pretty good. In that sense, there was this kind of marriage between analysis and algebra that I got from Serre. I really thought that was a formative period.

Q: Thank you for sharing this. Now let us move to some other theme. So today, you (Kumar Murty)
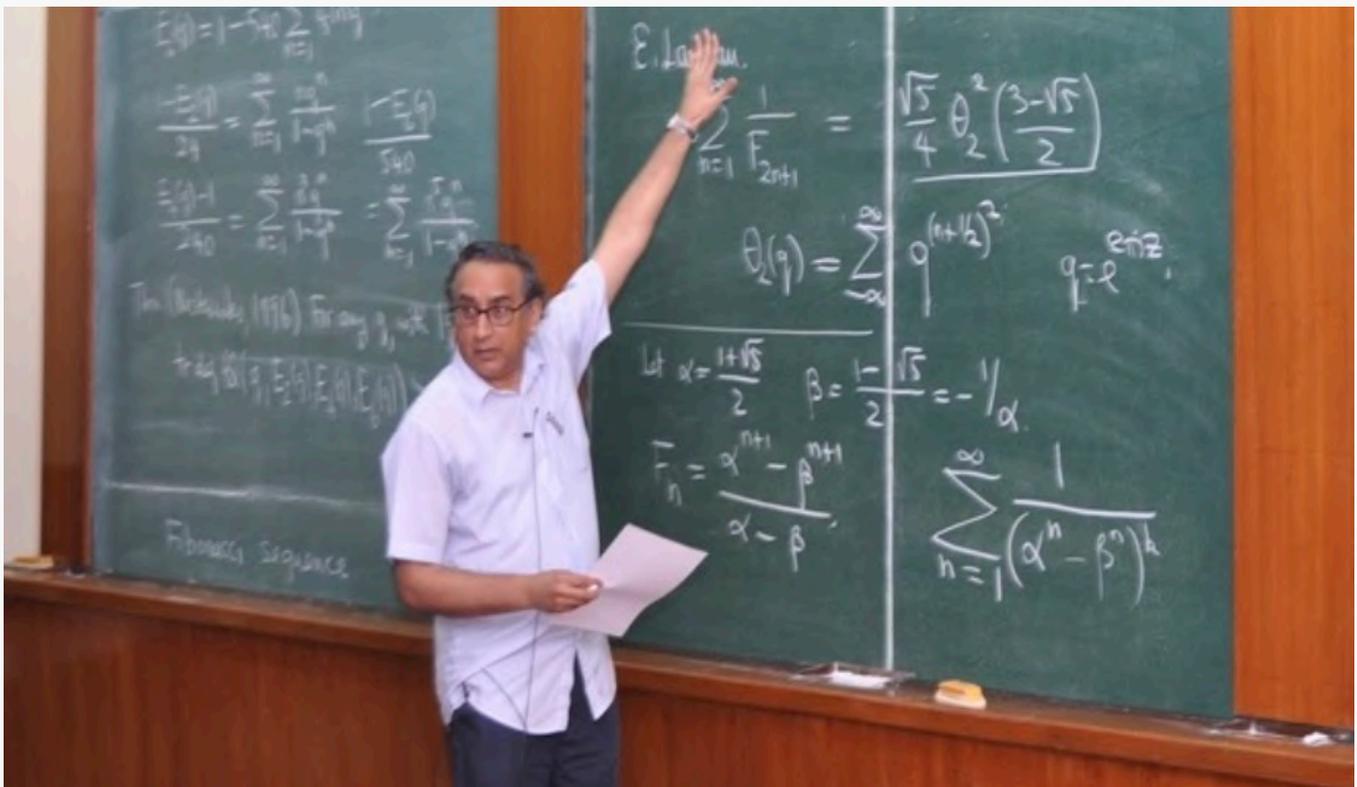

FIG 4 : *Ram Murty at the Tata Institute of Fundamental Research, 2011*

have mentioned cooperation as a basic positive human behavior several times. In fact, Prof. Ram Murty mentioned that you both learned calculus at an early age. This is a great example of cooperation. So, how has your collaboration evolved over time? And of course, from that, what can we learn about collaborative efforts in mathematics in general?

RM: Could I say something or two? So, if you're a graduate student, you can have a collaborative seminar. But it's probably not a good idea to have collaborative research with other graduate students or people at that stage. You could in some sense. The PhD thesis could be a collaboration with the advisor. But the advisor, you know, shouldn't be the one who's giving all the ideas. There must be some sort of give and take. And that's okay. I mean most of the time, the advisor supervises the student, and after the PhD thesis is over, maybe they will write two or three papers together. That's fine. But it'll be problematic if you did research with another graduate student, which of course you can. But it can't be part of a thesis. And so, there's a lot to be learned in the process of collaboration with someone who has a bit more experience than you do. I'm not saying two graduate students can't get together and try to work on something. That's certainly fine, but it can't be the PhD thesis. Because there will be true problems there because who is going to put that theorem in there? So, there are all these issues. That's one thing. But collaboration is one of the greatest things that we have access to. But the collaboration also has a lot to do with compatibility of personalities. So, you can't collaborate unless there's some sort of compatibility, you know. We've written more than 25 papers together. There's a lot of collaboration, and it's fun. There's no doubt about it. I've collaborated with other mathematicians. I've collaborated with my students. I found that, to put

it bluntly, it's easy to collaborate with someone who you can talk freely with, and nobody is putting on airs. You don't have to pretend you know something when you really don't. So that pretense is gone. Certain freedom emerges. And in that freedom, perhaps some new ideas come.

KM: Yes, because most of the time that is spent in collaborative discussion is not always productive. Most of the things we try don't work out, you know. But you keep trying and eventually something will click. So, if I'm afraid of what you think or how I will appear to you, I won't take that chance. But when we collaborate, we both say, you know, "Let's just try something." It could be nonsense, doesn't matter. Oh, it doesn't work. Okay, so then, we try another thing. But there's no... basically, there's no ego. And so, there's a good exchange of ideas. And, and again, this is a proven method. Whenever we proved a theorem that I think I'm proud of for a long time, it came about that way. That we just get together, just start. We know what we want to try and do, then we try, somebody tries, one of us tries something, the other person is taking notes, and then we realize this doesn't work. Okay. So, then we change places. The other person goes to try something ... doesn't work. And eventually, you'll be surprised how these things evolve into very deep work that you can be proud of for a long time. But collaboration is basically amplifying your own brain. It's like double the brain power. And if you allow it to work in a frictionless way without resistance of ego, you're magnifying your own power. So, it's a very, very useful tool for discovery. But, as he says, the thesis time is a special time. Because you need to prove to the world that you can do something on your own. So, that has to be protected. But beyond that, you can do all sorts of other collaborations.

FIG 5 : *John Tate (1925-2019), one of the greatest architects of modern Mathematics, was the doctoral supervisor of Kumar Murty.*

RM: We were discussing Hadamard's book on "The Psychology of Invention" or whatever the book is called. And in that, I think he does mention Henri Poincaré's method of discovery. He says when you try and solve a problem, somehow these thought molecules are moving around in your head, and sometimes, there's a sort of stable combination that comes out. And from that, some idea of a solution shows up. And I think I believe that. I believe something like that is going on. And I think the seminar is a venue for that. Collaboration is a venue for that. So, where you are freely talking more in collaboration because there's more freedom in the collaborative thing. It's not a seminar, it's a private discussion. But even the seminar can be a place where this comes up because that's why I always encourage my students to, whatever paper they're studying, they should explain it and give a talk. And during that, at least they're thinking about it. And I think there is some psychology of discovery connected with the movement of thought particles, as it were. Ideas related to a particular theme. So, when you put something on the blackboard, try to organize your thoughts, some order and arrangement take place, and so maybe a chance for a new idea to come and form a larger thought molecule. And then a new theorem is proved. And I've had this happen. This has happened. That's why I like collaboration. Yes. When I'm trying to explain something to a person, I feel I understand it better and in the process of understanding it better, I do end up making a new discovery myself. So, I think that's what's needed. And coming back to the study of papers, you know, in detail, each paper probably has a secret. But that secret is not open. It's hidden. It's not out in the open. A person can pad the paper. It could be a 100-page paper, but maybe there's only one idea. So, you must learn how to read papers. And it's not like reading literature. So, you read the paper, you let it kind of incubate in your mind. Over time a kind of a flash comes and says, "Aha, this is the key idea." Once you understand that key idea, everything kind of falls into place. Otherwise, all this technical jargon that the paper begins with is going to

FIG 6 : *Jean-Pierre Serre (1926), a pioneer of modern Mathematics and one of the mentors of Ram Murty.*

discourage you from learning things. So, there's a secret. The Goddess holds a secret. You know, the Goddess holds a secret, and that secret will not be divulged to you unless you make an effort to study the idea. Make that effort. The effort then is rewarding. It somehow allows in a new idea that helps to find out the secret. Serre once said to me, "How do you read a paper?" "You read it backwards," he said. I immediately understood when he said that.

Q: So is it like you want to prove some theorem … suppose it is a direct proof, so A implies B, then, do you mean that you want to show B? How to show B? You prove something–

RM: You need this. You need that. Yeah. He doesn't mean literally backwards. He means that somehow you have to read the paper, and then focus your attention; how do you get to the goal. How did you get B out of A? So, what are the steps that were there? And once you understand the steps, maybe there's a technical lemma which you shouldn't bang your head on. But you know these steps, and then you know how the whole proof is put together. Then once you understand the big picture, then you'll be willing to spend the time to learn the 20-page proof for a technical lemma. But if I start your lecture with technical stuff, then everybody will walk away.

Q: One more question when choosing the paper, as you mention, is that you should take one paper and read that. So, it seems selection is important. How do you select?

RM: You must learn how to browse through the library. You must learn how to browse … on the internet it's not easy. It's easier to browse in the library. Sit there just for the fun of it, with no specific goal in mind. The serendipity factor, if you want to call it that. You must have that attitude. See, this world is full of distractions. "I have to do this, I have to do that, I have to do…" You know, you're always filled with the "I have to do." I think it's important to sit in the library with a feeling that I don't have to do anything. And just read for the fun of it. And you're not even trying to prove a theorem. You're not even saying, "I want to generalize this guy's work." I just want to understand it. Like you would with a piece of music. Before you become a great musician, you must enjoy good music. You don't listen to music saying, "I'm now going to produce the next concerto or next symphony." This is not the way you do. You just enjoy music for the sake of enjoying music. You take a walk just for the enjoyment of the walk. You go to enjoy some scenery, watch these mountains just for enjoyment. You don't want to grab them. You just enjoy the beauty. So that kind of enjoyment, is I think what the Rishis were talking about when they say, "Ananda." Ananda refers to this enjoyment of life. We have forgotten how to enjoy life. So, you must learn how to enjoy ideas just for the fun of it. Otherwise, you're not an academic.

Q: Thank you. These days we are experiencing a pandemic of poor mental health among students, especially doctoral students. Is there anything you'd like to say upon improving the situation?
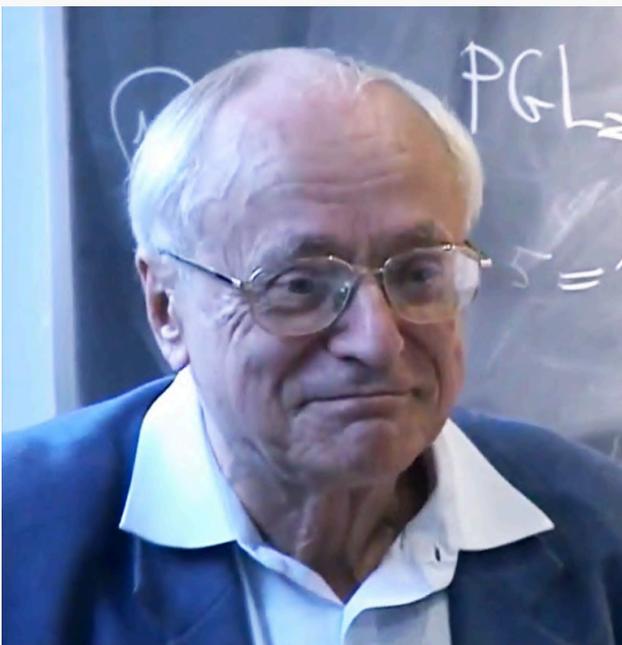
KM: Well, firstly, be a little kind and gentle. Because you see, the point of the doctorate is very, very demanding. That's not because it's hard. It's because you've never done that before, in most cases. So, just think of how you came to the point of doing the doctorate. Okay? You went through school, then through undergraduate, and then you did a master's degree, and you got into the doctorate program, you finished all the coursework, and now you have to write a thesis. A thesis is the first time you have to work on problems where people don't know the answer. Do you see? Even at a master's level, you take courses, you have assignments, there's an exam. Somebody knows the answer. If they put a question on your assignment or an exam where they don't know the answer, you have a legitimate complaint. Right? Somebody puts an unsolved problem on the exam- … you can complain. The exam is invalid. So, up to the point you're doing all this coursework, it's all known. For how many years? 15, 16, 20 years? Yes. You were doing that, and then, then suddenly people say, "Now you have to discover something new." It's a shock. If you're not used to it, if you haven't built that culture of thinking originally, it's a big shock. Psychological shock. I've had about two dozen PhD students. The difficult part I kept telling them, is not the math. Math is not the problem. You have convinced yourself you cannot discover anything new, because you've told yourself that and others have told you that. You think, basically, you've trained yourself to be a teacher, because you know how to stand in front of a class and talk about what's in the textbook. Right? And so, the impediment for you to succeed now, to cross this final hurdle and get your thesis, is not math, is not that

FIG 7 : *Kumar Murty in **Montreal, circa 1990**.*

you don't know enough math. It's that you don't have faith in yourself. So, I must gently lead you to the point where you believe you can do it.

RM: I mean, if you think about an Olympic athlete. If the Olympic athlete tells himself or herself that, "I can't do the high jump." They won't do it. And then when they try to do it, what do you think is going to happen? They won't do it. So, there's a kind of trade-off also. It's not necessarily inflation of ego that we're talking about when we say self-confidence, faith in oneself. It's essentially a statement of fact. So, firstly, we're all given a package of mental energies. It's just that we tend to put blockages on these energies, you see? That's the problem. We like the difference between a person who can do the high jump versus a person who cannot do the high jump. The person who cannot do the high jump has convinced themselves that they cannot do it. But with training, they can. You must start with the training. How can I teach the student how to do the high jump if every day the student comes and says, "I can't do it." So, you have to put away these ideas somehow and make the effort. It's a negative process. You are putting blockages on yourself. Exactly. You know? So, it's not an inflation of ego. In fact, it's a form of de-hypnosis, you know? You have been hypnotized. Maybe parents told you, maybe teachers told you, maybe society tells you that you can't do it, and therefore, you now believe you can't do it. But if you check the autobiography of great athletes even, they had to undo that. And psych themselves that they could do it. And they could. But it's not an ego factor. It's a kind of undoing of conditioning that you have been brought up on. And the other thing that's important is … I'm more a philosopher as such and the path of Jnana Yoga, which is just inquiry and pure thought. For every person who is trying to do research, there are a lot of factors in your favor. Firstly, time. The time that you're living in. So, you may say, "Oh, but Gauss worked on this problem. How can I solve it?" Well, Gauss worked on it in the 18th century. Time has changed. There may be more tools. Right? Just so you must look at it from a practical standpoint. And the mere fact that saying Gauss worked on it, you recite yourself into your incompetence. So, it's not some quality of respect for Gauss. In some sense, it's a certain disrespect for yourself. So that kind of argument or that Grothendieck worked on it, and he… well, Grothendieck didn't know analytic number theory, let's say. You, you see my point? Serre didn't do it. Well, he didn't know this stuff. It is not given to any single individual to be omniscient.

KM: Exactly. Nobody has a monopoly on talent. Nobody has a monopoly on your results. Everyone can discover something.

RM: Exactly. So, you find what interests you. You go deeply into it. You love it. You savor it like you savor a dish. You enjoy it like you would enjoy a piece of music. And one day, out of the blue, a new idea will emerge.

KM: I used to, for some time, make it a point to ask my PhD students to see Kung Fu Panda. Yes, that's funny. Kung Fu Panda, you see, is a very interesting movie for people who are trying to do research. Because it teaches you two things. Firstly, success doesn't necessarily follow conventional wisdom. You know? He says, "He doesn't even know kung fu and you call him the Dragon Warrior." And yet he was the one who could defeat the enemy that all the other experts couldn't do. We were
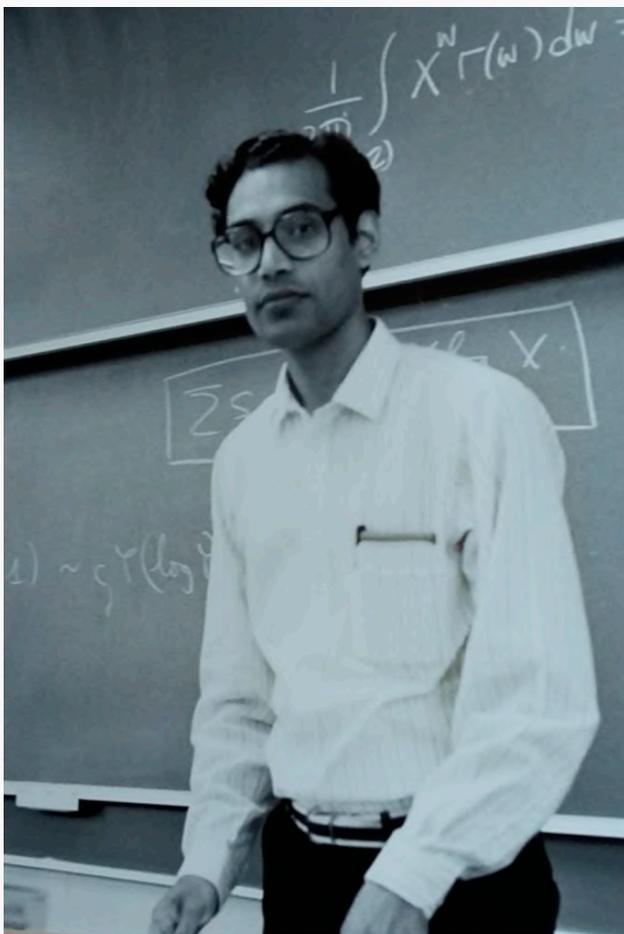
talking about Gandhi. It's similar. You see this little man with a frail body, marching thousands of kilometres. Yet he did it. So conventional wisdom is often wrong. So, you have to have those kind of dramatizations or actual examples in history, or your teachers as exemplars to say, "They could do it, why can't I?" How am I any less? Maybe you prove the Riemann hypothesis and maybe I'll prove a little non-vanishing theorem that's different, but I'll prove something. There's this statement of Nachiketa in Katha Upanishad. He says, "Among some, I'm the best. Among others, I'm in the middle. But nowhere am I the last." I can also do something. I also have the capability of doing something." There are many stories, that's why actually, you know, besides whatever discipline you're studying you must fill your mind with inspiring stories. Inspiring stories, motivational stories. Whether they are mythological, historical, or social, it doesn't matter. You fill your mind with those kinds of ideas to help you to understand the truth of the statement that you have ability, you have potential, you have creative talent, and that you can manifest it. I found amongst my students, the ones who listened to me when I told them these things, they succeeded. The ones who resisted, eventually they came back and accepted it, but the ones who resisted took the longest. For these, anything I told them just didn't click. In the end, they came back and did exactly what I told them to do 10 years ago. And it worked, interestingly. So, there's time for everything.

RM: So, each person is on their own personal journey of evolution. And you must find your own calling. That's why I say that as teachers, what we can do is give freedom; freedom to learn, freedom to explore, freedom to find what you find beautiful. And maybe the teacher can only guide. The teacher cannot dictate anything.

KM: And, you know, after they succeed, then they thank us by saying, "You had confidence in me before I proved anything." We say, "Yes, that's the job of the teacher." And then they write, you know, "I want to thank so-and-so for having believed in me when I didn't have a result." That's what your teachers are supposed to be able to do for you. You know, give your confidence a little boost. And they can see what the future kind of holds before you can see it. And so, give you that extra boost of confidence so that you can go over the top and prove your theorem.

RM: See, I hope you've understood what we're saying. When you sit down with a paper … for a month (say) and study it in depth to the point that you can lecture without

notes, you may try to solve another problem using the techniques and still fail after a month. But you can't tell me that you didn't discover anything. You must have discovered something. Perhaps you solved something else. Find out. You must have solved something else. And so that maturity must be there. Some new problems emerge which were never thought of before. They happen to be coming out of your own intelligence.

KM: You can say, "I have a problem and I'm looking for a solution." But another way of putting it, is what Atiyah used to tell us. He said, "I have a solution, but what problem does it solve?" He just studies a subject. And then, it throws light on certain problems. He wouldn't start with the problem. If you read the interviews with Atiyah, he'll explain his way of thinking about mathematics.

RM: And, you know, another interesting thing about this idea of solutions is that if you take up as it is said you have a solution looking for a problem, what is the difference between a great mathematician and a mediocre mathematician? And the answer is, it's just a joke of course. The answer is the mediocre mathematician is working on seriously unsolved problems, whereas the great mathematician is looking where the methods he has mastered can be applied. It's almost like the solution looking for a problem, or proof looking for a theorem. He already has the proof. In the sense that the methods are there. Yes. And it's looking for a theorem. It's a kind of funny, interesting psychology, isn't it? I mean, so that is the difference. So, sometimes time is ripe for certain ideas to be developed. And maybe all the tools or many of them are there. Like Fermat's Last Theorem. It just happened that this was the time. It was not a century ago. You know, the time was right. There's a certain inevitability.

RM: You know in some sense maybe we're talking about mental health, maybe there should be some sort of non-credit course, in which some sort of tips on how to study should be taught.

KM: Well, this talk could be turned into a course. Because each of those points can be expanded with examples too.

RM: But also habits, you know? And so maybe some sort of non-credit course about this kind of thing will attract… maybe we'll meet once a week for one hour or something, that may give us focus. And then students can also try to express their concerns, what their questions are. Some experienced researcher kind of leading the discussion could help alleviate some of the mental stress that people feel. You see, this whole thing about you entering the PhD program, now you have added stress to prove something. That in itself is a problem. We certainly don't want you to goof off. But at the same time, we don't want you to be stressed out to the point that you go bonkers. We need to strike a balance. You need to make an effort, you need to learn something, but this idea of ananda, enjoyment, may be torture at the beginning, but it may be a certain form of enjoyment later when you have mastered the idea. And it takes work, just as it takes work to learn the high jump. The marathon is not run in a day. It requires training. So, once you do it, you enjoy it. See all these people who run the marathons. When the thing is over, they're taking a victory lap. Well, how did that come? So, this is the enjoyment, that's why they're taking a victory lap. And there's the same thing in research; the same

FIG 8 : *Ram Murty and Kumar Murty with Henryk Iwaniec*

mental phenomenon is there. Training of the mind. The training of mental energies.

Q: What do you do when you are not in academic work? Actively?

KM: I have a lot of interests. Reading is fun. Reading non-math books. I do read quite a bit of philosophy books, and I also like writing, music, and singing. I like economics. I also like innovation, technology, ... there are many subjects which are very inviting. And it's also interesting to see, to put things together– you know that people didn't put together before. So, I think maybe that to summarize the whole thing, I like the joy of discovering. Discovering new things is fun.

Q: Do you think taking up something apart from your academics is a great form of not only enjoyment, but also a necessity?

RM: Absolutely. It comes back to hobbies. I think this is the problem. If I must go back and look at all the sad cases and the mental disasters in all of those cases, there was no healthy hobby.

KM: If you equate yourself with your academic achievements, you're in trouble. Because sometimes you will do well, sometimes you won't do well. Firstly, when you do well even, that's also dangerous. You get giddy. And when you don't do well, you think you're not good at anything. You know, these are extremes. You must be sensitive. You must develop some ways of thinking and living so that you're grounded, you know, rooted – look, some things may go well, things may not go well, it doesn't mean I'm any less. My worth has not gone down or gone up. It was always at a good level. I'm still capable of doing things. But that grounding is necessary,

FIG 9 : *Kumar Muntry with a statue of Kung Fu Panda at Brigham Young University (BYU), 2010.*

FIG 10 : *Ram Murty and Kumar Murty during interaction with school students at IISER Kolkata.*

that's all... That's really a good point you're raising. Grounding is necessary. That's how you create mental hygiene, people call it mental health. Because hygiene involves habits. So, you brush your teeth regularly, you take a bath regularly. These are habits that keep you in good health. Personal hygiene. Mental hygiene is also necessary. You think that you can put all the garbage you want in your head and you'll be healthy? You can't. Just like junk food: how much can you tolerate? And the body retaliates. And so, the mind will retaliate too. And mental retaliation is much more severe than physical retaliation. You might get an upset stomach, that's one thing. But mental unrest, that's created by all the garbage we ourselves put in, is harder to deal with. So, you need to make sure what you're taking in is healthy ... healthy ideas. And secondly, you need a way to neutralize anything negative that's already there. This is deliberate. These are very deliberate things that must be cultivated. They don't happen accidentally. You know, for example, about what you take in... yesterday was the Janmatithi of Shri Shri Sarada Devi. What was her main teaching? Don't put negative thoughts into your head by seeing negativity in others. Just if you could practice that one thing ... Your interaction with everybody is positive.

Q: How do you manage your time for studying and research while maintaining a high level of productivity? Are there specific habits or routine that you would like to suggest to the students?

RM: Each person must figure out how to use the 24 hours of the day. Everyone is different. You must determine when your mind is fresh and energized. That is the time for creative learning and research. For me, early morning hours are best, say between 5am and 9am, before any formal duties of employment intervene. I find browsing in the library is best in the afternoons.

KM: Besides browsing, it is important to be realistic about how much can be done in a day. For each day have a goal of one or two or three things that you will work on. And focus on those, and don't confuse yourself by working on other things.

Q: We know that teaching is an important component of learning and understanding. From the teachers' perspective, what would you suggest for improving the process of teaching?

RM: The teacher must make complicated ideas simple. This requires work. It requires solitary learning and writing from the teacher. Again, one must find the time of day for this work, usually one to two hours suffice.

KM: The teacher should awaken a love and enjoyment for the material. Once that love and enjoyment is awakened in the student, they can teach themselves all the details.

Q: You mentioned that the age of AI forces each of us to face questions like "Who am I? What's my purpose?" to all of us. Are there any specific books on Psychology or Philosophy which you would recommend for understanding more about human mind and life?

RM: Personally, I found the writings of Vivekananda, especially his books on the four yogas, to be helpful. This may be useful to some. If this doesn't work, then some "self-help" books may. Dale Carnegie's books on public speaking are full of fascinating stories and helpful advice.

KM: Vivekananda is certainly the person of our time who spoke most about faith in oneself and about believing in and manifesting one's own potential, which he said is infinite.

Q: While grappling with a new subject or a new book, many students get discouraged by either the terseness or the sheer volume of a book. Although there may not be any formula or algorithm to read and learn from books effectively, could you still mention a few important points to keep in mind while reading and understanding?

RM: Yes, the size of the book on the subject can be daunting. The secret is to find something that absorbs your attention. It begins from there. A little bit of time (say an hour) each day can go a long way and within a week or two, you will see results in mastering the topic. Note taking while you learn is also essential.

KM: Yes slow steady progress can be made if it is regular. Even a big book is made of individual pages. Reading a page a day should be possible. And before you know it, you would have mastered a significant portion of the book. It is certainly possible.

FIG 11 : *Ram Murty and Kumar Murty: In the company of ideas: A Mathematical Duet.*
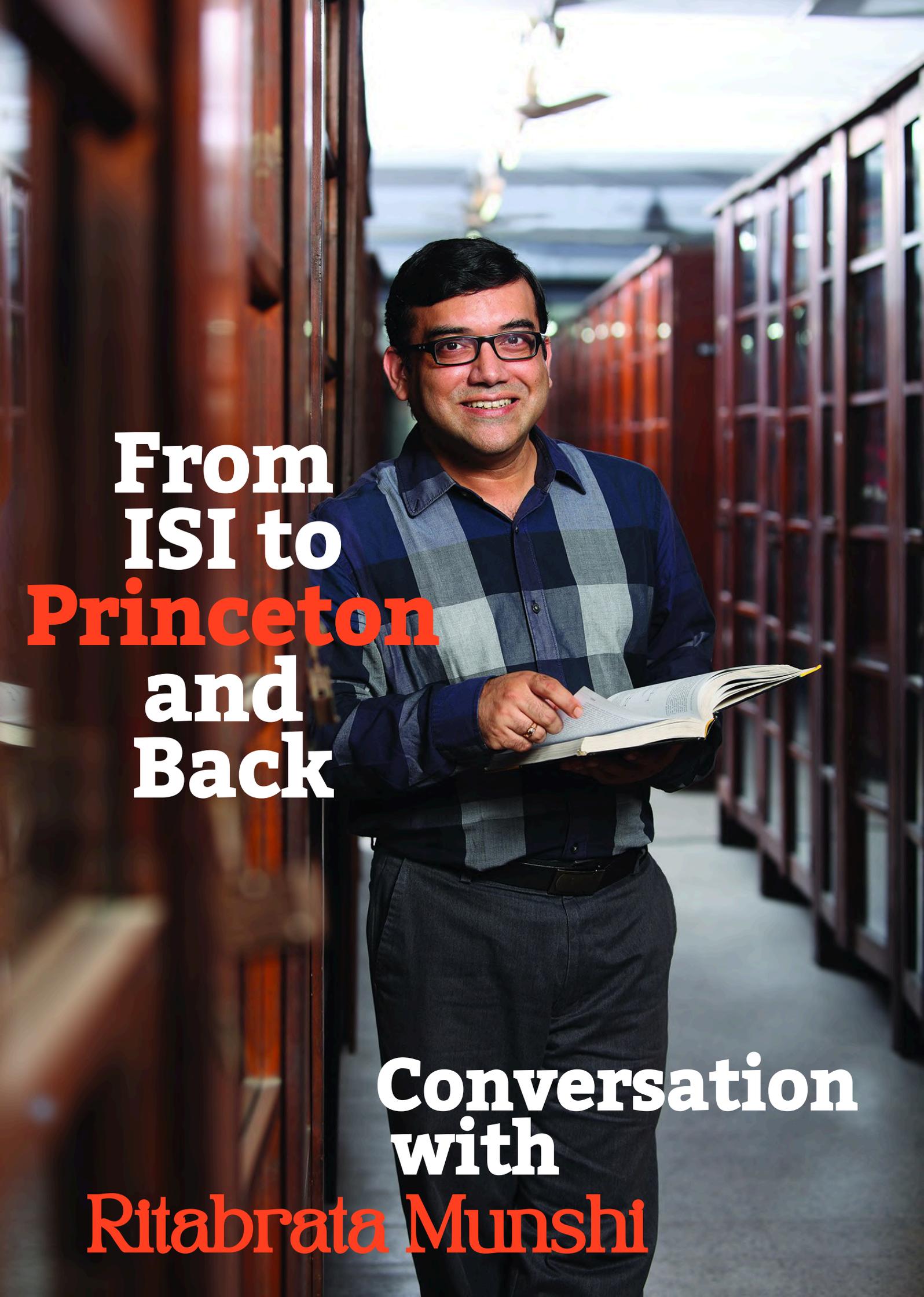
From ISI to Princeton and Back

Conversation with Ritabrata Munshi

From ISI to Princeton and Back: Conversation with Ritabrata Munshi

*interview by **Swarnendu Saha** (IISER Kolkata)*
December 10, 2025

In this wide-ranging conversation, Prof. Munshi reflects on a life shaped by mathematics—from early fascination with symbols to research at Princeton and a return to ISI Kolkata as faculty. He discusses number theory as an artistic pursuit, the changing philosophy of academic institutions, and why mathematics survives when fashionable disciplines fade. The interview offers rare insight into mathematics as both a personal vocation and a centuries-old human enterprise.

SS: Hello sir, I am Swarnendu Saha from Team InScight, and it is an honour to have you with us today.

RM: Thank you. It's a pleasure to be here.

SS: To begin with, could you take us through your journey of becoming a mathematician? Perhaps a brief overview of your academic path.

RM: Yes. It has always been my dream to be a mathematician. Even when I was very young, during my school days, I wanted to become either a mathematician or a theoretical physicist.

When I was in high school, I used to read my father's journals. My father was an engineer and received journals from the Institution of Engineers. I was fascinated by the symbols—especially the mathematical symbols—that appeared in those journals. I loved looking at them and trying to understand what was actually going on. That curiosity gradually turned into a passion. Even in classes seven and eight, I started maintaining a diary in which I would write down mathematical ideas and thoughts.

Prof. Munshi is a mathematician specialising in number theory. He was awarded the Shanti Swarup Bhatnagar Prize for Science and Technology for the year 2015 in mathematical science category. On 8 November 2018 he was awarded the ICTP Ramanujan Prize in a ceremony held at the Budinich Lecture Hall, ICTP. He is affiliated to Tata Institute of Fundamental Research, Mumbai, and the Indian Statistical Institute, Kolkata

FIG 1 : *Prof. Ritabrata Munshi at his dorm room in New Graduate College, Princeton, Aug 2001*

After finishing school, I faced an important decision —whether to pursue engineering, possibly through the IITs, or to study mathematics. For me, it was a clear-cut choice. There was no confusion. I chose mathematics. At ISI, I received excellent training in mathematics. In my view, it was—and still is—the best place to study mathematics. From there, my journey truly began. I later went to Princeton and worked under Andrew Wiles, which was a dream come true.

SS: At that point in time, since your father was an engineer, was there pressure on you to pursue engineering? Do you think it would have been an easier route, given the familiarity?

RM: No, actually, I think my parents realized quite early on that I was not really interested in anything apart from mathematics. I also used to draw and paint a lot, so those were the only two passions I had—mathematics and painting. So, for me, the choice was quite easy.

SS: So, you were a student at ISI?

RM: Yes, I was a student at ISI. I did B. Stat. and M. Stat.. There was no B. Math or M. Math back then. I did my B. Stat at ISI Kolkata—that was the only option at the time. For M. Stat, I completed the first year in Kolkata. The second year I did in Bangalore, because it depended on the specialization one chose. Since I was more interested in mathematics than statistics, I opted for advanced probability. For that specialization, I had to go to Bangalore.

SS: Today, you are a faculty member at ISI Kolkata, and you were once a student at ISI. Having experienced the same institute from two very different perspectives, how do you see it now?

RM: First of all, ISI has changed quite a bit. When I was a student—from 1996 to 2000—I was at ISI Kolkata. I returned as a faculty member in 2015. So, there is almost a 15-year gap between the two phases. A lot has changed. When I was a student, ISI was a very small place. There were very few students and very few faculty members. The focus was entirely on performance, education, and knowledge—nothing else. Now, there are many more students, and I think the diversity and variation among students have also increased. For instance, when I was a student, my batch had only 13 students.

SS: Thirteen. You mean, one and three ?

RM: Yes—thirteen. One and three. Now there are around fifty students. So you can imagine the increase in variation.

SS: So, roughly four times the size.

RM: Yes. Our batch was different—quite exceptional. In fact, one of my classmates is here today; he is now at the University of Georgia. I am meeting him after almost twenty years. It was a very small group, and we were very closely knit. We helped each other with studies and almost everything else. We all lived in the hostel back

then. Now, most students stay outside campus. I think the atmosphere has changed. I find it difficult to precisely articulate what has changed, but something certainly has.

SS: Would you say the change has been for the better or for the worse—both from a student's and a faculty member's perspective?

RM: If you ask me personally, I would say that I would rather be a student in the 1990s than now. But this is a different generation, and their priorities are probably different. Back then, interaction between faculty and students was much stronger. Our professors were always very welcoming.

SS: And how is it currently? When I joined in 2020 at IISER Kolkata for my BSMS, my batch had around 200 students. When my juniors joined, it increased to around 230. In that context, you are saying that 50 students in ISI is a large number. But in many institutes, a single department has more than that many students majoring in mathematics. So why do you think 50 students at ISI is a huge number?

RM: I think the key point is how ISI was originally designed. ISI was built to function with small groups. It is not like an IIT or an IISER. The philosophy behind ISI was fundamentally different.

SS: How is it different? What was that philosophy?

RM: The idea was to have a very small, carefully selected group of students, and teachers who would devote a significant amount of time to training them. With larger numbers, that becomes difficult. You simply cannot have one-to-one interaction with everyone. Earlier, that level of interaction was possible.

SS: What is the current teacher-to-student ratio?

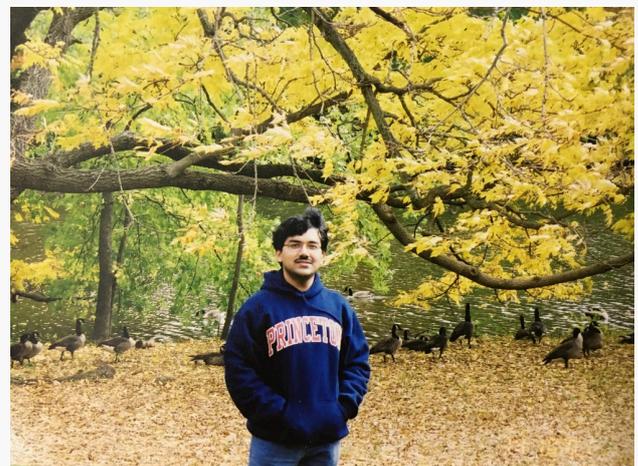RM: I don't know the exact numbers, but—

FIG 2 : *Prof. Ritabrata Munshi at Beside Carnegie Lake in Princeton, Aug 2001*

SS: I'm asking because if there are about 50 students, one would expect at least 20–30 faculty members.

RM: It's about 50 students per batch in B.Stat. Then there is M.Stat and M.Math. So roughly, you are looking at around 150 B.Stat students, about 100 M.Stat students, and around 70–80 M.Math students. And then there are PhD students as well. I personally supervise seven PhD students.

SS: So altogether, it would be around 250 students or more.

RM: Yes, altogether. But another important point is that not all faculty members at ISI are involved in core teaching. For example, in the Statistics and Mathematics Unit, all professors teach at least one course every semester and are deeply involved. However, that is not the case in all departments, simply because it is not always feasible.

SS: When ISI was founded, as far as I understand, its primary focus was mathematics and statistics, right?

RM: When ISI was built, the main focus was statistics— and anything closely connected to statistics.

SS: Yet, interestingly, one of the few preserved dinosaur fossils in the Kolkata region is housed on the ISI campus. It's quite old, and if I am not mistaken, it comes from the Tungabhadra Basin, discovered around the time of Tagore's centenary.

So ISI does not seem to be limited strictly to mathematics and statistics in the way we often perceive it. From both a student's and a teacher's perspective, how do you connect this broader academic presence? I'm trying to understand the background.

RM: Yes, ISI does have a Geology unit. In general, any discipline that has a strong connection with statistics finds a place at ISI.

SS: So how is geology connected to statistics?

RM: You can apply statistics to geology, anthropology, or almost any discipline. Wherever there are measurements, data collection, and the study of data and numerical distributions, statistics becomes relevant.

That is why, when Mahalanobis founded ISI, he had all of this in mind. His idea was that wherever statistics could be applied, that discipline should have a place at ISI.

There is also a unit of computer vision and pattern recognition. You need computers to perform statistical analysis, which is why there is a Computer Science Department. You need mathematics to develop the theoretical foundations of statistics, which is why there is a Theoretical Statistics and Mathematics Unit.

Statistics can be applied to quantitative economics— hence the Economics Department. It can be applied to the

study of language—hence Linguistics. It can be applied to biology—so there are Biostatistics and Genetics units. In that sense, everything is connected.

SS: Were these developments already in place when you were a student, or did they come up later?

RM: They were already there. These ideas have existed since Mahalanobis's time.

SS: So over the years, how do you see the growth and expansion of ISI in terms of education and academic research?

RM: Today, the primary focus is on four core subjects —Statistics, Mathematics, Economics, and Computer Science. These are the subjects for which ISI offers degrees.

However, when ISI was founded, the degree was only in Statistics. Mathematics was treated as applied statistics at that time. Over the years, other units have, in my opinion, lost some of their earlier significance. They were more central when Mahalanobis himself was around.

SS: So would you say there has been a decline in diversity at ISI?

RM: In a sense, yes—at least in terms of relative importance. Take mathematics, for example. Mathematics has become more important because it is now formally taught as a subject. For B.Stat and M.Stat, mathematics is essential, since it is a core component.

And now we also have B.Math and M.Math programmes. This has attracted more students who are specifically interested in mathematics. As a result, the Mathematics department has gained prominence.

SS: What is your primary field of research?

RM: Number Theory. Number theory is essentially the study of numbers—primarily integers. But the interesting part is not the numbers themselves; it is the structure they carry. Numbers have an additive structure —you can add them. They also have a multiplicative

FIG 3 : *Prof. Ritabrata Munshi at Ramanujan Prize Ceremony in International Centre for Theoretical Physics, 9th November 2018. [source: ictp.it]*

structure—you can multiply them. And number theorists are interested in understanding how numbers behave as you move along the number line—say along the positive integers—with respect to this multiplicative structure.

For example, consider divisibility. The number 1 has no divisors other than itself. The number 2 has no divisors other than 1 and 2. The same is true for 3. These are called prime numbers. But 4 is 2 × 2, so it has a non-trivial factor. Five is prime. Six is composite. Seven is prime. So primes appear irregularly, but they are fundamental. Prime numbers are the multiplicative building blocks of integers. Every integer can be written as a product of primes. What we are really interested in is understanding the interplay between the additive structure and the multiplicative structure of integers.

SS: What do you mean by "interplay"?

RM: As you move along the integer axis, you are progressing additively—you keep adding one. But at the same time, the multiplicative properties of numbers—such as primality and factorization—change in complex and fascinating ways.

Understanding this relationship is at the heart of number theory. Every step is essentially adding one. And what we want to understand is what happens to the multiplicative structure at each step—whether the number you reach is prime or composite.

SS: So in that sense, subtraction would just be adding minus one.

RM: Exactly—adding minus one. That also comes under the additive structure.

However, when studying multiplicative structure, it is not very important to look at negative numbers. One can restrict attention to positive integers, and the questions remain just as interesting. So, for example, you can ask: how many prime numbers are there? If you go up to a large number—say 10,000—how many primes are there below it? More generally, how many primes are there up to a number ?

We would like to have a formula for that. Gauss came up with a conjectural formula for this around 1796. He suggested that the number of primes less than or equal to x, is denoted by pi(x), is approximately $x/\log x$. This became a central problem in number theory, and mathematicians spent nearly 100 years trying to prove it. Finally, between 1796 and 1896, the problem was completely resolved.

What is remarkable is that, in the process of trying to prove this single statement about the distribution of prime numbers, mathematicians developed entirely new areas of mathematics. Complex analysis was developed, new applications of Fourier analysis emerged, and combinatorics and real analysis advanced significantly. In mathematics, we often say that the most interesting problems are those that generate more mathematics. This problem—understanding the distribution of prime numbers—did exactly that. It led to deep developments across multiple fields.

That is why prime numbers remain such an active area of research even today. As you can see, this conference itself

is titled "Primes, Patterns and Propagation." We are still deeply invested in understanding primes.

SS: When you started your career, as far as I remember, you wrote a proof of the Jordan Curve Theorem. In that regard, I have two questions. First, were you interested in number theory at that time, or did your interest develop later? Second, in the introduction to your work, it is mentioned that you were not very interested in computers at that time—has that opinion changed?

RM: Yes, I wrote a proof of the Jordan Curve Theorem during my first year of B.Stat. At that time, I was quite interested in topology.

However, my interest in number theory began even earlier. I remember that when I was in Class 10, while preparing for my board examinations, we went to the book fair and bought David Burton's Introduction to Number Theory. That book fascinated me. It is written in a way that even school students can read and appreciate it, and it provides remarks and notes that connect naturally to higher mathematics.

That is really how my interest in number theory developed. Even before that, of course, I had read about Ramanujan—which is a fairly standard Indian student story. You read about Ramanujan when you are ten, you try to become Ramanujan, and by the time you are sixteen, you realize that you cannot become Ramanujan.

Regarding computers—yes, at that time I was not very interested in them. Back then, computers felt rather boring. Things have changed since then, and of course I have also aged over the last 25 years. But even today, if you give me a computer and a mathematics book or a mathematical problem, I will choose mathematics any day.

SS: Currently, ISI Kolkata is facing certain challenges. As a former student and now a faculty member, how do you see these changes? How might changes in the institutional structure affect

FIG 4 : *Prof. Ritabrata Munshi delivering his talk on number theory at Ramanujan Prize Ceremony in International Centre for Theoretical Physics, 9th November 2018. [source: ictp.it]*

research and academics, given that the community and resources largely remain the same?

RM: I sincerely hope that the new structure will not affect the academic functioning of ISI. Academically, ISI is doing very well, and it does not need changes in that direction.

Of course, we always need more resources to grow and develop. But my main concern with the proposed bill was the restructuring of the governing council and, more importantly, the academic council. The academic council is central to maintaining academic autonomy. Fortunately, in the revised or modified version of the bill, that particular aspect has been addressed, so I am more comfortable with it now. As for the governing council, some changes were expected—I had been hearing about such possibilities for some time. What impact these changes will ultimately have on ISI remains to be seen. It could turn out to be beneficial, or it could bring challenges. But my firm view is this: as long as the academic structure is left untouched, ISI will continue to function well.

SS: As we approach the end—perhaps the last five minutes—I want to ask you something specific. You completed your undergraduate and master's education in India at ISI, arguably the best place for mathematics in the country, and then went to the United States for your PhD.

How do you compare the study, training, and thinking in mathematical research between India and the West?

RM: Overall, the training in mathematics in India is quite good. In fact, if you compare an average Indian student with an average American student, I would say the average Indian is better placed mathematically.

SS: By "average Indian," do you mean someone who has completed their bachelor's and master's from a public university?

RM: Yes. If you compare them with an average American who has done a major or minor in mathematics—especially if you include community colleges and exclude

the very top universities—then I think the average Indian student stands higher.

However, the big difference lies in flexibility and freedom. In the US, if you are a promising student, you are strongly encouraged and given access to opportunities. From a very young age, students have access to resources, mentorship, and even university-level courses. You can talk to professors, attend college classes, and sometimes even fast-track your education.

You might start college-level mathematics at the age of 15. Mathematics is largely a subject for young minds—much like chess. When you are young, your mind is fresh, and that freshness is crucial for mathematics. So it is very important to identify talent early and guide it properly.

SS: One of my final questions—and you may disagree with me, or even get angry—is this. Many of us chose science over engineering or medicine and faced questions about that choice, given the current situation in the country.

Now, after studying science, I want to ask you—someone who has devoted his life to mathematics—how do you see the subject today? Do you see it flourishing? Do you see students choosing it out of genuine interest, or merely as a degree?

And more broadly, many people say that science—and especially mathematics—is a "dead" subject. What is your take on that?

RM: People say many things. That does not mean they make sense.

SS: It is our responsibility to explain what is correct.

RM: Exactly. Mathematics is one of the oldest human intellectual pursuits. You can trace it back to ancient civilizations—the Vedic period, the Babylonians, the Greeks. Mathematics has existed for thousands of years.

What is remarkable is that even after 2,000–3,000 years, it is still alive and thriving. People continue to work in mathematics and discover new results. How many other subjects can claim that kind of longevity?

FIG 5: *Prof. Ritabrata Munshi invited as a speaker at International Congress of Mathematics 2018 in Rio de Janeiro. [source: Rio ICM2018]*

Take artificial intelligence, for example. Do you think AI will survive for 2,000 years?

SS: It may evolve or transform.

RM: I don't think it will survive in the same way. When I was deciding between engineering and mathematics, computer science, electronics, and information technology were "in fashion."

SS: But those fields still exist, right?

RM: They exist, but not in the way they once did. The excitement, the structure, the focus—it has changed. Mathematics, on the other hand, has endured, although its prominence has not been uniform throughout history.

SS: By "not uniform," do you mean temporally or subject-wise?

RM: Temporarily. If you look at ancient Greek mathematics—Euclid, Pythagoras, Archimedes—it was a golden era. Then there were periods with relatively little visible activity. Later, from around the 12th century onward, algebra and new mathematical ideas emerged. Again, there were quieter periods, followed by renewed bursts of activity.

SS: Why do you think this pattern exists? The need for mathematics—whether practical or intellectual—should always be present, shouldn't it?

RM: I think before the Industrial Revolution, mathematics had limited application in everyday human life. After the Industrial Revolution, people realized how mathematics could be used to optimize systems, solve practical problems, and drive technological progress. But the Greeks were not doing mathematics because it was useful. Mathematics was, in a sense, a "useless" pursuit back then—or perhaps a better word is art. At its highest level, mathematics is art, not science.

SS: And that still holds—that higher mathematics is a beautiful but "useless" art?

RM: Yes, absolutely. So it remains. At the highest level, mathematics is purely art. There is no science in it—it is pure imagination. At the same time, mathematics is also the unifying language. It is the language we need to understand underlying reality.

SS: By reality, you mean—

RM: The universe, physics, and all of science. Mathematics is the correct language to describe them.

That is why mathematics has survived for thousands of years. I do not think it has lost any of its charm. In fact, I doubt whether many of the subjects that are fashionable today will survive even a hundred years. Things will change so much that people may forget them altogether.

SS: One final question. As an experienced mathematician, what would you like to say to younger students—those who have already taken up mathematics, those who are thinking of taking it up—and also to the general public?

RM: To students who are choosing mathematics, I would say this: you are making an excellent choice. But be prepared for a lot of frustration. Doing research in mathematics—especially at a high level—is like trying to move a wall. And when it moves, you are lifted to a completely different level. But most of the time, it does not move. So mathematics is, in some sense, 99% frustration and 1% success.

SS: So we have to enjoy the journey through that frustration?

RM: Exactly. The real joy of mathematics lies in the attempt itself—in trying to move the wall. That is where the fun is. When the wall finally moves, you feel immense satisfaction—but there is also a strange sadness, because the struggle that defined the journey has ended.

FIG 6 : *Prof. Ritabrata Munshi with his classmates of St Thomas Primary School, Chandannagar, West Bengal (front row, fourth from right). [source: Department of Science and Technology, India]*

Seeing the World Mathematically: A Conversation with Prof. S. D. Adhikari

interview by **Swarnendu Saha** *(IISER Kolkata)*
January 07, 2026

In this conversation with our representative, Prof. Sukumar Das Adhikari connects number theory, combinatorics, physics, and Indian philosophy to argue that mathematical structures emerge from human cognition as much as from the external world. The interview offers fresh insights into how abstract ideas—pursued for beauty alone—later become foundational to physics, computing, and AI, while also revealing rarely discussed personal experiences with mentors, global collaborations, and unsolved problems.

Prof. Adhikari started his academic journey from B.B. College Asansol and spent time with Prof. Balasubramanium at Tifr and IMSc Chennai. He is a former member of National Board of Higher Mathematics, Fellow of National Academy of Science. Prof. Adhikari is presently the dean of school of mathematical Sciences at RKMVERI, Belur.

SS: Hello sir, I am Swarnendu Saha, and I welcome you to this interview session with InScight. My very first question is inspired by something a professor once told me: "Mathematics, in its highest form, does not remain science; it becomes useless—but fantastic—art." What are your thoughts on this?

SDA: Actually, this idea comes from a quotation from Bertrand Russell. You've paraphrased it a bit differently, but Russell compared mathematics when rightly viewed as beautiful as a sculpture. He emphasized that mathematics is not only useful but also profoundly beautiful. Take the distribution of prime numbers, for instance. The pattern of primes in general, is beautiful in itself. There are many such patterns in mathematics that possess aesthetic value beyond utility.I once attended a lecture of Sir Michael Atiyah where the speaker posed an interesting question: Why does the external physical world follow mathematics? The answer given was that it is not the external world as it is, but the external world as it appears to us through our senses. Our senses are governed by logic and mathematical thinking. Therefore, the world appears mathematical to us—quite naturally.

SS: So, you mean to say that it is not mathematics governing reality, but rather our perception of reality that is mathematical?

SDA: Exactly—the world as it appears to us. Consider sound: it has frequencies, but we can only hear within a certain range. Frequencies below or above that range are inaudible to us. Similarly, many animals cannot perceive colors; for them, the world is essentially black and white.So reality is not necessarily what is, but what we are capable of perceiving. For example, if you wear yellow-tinted glasses, everything appears yellow. The external world remains the same, but your perception changes.

SS: In that sense, it resonates with Tagore's words:

আমারই চেতনার রঙে পান্না হল সবুজ,
চুনি উঠল রাঙা হয়ে /
আমি চোখ মেললুম আকাশে,
জ্বলে উঠল আলো"

*(It is my senses which colour
Emerald green and ruby red.
I look at the sky –
Light appears;)*

SDA: Exactly. All thinkers have essentially said the same thing. It is all because of consciousness. If there were no consciousness, who would perceive anything?

FIG 1 : *Prof. Adhikari completed his BSc in Mathematics at* **Banwarilal Bhalotia College in Asansol**, *where he developed deeper interests for the subject*

And beyond that—what would even exist? No one really knows. Reality varies depending on the way it is perceived. We see the world through the lenses of desire and fear. I desire something, and because of that desire, I fear losing it. Desire and fear are two sides of the same coin. Only when one transcends both desire and fear does one begin to see reality as it truly is. That is why yogis and jñānis perceive a world very different from the one we live in. They inhabit a world where there is only bliss.

SS: Because they have nothing left to seek.

SDA: Yes, and they are not afraid of losing anything. In a world where everything is constantly being destroyed, the one who perceives that which is underlying and indestructible—that person sees reality.As you see in Bhagavad Gita (Chapter 13, Verse 28):

Vinashyatsu avinashyantam yah pashyati sa pashyati

SS: My next question is this: often physicists and physics professors say that physics is the world itself, and mathematics is merely the language used to describe it. In that context, physics is reality, and mathematics is its language. How do you view this idea?

SDA: As I mentioned earlier, it can be said that way. But again, I'll go back to Atiyah—Michael Atiyah. He spoke about this idea in some interviews; I don't recall exactly where. He gave a simple analogy. Suppose two people come to a river. One person wants to cross it, so he jumps from one stone to another—this stone, then that stone—and eventually gets stuck midway. Then he notices that someone else has built a bridge connecting the stones, and he simply walks across using the bridge. The person who built the bridge is a mathematician. He is not merely trying to cross the river; he is constructing a bridge—often a beautiful one—from one stone to another. That bridge may or may not have immediate use, but it can become useful for someone who wants to cross.

Take Riemannian geometry, for example. When it was developed, the motivation was not application.

Mathematicians were exploring ideas like modifying the parallel postulate, trying to understand geometry in different ways. There was no practical application in mind at that time. Yet later, it became central to physics. In general, anything that is beautiful tends to find application eventually. Recently, I was lecturing on the Davenport constant. It originated from problems in factorization within number fields. Much later, it found applications in proving results such as the existence of infinitely many Carmichael numbers. So one good idea often ends up being useful in many different contexts. Someone, somewhere, eventually finds a way to apply it to something else.

SS: So does that mean that mathematics in its purest form is not immediately applicable to reality?

SDA: There are two possibilities. Sometimes, you want to achieve a specific goal, so you create the necessary mathematics—that happens quite often. At other times, mathematicians pursue ideas purely for beauty, logical completeness, or structural consistency. Later, those ideas become useful—not necessarily in explaining nature as it truly is, but in explaining how nature appears to us. And that appearance follows a certain logical structure. Both approaches arise from the same source: the human brain. Mathematics is built from our intuition, just as our understanding of the world is shaped by how it appears to us.

SS: Let me shift the discussion slightly now. On a more personal note—how did you end up becoming a mathematician?

SDA: As I mentioned earlier, I was interested in both mathematics and physics. At that time—after passing Class 11—I was slightly more inclined toward Physics than Mathematics. In those days, there was no Class 12. We had Class 11, and then we directly went to B.Sc. I passed my school in 1973.

SS: My mother was born in 1971 (*chuckles*).

SDA: So, what happened next was this: I appeared for the entrance examination at B.B. College. In physics, I was placed 11th (1st on the waitlist), but they took only 10 students. In mathematics, however, I ranked first. So naturally, I joined mathematics. After about one or two months, two students left the physics program to join engineering. I was asked whether I wanted to shift. But by that time, I had already developed an interest in mathematics.

SS: And that's when you continued with mathematics.

SDA: Yes, because earlier we didn't really know what a subject truly contained. What we study as mathematics in school is very different. But once I encountered advanced topics—real analysis and other higher-level courses—I became deeply interested.

We were also fortunate to have some excellent teachers at B.B. College at that time. B.B. College refers to Banwarilal Bhalotia College in Asansol. I am still in touch with

FIG 2 : *Prof. Adhikari completed his MSc at **Burdwan University**, where he secured the **first rank in Pure Mathematics** and was awarded a fellowship.*

some of my teachers. Only one of the college teachers in Mathematics is still alive. He called me recently and wants to meet me—I'll be visiting Asansol in February. He helped me immensely and had great affection for me. His name is Rammoy Haldar. He is now about 90 years old and lives in Asansol. If I forget to call him, he calls me himself. One of my chemistry school teachers is also still alive and lives in Durgapur. He, too, calls me. I shared a very warm relationship with my teachers.

SS: After completing your bachelor's degree—where did you pursue your master's?

SDA: I completed my master's degree at Burdwan University. After that, I secured the first rank in Pure Mathematics and was awarded a fellowship.

SS: And you joined IMSc, Madras?

SDA: No. I received a fellowship at Burdwan University.

SS: For your master's degree?

SDA: No, for my PhD. I joined Burdwan University for doctoral research. However, not much research activity was happening there at that time. So I tried to move elsewhere and even explored the possibility of joining Chandigarh University. Then someone advised me to apply to TIFR. I joined TIFR, but after about two and a half years I found someone willing to be my supervisor; Professor Balasubramaniam gave me a few things to study. But soon after that, he was moving to Chennai, and fortunately, I was able to move in with him. That turned out to be very good luck for me. He was an exceptional teacher.

SS: Two and a half years?

SDA: Yes. I was on a TIFR fellowship during that period. I was studying and preparing—that was essentially what I was doing. Study.

SS: Coursework?

SDA: Yes—coursework, and also deciding which direction you want to pursue. Around that time, Professor Balasubramaniam returned from abroad and asked me to read certain material. As I said, shortly after that, he was preparing to shift.

SS: And you shifted with him?

SDA: Yes. He gave me that option, and I chose to go with him. In hindsight, it was a very good decision.

SS: Why do you think it was a good decision?

SDA: Because of the teacher—purely because of the teacher.

He gives a lot of freedom and a lot of encouragement. He allows you to work independently, and at the same time, he helps you exactly when it is needed. Even now, I remain in touch with him. I visited him again in August when I went for a family function—he invited me, and I went. I also met him again in Chandigarh during an event connected to Professor Bambah. There was a conference organized in Professor Bambah's honour, where about twelve people were invited speakers, all of them are rather senior. Among them were Professor Balasubramaniam, Professor Raghunathan, and Professor Kulkarni, who was a former director of HRI, Allahabad. Professor Kulkarni was 83, Professor Balasubramaniam was 75, Professor Raghunathan was 84, and I was 69. Professor Bambah passed away in May 2025, about four months before his 100th birthday. So the conference was held in his memory, and I met Professor Balasubramaniam again there, on September 30.

SS: At one point, you mentioned that you joined TIFR and spent about two and a half years doing coursework etc. before moving to Chennai. But that reading phase alone wasn't for two and a half years, right?

SDA: No, the two and a half years include everything—coursework and preparation for research in Number Theory together.

SS: So how much of that time was purely coursework? Around two years—that's what I was trying to clarify. Now, when students like me, my juniors, or my immediate seniors look for PhD positions around 2025, things are quite different. There are SOPs, structured projects, and in Europe or Germany, for example, PhDs are often three-year project-based positions where you apply directly to a defined topic.

SDA: Yes, things have changed a lot. I also received a fellowship offer from the US at that time, but I didn't go because I already had a good guide by that time.

SS: So how do you see this change? You joined through a program, did coursework, and then spent about two years on research.

SDA: After moving to Chennai, I finished quite quickly and worked intensely. To some extent the degree got delayed due to the formal requirement that my guide had to be registered at Madras University. After that I received a postdoctoral position at TIFR again and after about one year I left for ICTP, Trieste. After that, I returned and became an NBHM postdoctoral fellow.

FIG 3 : *Prof. Balasubramaniam, former director of IMSc,Chennai and Padma Shri awardee, was a key mentor during Prof. Adhikari's PhD years at TIFR and later at IMSc.*

SS: National Board of Higher Mathematics ?

SDA: Yes—an NBHM postdoc at SPIC. At that time, CMI was known as SPIC. After that, I joined what is now the Harish-Chandra Research Institute.

SS: Which was earlier known as the Mehta Research Institute.

SDA: Yes, MRI at that time.

SS: So how do you see the changes in mathematics academia in India—from when you were a student to today?

SDA: Today, there are many more institutes. Before Professor Balasubramaniam moved to Madras, Prof. E C G Sudarshan came in and took over as its Director and at his invitation, Prof. C. S. Seshadri was taking care of Mathematics. But overall, the academic ecosystem has expanded significantly.

SS: I'd like to move to a different topic. I've heard that Pāṇini's grammar in Sanskrit is often viewed by scientists and mathematicians as a sophisticated collection of algorithms.

SDA: Yes, it is extremely mathematical. I attended some lectures at RKMVERI, where there is a strong Sanskrit department.

Mathematicians gave lectures connecting Sanskrit grammar and mathematical structure. I am interested in Sanskrit, but I haven't studied its grammar in depth. Still, it is undeniably very mathematical.

SS: At its core, it's about logic, right? Could you explain that briefly for students?

SDA: I wouldn't want to explain something I only partially understand. I know a little, but not enough to do justice to it.

SS: Let me ask about your academic work. After your postdoctoral years, what was your main research area?

SDA: My PhD was in analytic number theory, which I continued working on for some time. Later, I began working on combinatorics and what I lectured on today—additive and combinatorial number theory. I also did some work in transcendental number theory. Over time, I worked across four or five areas within number theory. But what I enjoy most are elementary methods. Elementary does not mean easy. It means avoiding heavy machinery like complex analysis or abstract algebra, and instead using real analysis, combinatorial reasoning, and first principles.

SS: From first principles.

SDA: Yes. Paul Erdős is my hero. I was fortunate to meet him in Hungary. I spent five days talking to him extensively. He gave me some problems, I wrote in my notebook, and I even spoke to him once on the phone later while I was in France.

Sadly, he passed away soon after that. Otherwise, we might have written a joint paper. Still, I have papers with people who themselves have papers with Erdős.

SS: Could you briefly explain the subjects you've worked on, for students?

SDA: At present, my main focus is combinatorics. The core combinatorial tools are induction and counting. These methods are used across many areas of mathematics. At the same time, combinatorics now borrows tools from topology, algebra, probability, and ergodic theory. For example, Szemerédi's theorem—originally conjectured by Erdősand Turan —was first proved using difficult but elementary combinatorial methods.

Later, Furstenberg gave an ergodic-theoretic proof. I even organized an Ergodic Ramsey Theory workshop in Pune in 1997 and invited Bergelson, a student of Furstenberg. So while my main strength—my "bread and butter"—is combinatorial thinking, I actively keep track of developments in other approaches. I sometimes use algebraic tools in combinatorial proofs as well.

SS: Let me bring the discussion back to the physical world. Earlier, you used the analogy of someone building bridges and someone else using them. But can combinatorics be directly involved in the material world?

SDA: Many mathematical questions arise from very natural curiosities—such as how many ways something can be done. At the school level, this is basic combinatorics. As these questions become more sophisticated, they find strong applications, particularly in computing. Combinatorics is extremely useful. I teach a course on it, and many students from computer science want to attend. In some years, the entire computer science batch attended the course along with mathematics students. Combinatorics has flourished over the last few decades largely because of

FIG 4 : *Tata Institute of Fundamental Research (TIFR), Prof. Adhikari worked twice here, once during the Ph D days when he started studying with Prof. Balasubramanian and later again as a postdoctoral fellow.*

its applicability to computer science. In many ways, computer science is combinatorics.

SS: Given this close connection between combinatorics and computer science, today we hear a lot about artificial intelligence.

Does AI research also rely on combinatorics?

SDA: Yes, AI certainly uses combinatorics. But there is an important distinction. Artificial intelligence works by following algorithms—algorithms that humans already know how to design. If I already know that choosing one path is less costly than another, I can encode that information and feed it into an AI system. The system will then follow those instructions efficiently. But when something genuinely new is required—when a new idea or a new algorithm has to be discovered—that still requires the human brain. Machines can perform calculations faster. They can add, multiply, and even make logical decisions very efficiently. But those decisions are based on prior human knowledge. The act of probing, analyzing, and discovering something new—that creative step—comes from humans.Computers are extremely useful for data collection and implementation. But the step from raw data to deep understanding often needs human intuition. That does not mean humans will be replaced entirely. For example, if there is radiation leakage inside a nuclear reactor, humans cannot go inside safely. We use robots to perform such tasks. In that sense, technology protects human resources rather than replaces them.

SS: But do you think that today mathematics—especially abstract mathematics—is not directly feeding the masses in a material sense, and therefore may not be attracting new generations?

SDA: No, no—that is not the right way to frame it.

There is a famous line of poetry—by Satyendra Nath Datta—

জোটে যদি মোটে একটি পয়সা খাদ্য কিনিয়ো ক্ষুধার লাগি'
দুটি যদি জোটে অর্ধেকে তার ফুল কিনে নিয়ো, হে অনুরাগী!

*(If you have one coin, buy food to satisfy hunger.
If you have another coin, buy flowers to nourish the soul.)*

First come necessities—food, shelter, daily needs. Mathematics certainly serves these needs today through computers, technology, and applications. After that, if time and energy permit, we also explore ideas driven by logic and beauty. And those ideas often become useful later. What is done today purely for intellectual beauty may find applications tomorrow. Take number theory—Hardy famously believed it would never be used in war. Today, number theory is fundamental to cryptography and coding.At the time, Hardy took pride in the fact that his work had no military application. But history turned out differently. The point is: you never know when a good, logical, or beautiful idea will become useful, or where it will be applied. Sometimes it is used in areas for which it was never intended. Even within mathematics, ideas developed in one field often become powerful tools in another. Algebraic concepts are frequently used to solve problems they were not originally designed for. That is how mathematics grows—and why it remains both meaningful and indispensable.

SS: So, would you say that a lot of this is a matter of luck and chance?

SDA: Yes—many things do involve chance. Someone happens to encounter a particular conjecture. If they hadn't seen it, they might have worked on something entirely different. Perhaps they were exactly the right person to solve that problem. In the world, many such chances are constantly at play. Suppose there are ten people who are all capable of solving a problem. Only one of them happens to meet the right question at the right time. The other nine could also have solved it, but they never encountered it. There are many people who stop formal education, yet they have tremendous potential. It's like farming—you need good soil, sunlight, and water. Even good soil may fail to produce anything if it doesn't get water.

SS: Following that, let me raise another concern—especially relevant to Bengal, where I have some data. In general science, and particularly in core subjects like physics and mathematics, fewer students are enrolling nowadays.

SDA: There are two aspects to this. One concerns teachers—but I won't go into political aspects of selection. If teacher selection is done properly, we get motivated teachers. If I am not motivated, how can I motivate you? If I am teaching only for money, I cannot inspire students. But if I genuinely love the subject, that love naturally gets transmitted. Of course, not every student has that spark—but even if a student does have it, an unmotivated teacher can kill it. Also, teachers are not only those who stand in classrooms. A good book can be a teacher. An experience can be a guru. And something must exist inside the student. Ramanujan is a perfect example. He found Carr's Synopsis and discovered patterns and beauty on his own. But something inside him drove that

FIG 5 : *International Centre for Theoretical Physics (ICTP), Trieste*, *where Prof. Adhikari pursued his postdoctoral research.*

exploration. Later, he was recognized and supported. If that recognition hadn't come, his story might have ended very differently.

SS: Ramanujan would not have been Ramanujan.

SDA: Exactly. Had Ramanujan gone through conventional mathematical training too early, perhaps he would not have been Ramanujan either. He might have been forced to think like the rest of us, without the freedom to think in his own way. Initially, he was allowed to explore freely. Later, structured training supported him. That combination was optimal. As Nisargadatta Maharaj once said, for anything to happen, the whole world must coincide. Nothing happens in isolation—everything must support it.

SS: At IISERs, we see batches of 200−250 students across subjects, but mathematics has the fewest enrollments. One reason seems to be the job market.

SDA: Yes, employment plays a role. Students see their seniors completing five years of PhD and still struggling for stable jobs. Even if they love mathematics, survival matters. So many students move into data science after an MSc in mathematics. The question then becomes: Can mathematics feed people? But "feeding" doesn't only mean immediate money. Teaching positions in schools and colleges are essential, but many posts remain unfilled, funding is limited, and selection processes

are sometimes flawed. Minimal institutional support is crucial. Without it, even good students struggle.

SS: In that context—what makes a good teacher?

SDA: A good teacher must first be a good student.

I don't believe in the strict division between teacher and student. There are only senior students and junior students. Everyone is learning. The moment I stop learning, I stop being a teacher. A teacher must have burning enthusiasm—to learn, to explore. In a classroom, I can only teach for a few hours. But if I genuinely see the beauty of the subject, you will see it in my eyes.

SS: So you see yourself as a student?

SDA: Absolutely. Everyone is a student. Otherwise, all problems would already be solved.

SS: You mentioned unsolved problems. Have you devoted significant time to such problems yourself?

SDA: Yes, yes—I have been fortunate. I've solved or substantially contributed to some good problems.

In one case, I worked with my teacher Professor Balasubramaniam on a conjecture—not during my PhD, but later. The conjecture was resolved, though gaps between upper and lower bounds remained. Later, in higher dimensions, I worked with Yang Gao Chen, whom I met in Hungary. In two dimensions, I worked with another group. Later, with Andrew Granville in Canada, we more or less resolved the problem in dimension two.

SS: Can mathematical problems truly be solved completely—like "two plus two equals four"?

SDA: Some can. Fermat's Last Theorem was solved. Waring's problem was solved. Van der Waerden's theorem was solved. Szemerédi's theorem—originally conjectured by Erdős and Turán—was solved. But once you solve a problem, you generalize it—and the generalization becomes a new problem. Mathematics never really ends. Szemerédi proved the theorem using difficult but elementary combinatorial methods. Later, Furstenberg proved it using ergodic theory. Different methods, same truth.

SS: You've quoted many philosophical figures during this conversation.

SDA: Not monks—I quoted the Gita and Nisargadatta Maharaj. He was not a monk; he was a householder in Maharashtra. Maurice Frydman—an engineer who knew many languages—recorded Nisargadatta's teachings and later became a monk. Nisargadatta did not.

SS: You're currently affiliated with Ramakrishna Mission institutions. Have these philosophies influenced you?

FIG 6 : *Chennai Mathematical Institute(CMI) known as SPIC during his time. Prof. Adhikari worked as a NBHM(National Board of Higher Mathematics) postdoctoral fellow here, after ICTP, Trieste.*

SDA: I've been reading such texts since Class 10. Not only Ramakrishna—also Ramana Ashram,Tiruvannamalai and Pondicherry Aurobindo Ashram . Advaita Vedanta resonates deeply with me. But philosophy is like swimming. Reading books won't make you a swimmer—you must jump into water. Books like are maps. Walking the path is realization. To realize the truth, one must go beyond desire and fear.

Drishtim jnanamayim kritva pasyet brahmamayam jagat.

We all wear spectacles of desire and fear. Clean them—and reality appears.

SS: How does this help you as a mathematician?

SDA: It teaches detachment. If I get a good result and become complacent, someone else may move ahead. If I fail repeatedly and become dejected, I may miss success just one step away. So neither excessive happiness nor excessive sadness helps. One must simply keep working. As I have already mentioned, Faiz said it beautifully: the journey itself is success. If you fix a destination and fail to reach it, you feel defeated. But if walking itself is the goal, you are always successful. That is karma yoga.

SS: You seem to enjoy learning languages as well.

SDA: Yes. I learn French every morning. I improve my Sanskrit every evening.

That's why I say – everyone is a student. There is no final teacher.

SS: Did you spend time abroad?

SDA: Yes, many times. I stayed in Italy for about two years. After that, I kept visiting Europe for one or two months at a time. I have also been to the US, Canada, China, and Japan. In fact, I have visited almost all continents — Australia, South Africa, South America. I have good friends and collaborators in many places. For

instance, during Durga Puja 2025 I was in Spain. After a two-day conference in Chandigarh, I came to Delhi and then flew to Barcelona. There, we completed a piece of work we had been pursuing for nearly a year. In July, I went to Austria—specifically to the University of Graz. There was a conference at TU Graz, and I also visited other universities. Last year again, I visited Barcelona and the Hungarian Academy. I still manage to travel to some extent. In March, I am going to Rome to visit an old collaborator, Francesco Pappalardi, who was a student of Professor Ram Murty. On the way, I will lecture at IIT Delhi, and on the way back I will lecture in Ashoka, as I have friends there and enjoy meeting people.

SS: Where do you stay now?

SDA: I stay on the campus of RKMVERI, near Belur Math.

SS: And your home?

SDA: I have a flat, and I also have a place where I meet people. Coming back to your earlier point, since I have visited many places, including very large institutes like IHES, Paris and Princeton University, I have met some of the top mathematicians in combinatorics. One of the greatest figures for me is Erdős—he is my hero. I was lucky to meet many people and interact with them. For example, I met Schmidt, a great expert in transcendental number theory, several times in India and Europe and he later invited me to Colorado.

SS: How does it feel to meet such great people and interact with them?

SDA: If you think someone is superior to you, you develop an inferiority complex. If you think someone is inferior, that is another kind of complex. But if you have no complex at all, then what is the problem? I respect those who know more than me and learn from them. If someone is thinking a bit slowly, I help them. Everybody is learning, and we help one another.

SS: But you called Erdős your hero.

SDA: Yes, a hero in the sense of his nature. He was always thinking about mathematics. He carried only a small bag and traveled constantly. Whatever money he got, he gave to poor students. Even if someone wanted to return the money after getting a job, he would pass it on to another needy student.

SS: Pass it on!?

SDA: Exactly—pass it on. He would take students out to dinner and spend the money on them. He had no fixed place to live—only mathematics. He wrote 60–70 good papers a year. Many ideas in probabilistic number theory originated from Erdős. If you read the book by Halberstam and Roth on sequences, you will see how much came from Erdős' questions and work. He and Selberg proved the elementary proof of the Prime Number Theorem. Despite all this, he had no arrogance. He spoke to me very naturally. Once, we were chatting, and after some time he politely said he would take a short rest. I went for coffee and came back—he was already

FIG 7 : *Harish-Chandra Research Institute (HRI) earlier known as Mehta Research Institute (MRI). After SPIC, Prof. Adhikari joined HRI.*

working again. He never said, "Don't waste my time." Such humility.

SS: Do you still have students?

SDA: My PhD students have completed, but I still work with people—through collaborations.

SS: How do you see the difference between academia in India and the rest of the world?

SDA: India is doing well. I am not saying otherwise. But in many places abroad, facilities are better—especially libraries. In India, some institutions lack strong libraries, though the National Board of Higher Mathematics (NBHM) has been trying to help by sending books and providing financial support. I served on the NBHM library committee for many years and was associated with NBHM for about 12 years. I know they contribute significantly. Senior mathematicians also initiated programs like MTTS (Mathematical Training and Talent Search), started by Prof. Kumaresan, and later senior student training programs under NCM. I was involved in MTTS for many years and taught there. Many people have contributed—some far more than me.

SS: Do you think going abroad for higher studies helps in returning to Indian academia?

SDA: Going abroad helps because you meet people and exchange ideas. Two brains working together often lead to results. For example, I once shared an idea with Ruzsa. He was listening casually, but Chen picked it up immediately and said it was correct. That led to a paper. However, I don't fully agree with the idea that foreign degrees should automatically carry more weight. Some people have done excellent work entirely in India. We have very good guides and teachers here.

SS: What is NBHM?

SDA: NBHM stands for the National Board of Higher Mathematics. They provide postdoctoral support, travel grants, faculty funding, books for libraries, and organize training programs. They support MTTS and advanced workshops through NCM. The Board was initiated by Prof. M. S. Narasimhan, followed by Prof. Raghunathan and others. Many of us were involved at different stages.

SS: How were your days at the Harish-Chandra Research Institute?

SDA: After DAE took over, I was among the first group to join. "First" here means four people were selected. One person joined after a month, but I happened to join first. So, in terms of station seniority, I was superior to everyone else. After the age of 60, I received a two year extension (with possibility of another two). But I left after six months and I came here to RKMVERI.

SS: How are the research lives at HRI and RKM different?

SDA: Progress is happening. Sometimes it is visible, sometimes not. But underneath, there is always some progress. Funding is an issue, of course. I think gradually private initiatives should also take responsibility. In the US, many excellent universities are private. In India too, that culture is coming up, and I don't think it is necessarily bad. Institutions like Ashoka University in Delhi, Shiv Nadar University, SRM, and others are private. Even Lodha has invested. Professor Kumar Murty is there —very good people are taking care of these institutions. Not everything will work perfectly from the beginning, but over time things will improve. Interactions will increase. Many senior mathematicians like Ram Murty and Kumar Murty frequently came to India and took students abroad. I visited them there, and many people benefited from long-term visits. One chapter of my PhD thesis was actually suggested by Ram Murty, who used to come every year to Chennai to lecture. He was a friend of my guide.

SS: As a teacher, what do you look for in a student who wants to work with you?

SDA: First and foremost, the subject must be natural to that person. I try to see whether their thinking is natural for mathematics. Sometimes a person may be very capable, but meant for something else. Second, the person must be hardworking and motivated. If that initial spark is there, I can help motivate further. But if the basic inclination is missing, I cannot push it. You see, if you take a square object, no matter how much you kick it, it won't roll. But a round ball will. So that initial nature matters. Everyone has some talent. I believe that in nature—God's world—everyone is meant for something. You have to discover what that is and follow it. By doing work that is natural to you, you are worshipping that work. And that itself is worship of God. As Bhagavad Gita (BG 18.46) says

Swakarmana tamabhyarchya siddhim vindati Manavah.

So work—work incessantly, as Swamiji said—but do not bind yourself. Bondage is terrible. Working with expectations of praise, success, or recognition creates bondage. Success will come. Failure will come. Praise will

come. Criticism will come. You continue working and enjoy the work itself. That is what Faiz also said:

Faiz thi raah sarbasar manzil
Hum jahaan pahunche kaamyaaab aaye.

SS: Finally, what would you like to say to upcoming mathematics students?

SDA: When I enter a classroom, I stand with folded hands—internally. You never know who is sitting in front of you. That person may become a great mathematician in the future, perhaps far greater than me. You are initiating someone into a subject. You don't know who will become what. So one should always remain humble, with folded hands before the miracle of God. We are doing very little, really. But this attitude helps both yourself and others. There should be no aham—no ego. An egoistic person thinks, "I did this." But in reality, I met the right person, I encountered the right question, someone else gave the right input, and together we solved it. So where is "me" in this? If nature is doing everything, I am only an instrument.

SS: Thank you sir for your time with us. We truly believe our readers shall have a great time to spend some time with you through the pages of InScight. With that thought, we end today's discussion. Thank you.
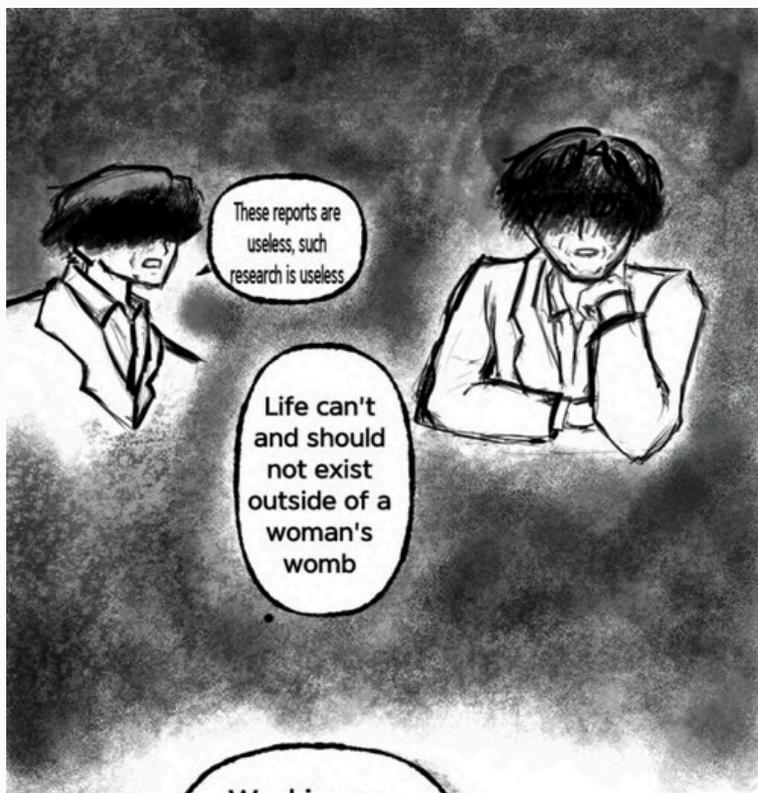

FIG 8 : *Prof.Adhikari is a professor at **Ramakrishna Mission Vivekananda Educational and Research Institute (RKMVERI)** and dean of School of Mathematical Sciences*

World's first ever IVF baby "Louise Brown" was born in England on the 25th of June.

Develop a cryopreservative technique for freezing, store and thaw the embroys before placing them in the uterus.
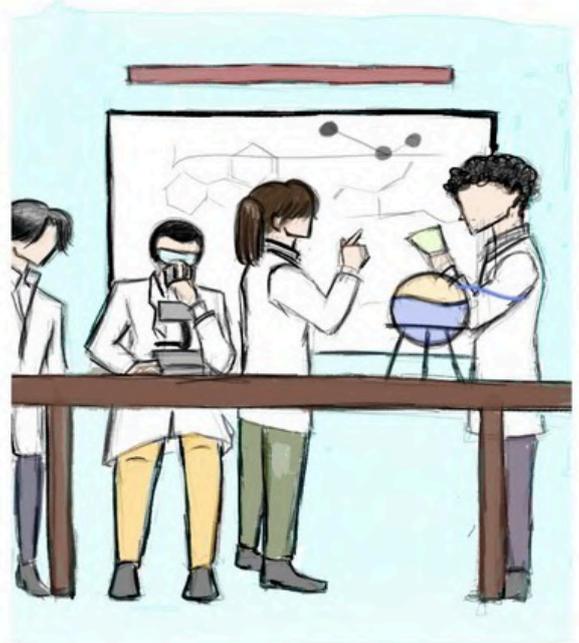
Meanwhile in India

Natural-cycle IVF, did not involve ovarian stimulation drugs and was a groundbreaking process at that time.
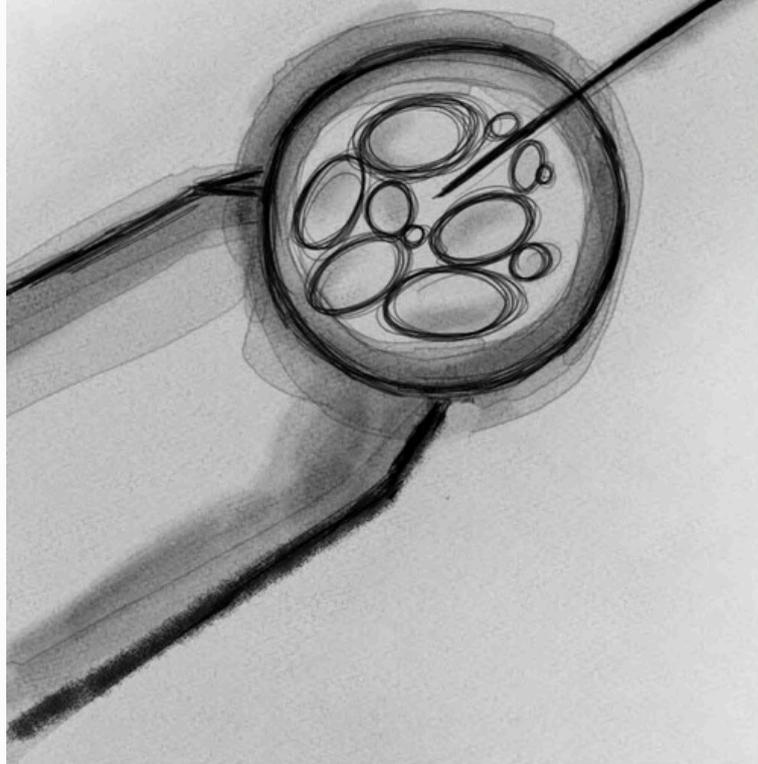

Use of gonadotrophins for ovarian stimulation prior to ovum pick up.

Use transvaginal route by colpotomy for harvesting.

In Vitro Fertilization: The retrieved egg was combined with her husband John Brown's sperm in a laboratory petri dish (often referred to as a "test tube" by the media).

"That day"

This ...this will change everything.

"Durga"
3rd October 1978

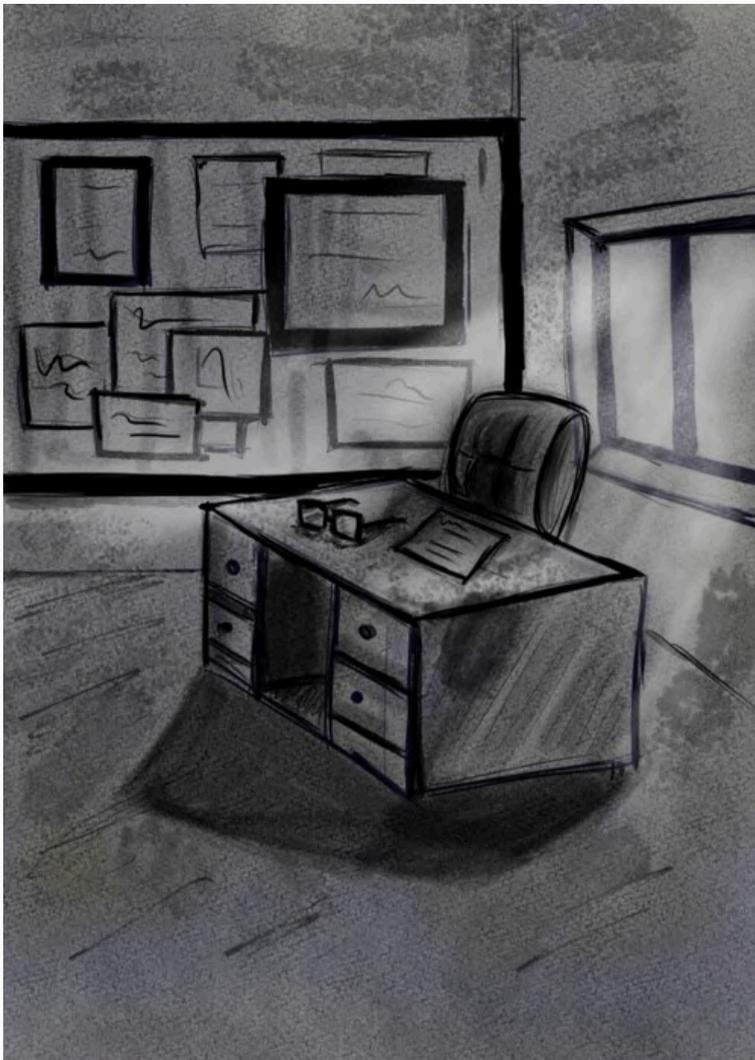**1979**

But his victory
turned to ashes

"He's lying...
He's desperate for fame..."

June 19,1981

" I can't wait to
have a heart
attack ".

The year was
1981, unable
to bear the
humiliation,
he ended his
own life at
just 50.

Decades later ,
the truth was finally
revealed.

In 2020, the
ICMR–National
Institute for
Research in
Reproductive
Health, Mumbai,
published a book
titled "Dr. Subhas
Mukherjee: A
Visionary and
Pioneer of IVF"

Dr. Subhash Mukhopadhyay

India's first IVF pioneer, remembered too late.

Kajori (right) is a student with curiosity in the sciences. Along with pursuing her interest in science, she also indulges in sketching and painting as hobbies. During the lockdown, she started getting into digital art and has since been drawing her favourite anime and comic characters. *Afreen* (left) has always been very keen about nature, particularly biology. She has always loved reading novels and comics, and as someone who nerds on fiction, she wanted to try understanding how these stories are written by creating this comic with Kajori.
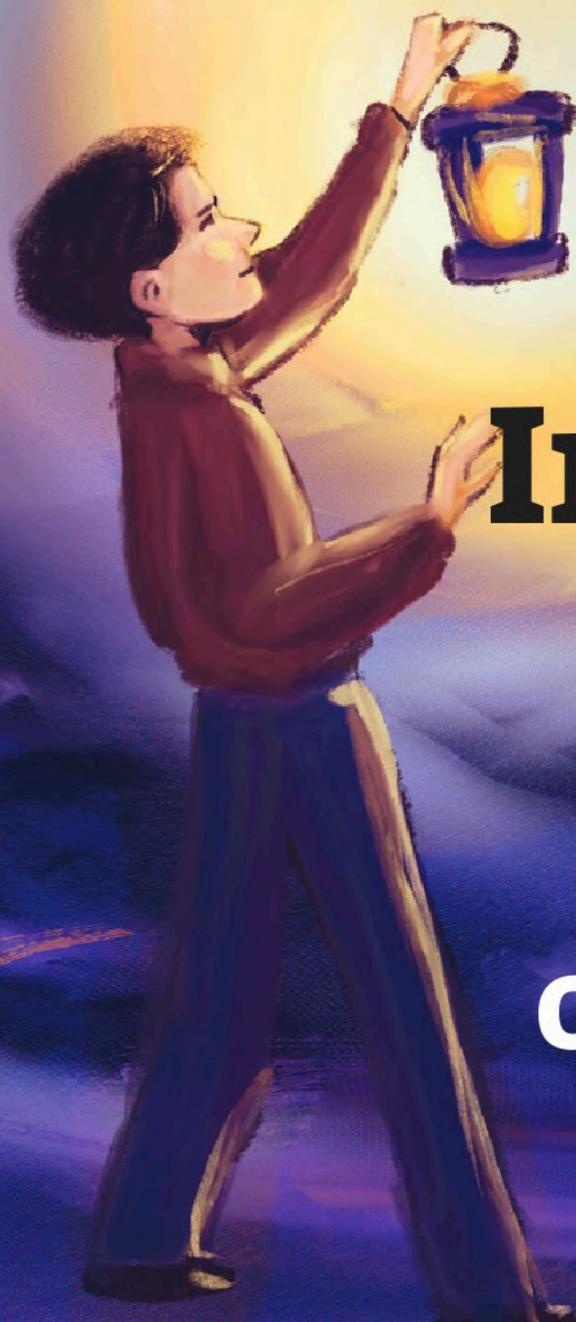
SILENT INDIAN ARCHITECT OF MODERN PHYSICS

– Lipsamayee

Debendra Mohan Bose (1885-1975) was a pioneering India physicist known for his significant research in cosmic rays, artificial radioactivity, and neutron physics.

As a child, he grew up in an environment rich in scientific ideas; some may be thanks to his uncle, J.C.Bose.
From a young age Debendra was fascinated from experiments and this curiosity shaped his approach to science.

Wilson Cloud Chamber

D. M. Bose excelled in his studies in physics at Calcutta University. Later, he worked in laboratories in Cambridge and Berlin. While working abroad, he designed and improved the Wilson cloud chamber, a device used to detect subatomic particles by making their tracks visible in a supersaturated vapour. These experiments placed him among the early pioneers in this field.

After returning to India, he joined Calcutta University. Along with Bibha Choudhuri, he conducted intense. research on cosmic ways. Placing them among first scientists to observe mesons and improve particle physics.

— *Lipsamayee*

Even during WWII, both physicists did not stopped. In a series of articles published in Nature, they identified a cosmic particle having an average mass close to 200 times the mass of electron.
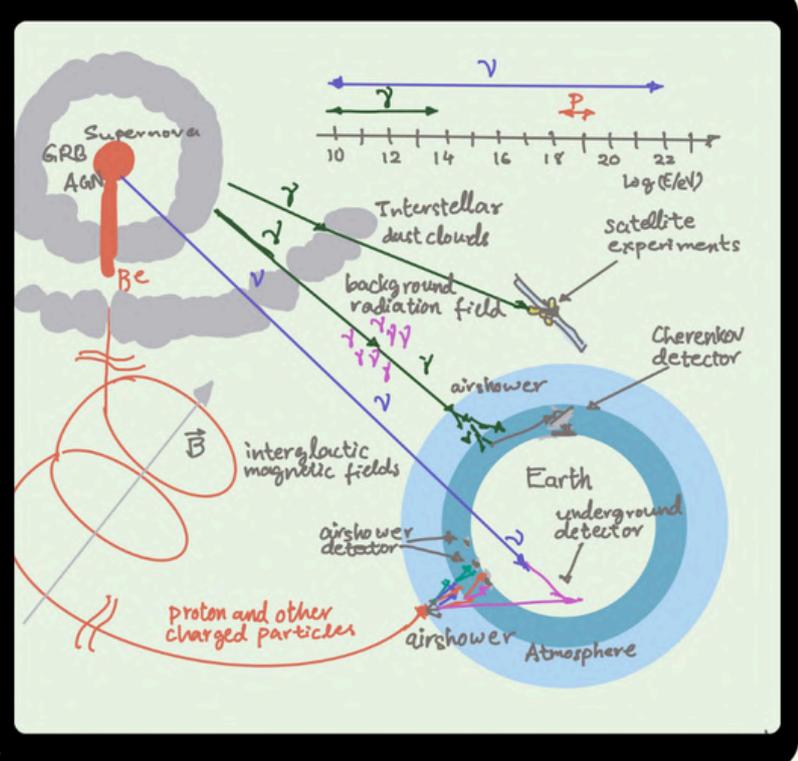
Noble Laureate Cecil Frank Powell was awarded for his study in mesons in 1950. Powell acknowledged the method developed by Bose and Chaudhuri as the first attempt in this field in his 1959 book.

Debendra Mohan Bose played a crucial role in nurturing India's scientific community. He actively supported younger scientists by sharing books, instruments, and ideas that were difficult to access in India at that time. Later, Bose's role as director at the Bose Institute created the academic ecosystem encouraging culture of scientific inquiry.

— Lipsamayee

D. M. Bose
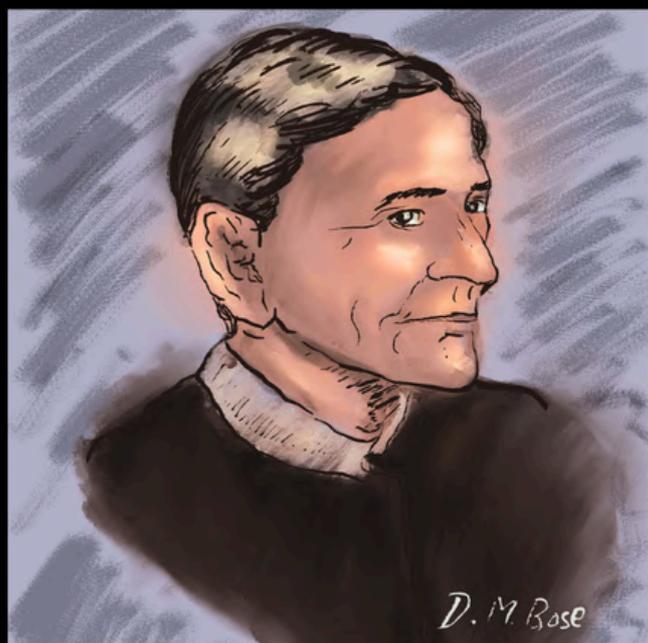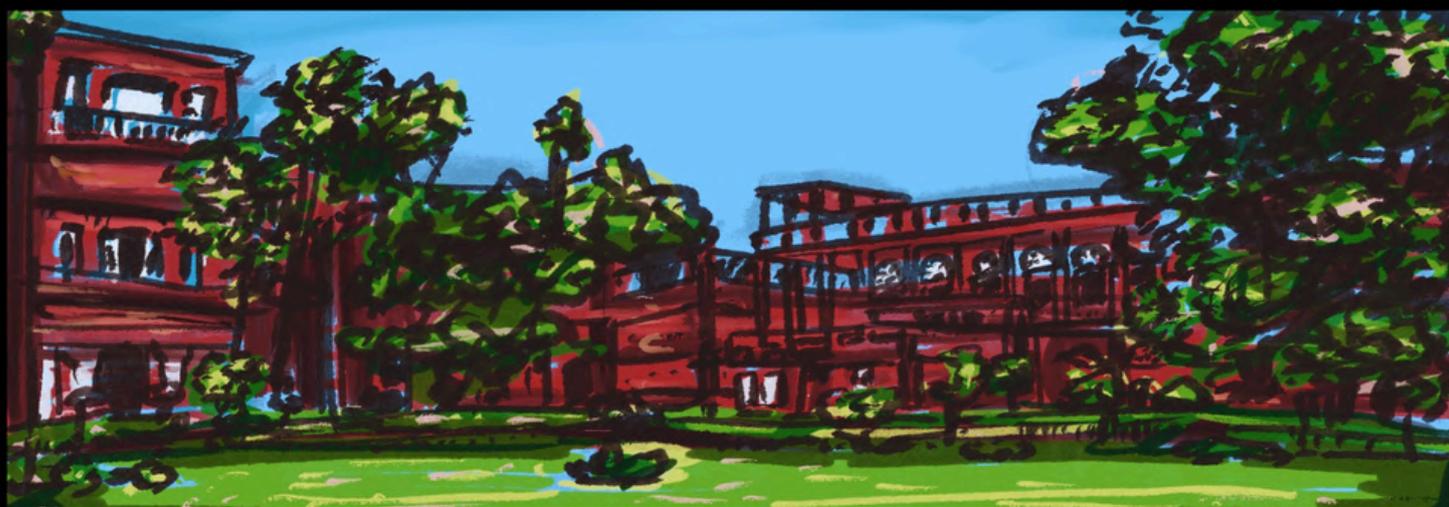
From a curious child fascinated by experiments to a respected scientist and longest serving director of Bose University, Debendra Mohan Bose never lost his curious self. He believed that science was not about fame or awards, rather about unraveling truth and sharing knowledge. His life was the breathing proof of it.

Lipsamayee Mishra *is a student in IISER Kolkata pursuing physics and maths. She is an art enthusiast who is passionate about illustrations and exploring new visual artforms.*
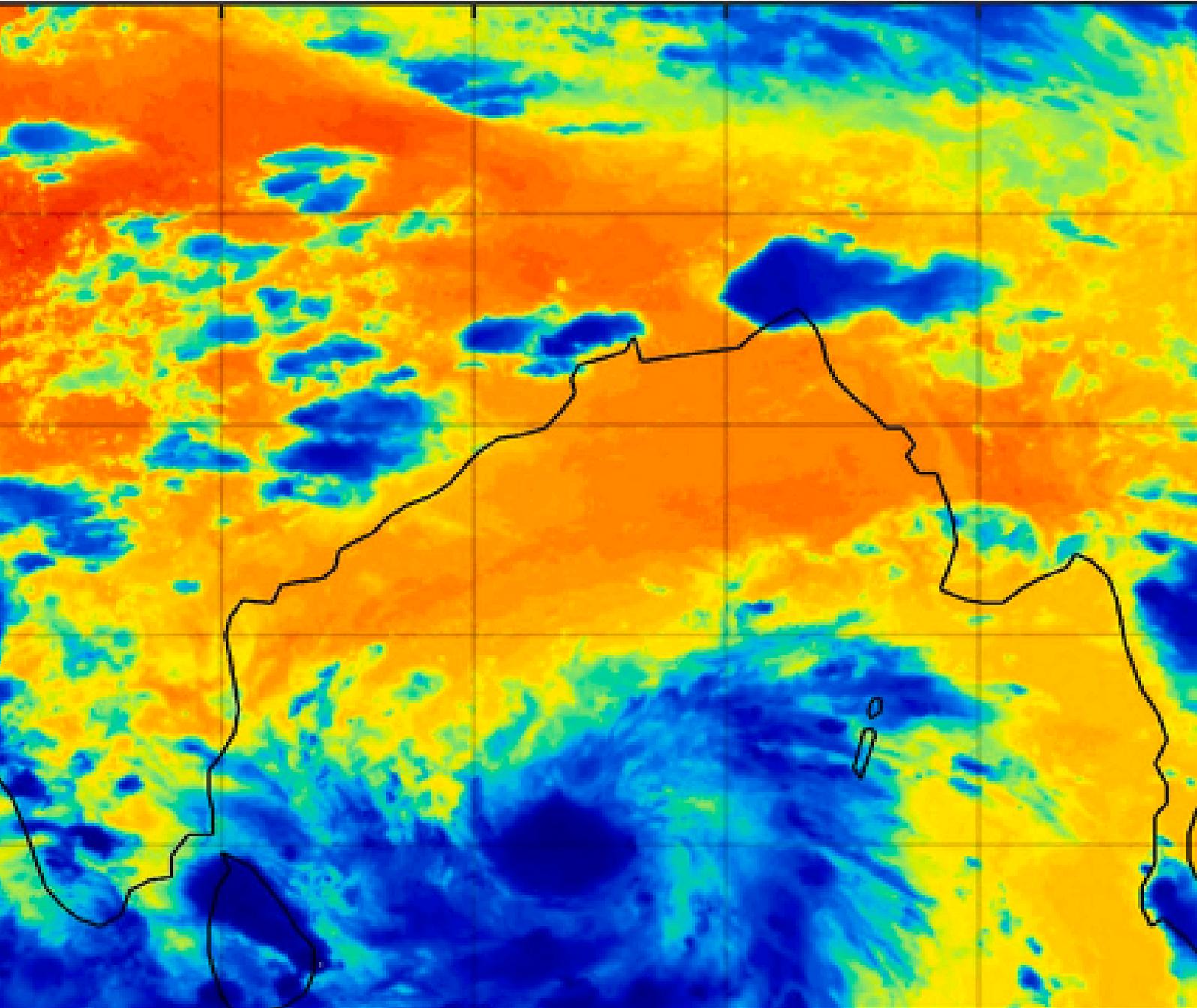
Insight Digest – Curated Science News

Fresh highlights from the frontiers of science

Sukalyan Deb Superconductivity in a 5' twisted bilayer WSe2

Abhinav Prabhu Ligadi The Microbe That Blurs the Boundary of Life

Aranya Mukherjee Finding the Right Balance for more Stable Qubits

Swarnendu Saha When a Cyclone Shakes the Sky: How Super Cyclone Amphan Disturbed Earth''s Upper Atmosphere

🔗 *Also available online, at scicomm.iiserkol.ac.in*

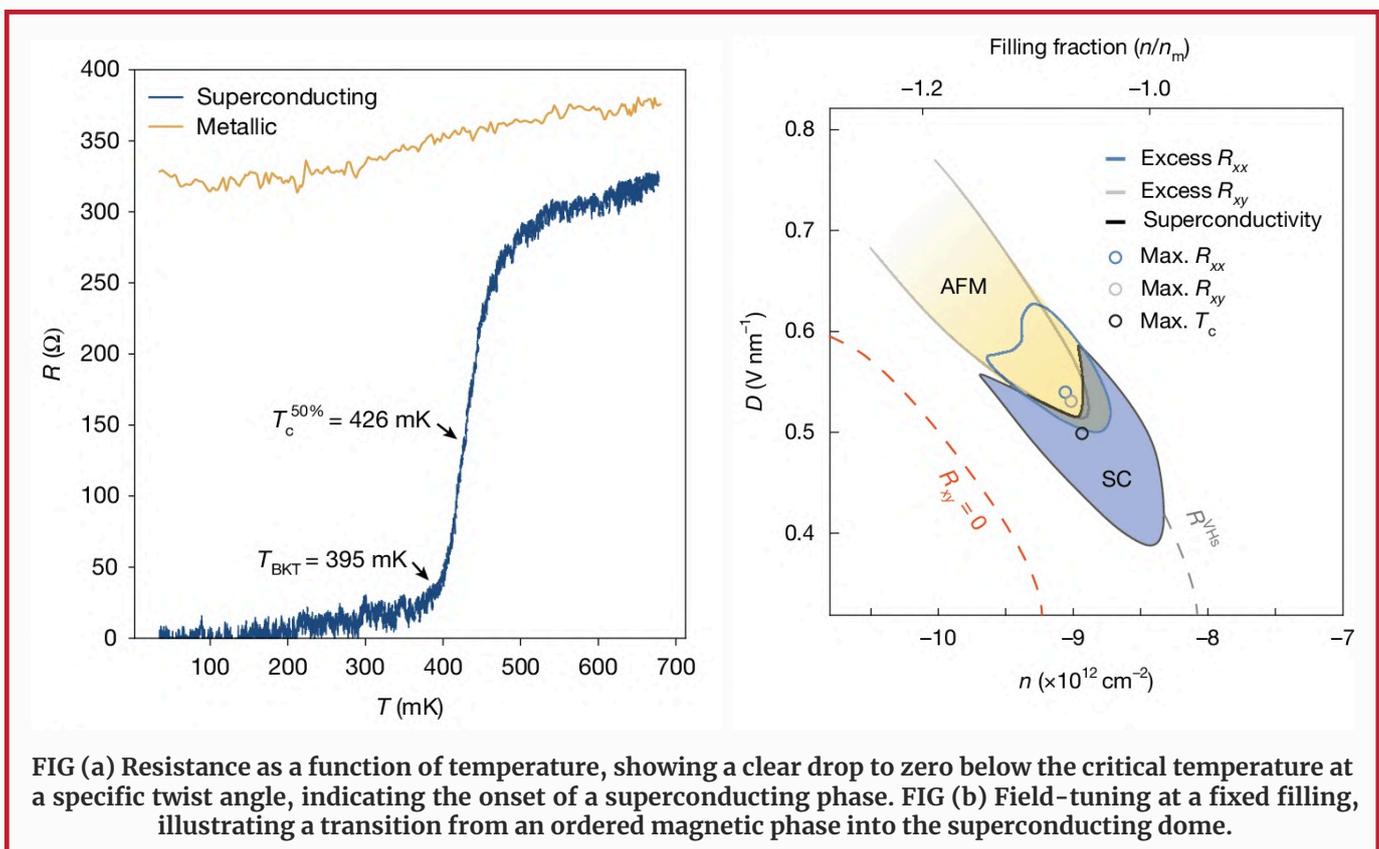# Superconductivity in a 5′ twisted bilayer WSe2

Contributed by **Sukalyan Deb (IISER Kolkata)**

Twisted bilayer graphene is a paradigmatic platform in the regime of the strongly correlated system, and "twisted" here refers to the slight rotation between the layers of graphene(1.1′) , known as the magic angle, which creates a unique electronic structure with 'flat bands'. The flat band suppresses the kinetic energy relative to the coulomb enabling rich phases like magnetic ordering, unconventional superconductivity, and Chern ferromagnets , and associated quantum anomalous Hall states which appears depends sensitively on carrier density, displacement field, twist-angle homogeneity, and substrate alignment. A closely related material is twisted bilayer tungsten diselenide (tWSe$_2$),formed from two monolayers of the transition-metal dichalcogenide (TMD) but it also brings crucial new ingredients: strong spin-orbit coupling, robust spin-valley locking. These features reshape the effective low-energy models (often mapped to extended Hubbard models with valley degrees of freedom) and provide powerful electric-field control of bandwidth, band topology, and valley polarization. Experimentally, unconventional superconductivity has been reported in tWSe$_2$ at a 5-degree twist with applying an external electric field.

At small twist angles, the two layers hybridize strongly and generate bands whose bandwidth can be comparable to the on-site interaction U , placing the system in an intermediate-coupling regime. In this window, neither a weak-coupling metal nor a fully Mott-localized insulator is guaranteed ; modest changes in control parameters can tip the balance. A perpendicular displacement field is especially powerful, distorts the bands, shifts van Hove singularities (critical points in the electronic band structure where the slope of the energy dispersion is zero) . Transport reflects this tunability ,without the twist the resistivity is metallic across dopings and displacement fields—decreasing with temperature and showing no sign of a gap. As the twist angle brings the system into the flat-band regime and the displacement field is tuned, the normal state can evolve from a conventional metal to a correlated metal with enhanced scattering . Upon approaching optimal fillings—where the density of states is elevated and screening is reduced—the system enters a superconducting phase typically forming a dome in the displacement-field-density plane. On the flanks of this dome , indicating that pairing emerges in close proximity to other interaction-driven orders.In flat-band graphene systems, people have proposed that ordinary phonon-mediated pairing could produce superconductivity, and similar ideas exist for tWSe$_2$. This close tie between superconductivity and magnetism points instead to a magnetically mediated pairing mechanism, e.g. via spin fluctuations, rather than conventional phonons.

The study offers a fresh perspective on the interplay between magnetic ordering and superconductivity that goes beyond the scope of standard transport measurements. It also provides a useful comparison with known graphene and TMD systems, highlighting the crucial role played by twist angle in shaping these correlated phases.

FIG (a) Resistance as a function of temperature, showing a clear drop to zero below the critical temperature at a specific twist angle, indicating the onset of a superconducting phase. FIG (b) Field-tuning at a fixed filling, illustrating a transition from an ordered magnetic phase into the superconducting dome.

The Microbe That Blurs the Boundary of Life

Ryo Harada, Yuki Nishimura, Mami Nomura, Akinori Yabuki, Kogiku Shiba, Kazuo Inaba, Yuji Inagaki, Takuro Nakayama. bioRxiv 2025.05.02.651781

Contributed by **Abhinav Prabhu Ligadi (Department of Biological Sciences)**
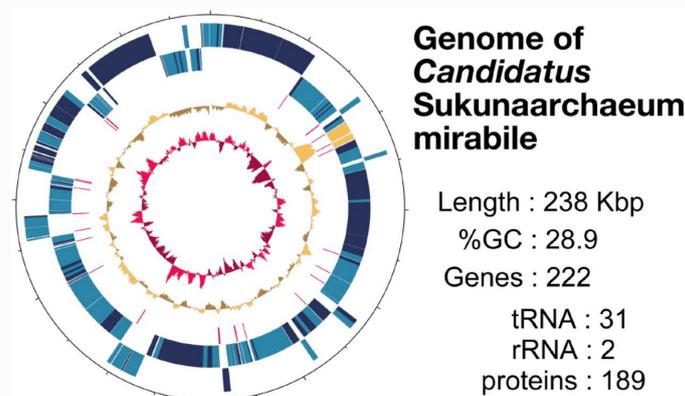
For more than a century, biology has maintained a sharp distinction between viruses and cells. Cells grow, divide, and metabolize independently; viruses remain inert until they hijack a host cell. But a discovery by Harada et al., reported in their seminal paper, has challenged this comfortable binary. They identified Candidatus Sukunaarchaeum mirabile—a novel archaeon so extraordinarily reduced in its genetic material that it exists in a conceptual twilight zone between living cells and viral parasites.

The discovery began almost by accident. Researchers at the University of Tsukuba in Japan were sequencing the complete genomic contents of a dinoflagellate—a marine plankton organism called Citharistes regius—which was already known to harbor symbiotic cyanobacteria. Among the expected genetic sequences, they found something entirely unexpected: a small, circular loop of DNA only 238,000 base pairs long. This was remarkable because the previous record for the smallest known archaeal genome stood at 490,000 base pairs. The newly discovered genome was less than half that size, yet phylogenetic analyses confirmed that it belonged to a cell—an archaeon—not a virus. The organism bearing this genome was named after Sukuna-biko-na, a small-statured deity in Japanese mythology, embodying both its diminutive scale and its position between two worlds: Candidatus Sukunaarchaeum mirabile (mirabile meaning "remarkable" in Latin).

What makes Sukunaarchaeum so extraordinary is not merely the size of its genome, but what that genome contains—and more tellingly, what it lacks. The 238 kilobase genome encodes a profoundly stripped-down, replicative core consisting of the machinery necessary for DNA replication, transcription, and translation. It contains approximately 189 protein-coding genes, nearly all dedicated to self-propagation. However, it lacks virtually all recognizable metabolic pathways. The archaeon cannot synthesize the amino acids that build proteins, cannot produce the nucleotides that build

DNA, and cannot generate its own energy. This extreme reduction places it in an unprecedented position: it is a cell in structure yet functions like a parasite outsourcing nearly every biological process except replication to its host, the dinoflagellate Citharistes regius. In this sense, it resembles a virus more than any previously known cellular organism—yet it retains the ribosomal machinery and genetic architecture of true cellular life. The implications of this discovery extend far beyond taxonomy. The genome organization of Sukunaarchaeum offers a rare window into fundamental questions about the nature of life itself. What are the minimal genetic requirements for an entity to be considered "alive"? Is cellular life defined by metabolic autonomy, or can an organism retain that designation by virtue of possessing its own replication machinery? Harada and colleagues argue that Sukunaarchaeum challenges conventional functional distinctions between minimal cellular life and viruses, suggesting that the boundary between these categories may not be as absolute as traditionally conceived.

From an evolutionary perspective, Sukunaarchaeum may represent a living snapshot of an ancient transition Many evolutionary biologists propose that early life originated from simple, highly dependent molecular systems—perhaps RNA-based replicators that gradually acquired metabolic sophistication and independence The authors suggest that Sukunaarchaeum could be a remnant of such an intermediate state, a surviving "evolutionary intermediate" that has managed to persist within symbiotic relationships over billions of years If this interpretation is correct, the archaeon offers unprecedented empirical evidence for a gradualist model of life''s emergence from non-living chemistry Furthermore, environmental sequence data analyzed by the research team revealed that this is not an isolated curiosity. For researchers working on synthetic biology and the design of minimal cells, Sukunaarchaeum offers both inspiration and caution. It demonstrates that a cell can function with far fewer genes than previously

Genome of *Candidatus* Sukunaarchaeum mirabile

Length : 238 Kbp
%GC : 28.9
Genes : 222
   tRNA : 31
   rRNA : 2
   proteins : 189

The genome map of Sukunaarchaeum. From outermost to innermost circle, the positions of protein-coding genes and rRNA genes on the +/-strands, tRNA genes, GC content, and GC skew are shown. Color codes for the outermost and 2nd outermost circle: Blue, genes of unknown function; light blue, genes of known function; yellow, rRNA genes.

Finding the Right Balance for more Stable Qubits

Bassi, M., Rodríguez-Mena, E.A., Brun, B. et al. Optimal operation of hole spin qubits. Nat. Phys. (2025)

Contributed by **Aranya Mukherjee (Department of Physical Sciences)**

Quantum computers are often discussed as futuristic machines, but their development depends on solving practical problems at extremely small scales. One such challenge is how to operate qubits–the basic units of quantum information–in a way that is both fast and stable. A recent paper published in Nature Physics addresses this issue for a promising type of qubit known as a hole spin qubit, offering new insight into how these systems can be controlled more reliably.
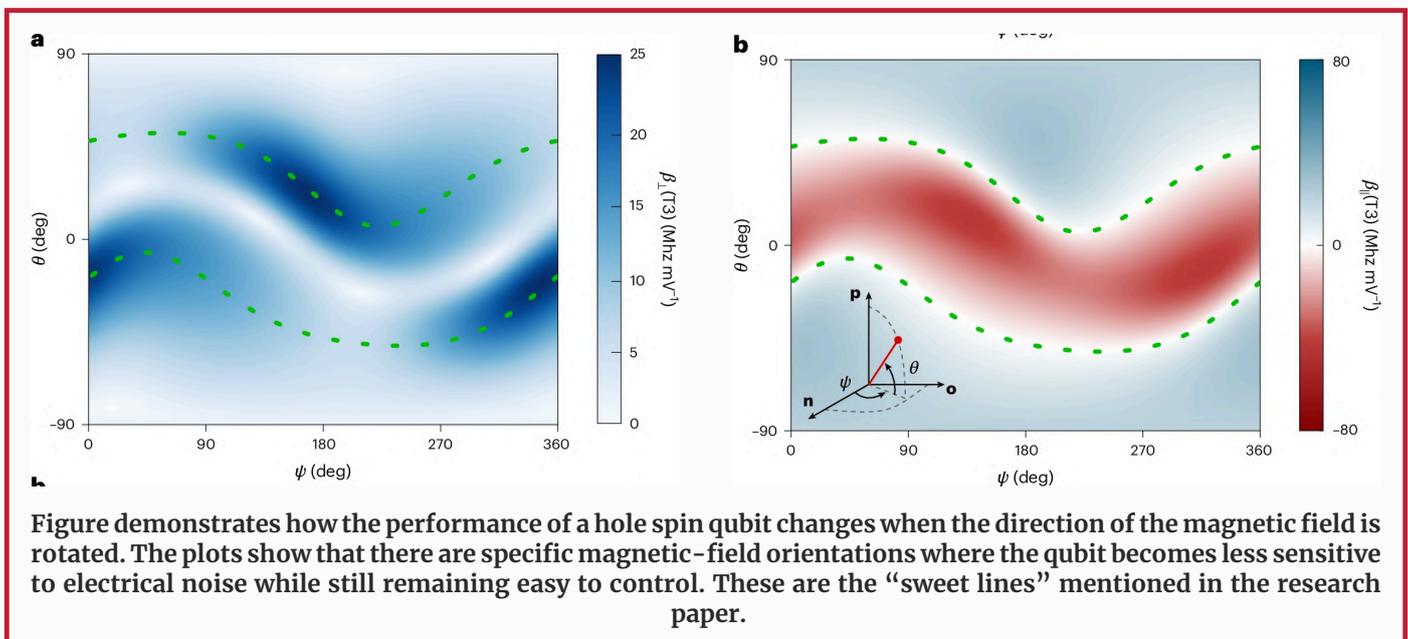
The work was carried out by researchers at Université Grenoble Alpes, CEA-IRIG (France), and collaborating institutions, led by Silvano De Franceschi and Vivien Schmitt. Their study focuses on semiconductor-based qubits, particularly those built using silicon. This is significant because silicon qubits can, in principle, be fabricated using existing industrial techniques, making them attractive for scalable quantum computing.

Unlike electron-based qubits, hole spin qubits rely on "holes," which are the absence of electrons in a material. These holes possess a quantum property called spin that can store information. Hole spins respond strongly to electric fields, allowing for fast electrical control. However, this same sensitivity also makes them vulnerable to electrical noise, which can disturb quantum states and reduce qubit performance.

To address this, the researchers explored how the qubits behave under different operating conditions. By carefully changing the orientation of an external magnetic field they identified specific regions, referred to as "sweet lines," where the qubits become much less sensitive to charge noise. At these operating points, stability improves while efficient electrical control is preserved striking an essential balance for quantum operations.

An important result of the study is that these optimal conditions are not fixed. By adjusting voltages on nearby electrodes, the position of the sweet lines can be shifted allowing multiple qubits to be tuned into low-noise regimes at the same time. This tunability is crucial for building larger quantum processors, where many qubits must operate together in a controlled and reliable manner. The research builds on years of experimental and theoretical work in semiconductor quantum systems and combines advanced nanofabrication, modeling and measurements performed at extremely low temperatures. Key contributors include Marion Bassi Esteban-Alonso Rodríguez-Mena, Boris Brun, Simon Zihlmann, and others, reflecting the collaborative nature of modern physics research. While this study does not present a complete quantum computer, it resolves an important operational challenge and provides clear guidance for how hole spin qubits can be used effectively Incremental advances like this play a central role in turning quantum computing into a scalable and practical technology.

Figure demonstrates how the performance of a hole spin qubit changes when the direction of the magnetic field is rotated. The plots show that there are specific magnetic-field orientations where the qubit becomes less sensitive to electrical noise while still remaining easy to control. These are the "sweet lines" mentioned in the research paper.

When a Cyclone Shakes the Sky: How Super Cyclone Amphan Disturbed Earth''s Upper Atmosphere

Contributed by **Swarnendu Saha (20MS, IISER Kolkata)**

Tropical cyclones are usually studied for the destruction they cause at Earth''s surface–strong winds, heavy rainfall, storm surges, and flooding. This study shows that their influence does not stop there. Using Super Cyclone Amphan (May 2020) over the Bay of Bengal as a case study, we demonstrate that intense tropical cyclones can also disturb the upper layers of Earth''s atmosphere, including regions that host satellites and radio communication systems.
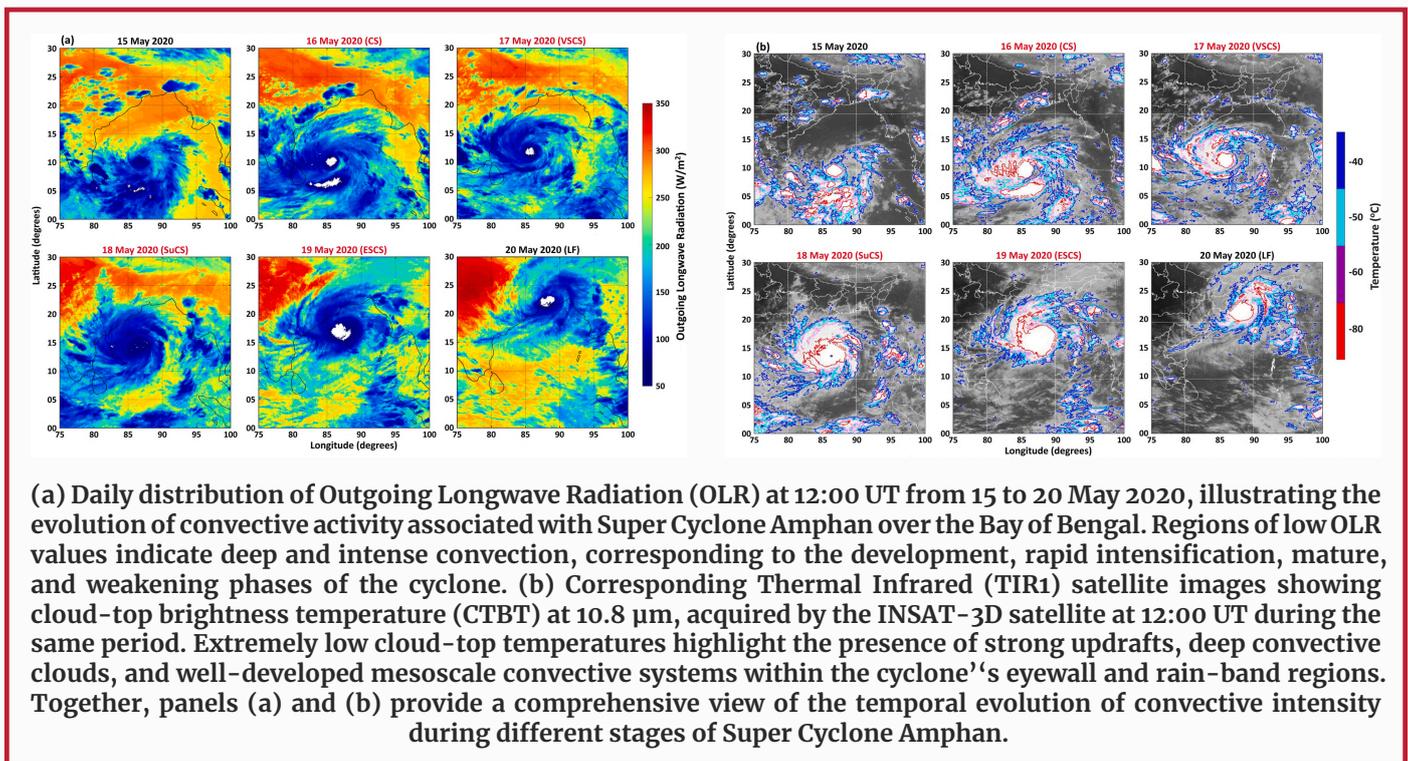
The most interesting finding of this work is the clear, multi-altitude link between intense cyclone-driven convection, lightning activity, atmospheric gravity waves, and disturbances in the lower ionosphere. By combining observations from several satellites, we show that Amphan''s powerful convective clouds and strong lightning activity generated atmospheric gravity waves– ripples in the atmosphere similar to waves produced when a stone is thrown into water. These waves traveled upward from the lower atmosphere through the stratosphere and mesosphere, reaching the lower thermosphere and ionosphere.

A particularly novel aspect of this study is its detailed focus on cloud-to-cloud (CC) lightning, which has been largely overlooked in cyclone studies over the Indian region. We find that CC lightning intensified during Amphan''s rapid strengthening phase, especially within the eyewall region, and that a large fraction of these flashes were high-energy and positively charged. This lightning behaviour closely followed the cyclone''s intensification and weakening, suggesting that CC lightning can serve as a valuable indicator of strong internal storm dynamics.

Why does this matter? For the space-weather and ionospheric community, this study strengthens the evidence that weather systems near Earth''s surface can directly affect the near-space environment through gravity waves. Such disturbances can influence radio wave propagation, satellite drag, and navigation systems like GPS. For meteorologists and climate scientists, the results highlight lightning and gravity waves as key but under-represented components of cyclone dynamics offering new pathways to improve storm monitoring and modelling.

More broadly, this work enriches Earth system science by demonstrating how tightly connected different atmospheric layers are–from the ocean-driven cyclone at the surface to electrical and wave processes over 100 km above Earth. By bridging meteorology, atmospheric electricity, and space physics, this study helps move us closer to a truly integrated understanding of how extreme weather events impact the entire Earth-atmosphere-space system.

(a) Daily distribution of Outgoing Longwave Radiation (OLR) at 12:00 UT from 15 to 20 May 2020, illustrating the evolution of convective activity associated with Super Cyclone Amphan over the Bay of Bengal. Regions of low OLR values indicate deep and intense convection, corresponding to the development, rapid intensification, mature, and weakening phases of the cyclone. (b) Corresponding Thermal Infrared (TIR1) satellite images showing cloud-top brightness temperature (CTBT) at 10.8 μm, acquired by the INSAT-3D satellite at 12:00 UT during the same period. Extremely low cloud-top temperatures highlight the presence of strong updrafts, deep convective clouds, and well-developed mesoscale convective systems within the cyclone''s eyewall and rain-band regions. Together, panels (a) and (b) provide a comprehensive view of the temporal evolution of convective intensity during different stages of Super Cyclone Amphan.

Science Games

Maths Quiz Quiz questions drawn from basic and advanced mathematics.

Themed Crossword | Nobel Prizes The theme for this issue are the Nobel prizes.

Linked List Link each term with the next, and complete the science word chain!

Who Am I? — Institutions of Science Can you guess the names of these science universities and institutions?

Pop Quiz — For The Maths Wiz

Q1. While studying atmospheric dynamics using early computer simulations, a mathematician noticed that extremely small changes in initial conditions could lead to vastly different outcomes. This sensitivity to initial conditions later became a foundational idea in chaos theory and is popularly known as the "Butterfly Effect." Which mathematician first identified this phenomenon?
- I. Benoit Mandelbrot
- II. John von Neumann
- III. Edward Lorenz
- IV. Alan Turing

Q2. A reclusive Russian mathematician stunned the mathematical community in the early 2000s by resolving the century-old Poincaré Conjecture using Ricci flow techniques. Despite this achievement, he declined both the Fields Medal and the $1,000,000 Clay Millennium Prize, citing ethical reasons and fairness toward other contributors. Who is this mathematician?
- I. Andrew Wiles
- II. Grigori Perelman
- III. William Thurston
- IV. Richard Hamilton

Q3. A famous unsolved problem in number theory proposes that every even integer greater than 2 can be expressed as the sum of two prime numbers. This conjecture is one of the Clay Mathematics Institute's Millennium Prize Problems, carrying a reward of $1 million. What is this conjecture called?
- I. Riemann Hypothesis
- II. Goldbach's Conjecture
- III. Twin Prime Conjecture
- IV. Collatz Conjecture

Q4. The term "googol," meaning a 1 followed by 100 zeroes, was coined in 1938 by the young nephew of American mathematician Edward Kasner. What was the boy's name?
- I. Milton Sirotta
- II. Alan Sirotta
- III. David Kasner
- IV. Samuel Newton

Q5. In the Fibonacci sequence (1, 1, 2, 3, 5, 8, ...), the ratio of consecutive terms approaches a fixed irrational number as the sequence progresses. What is this number called?
- I. Euler's Number
- II. Pi
- III. The Golden Ratio
- IV. Euler-Mascheroni Constant

Q6. One of the most important mathematical tools in science and engineering was developed independently in the 17th century by Sir Isaac Newton and Gottfried Wilhelm Leibniz. What is this tool?
- I. Linear Algebra
- II. Differential Equations
- III. Calculus
- IV. Probability Theory

Q7. In the decimal expansion of π, the first occurrence of a digit repeating six times consecutively appears at the 762nd decimal place. Which digit repeats, and what is this point famously called?
 I.   Digit 8, Euler Point
  II. Digit 7, Ramanujan Point
  III. Digit 9, Feynman Point
  IV. Digit 6, Gauss Point

Q8. One of the most celebrated formulas in mathematics links five fundamental constants – e, i, π, 1, and 0 – in a single elegant identity, $e^{i\pi} + 1 = 0$. This equation is often regarded as the most beautiful in mathematics and arises from a deeper exponential relationship in complex analysis. Which mathematician first formulated this relationship?
  I.   Carl Friedrich Gauss
  II. Leonhard Euler
  III. Joseph Fourier
  IV. Augustin-Louis Cauchy

Q9. An ancient Greek mathematician, often referred to as the "Father of Geometry," authored the influential work Elements, which served as the standard geometry textbook for more than 2,000 years. Who was this mathematician?
  I.   Archimedes
  II. Pythagoras
  III. Euclid
  IV. Apollonius
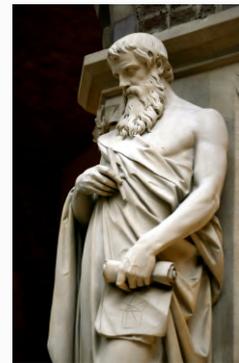
Q10. Amicable numbers are pairs of integers where each number equals the sum of the proper divisors of the other. In ancient times, these numbers were believed to promote which human emotion?
 I.   Wisdom
  II. Loyalty
  III. Love
  IV. Courage

Q11. The development of non-Euclidean geometry in the 19th century profoundly altered our understanding of space and curvature. This mathematical framework later became essential to which of Albert Einstein's major theories?
  I.   Special Relativity
  II. Quantum Mechanics
  III. General Relativity
  IV. Grand Unified Theory

Q12. In this picture you can see this has only one side. An ant walking along it covers what seems like both sides before returning to its origin, flipped, showing no distinction between "top" and "bottom".
  I.   Mobius strip
  II. Klein Bottle
  III. Projective Plane
  IV. Twisted Cylinder
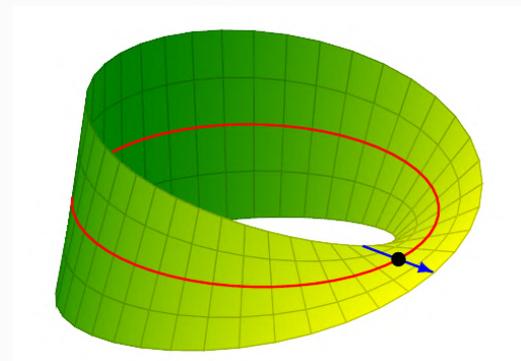
Answers can be found at the end of the issue. For an interactive version of the quiz, check out our __website__

Who Am I? — Name These Institutions of Science

Guess the names of the scientific institutions/universities from the hints.

Which research institution am I?

· I began as an industrial research facility in the United States.

· I produced breakthroughs that reshaped communication technology.

· The transistor was developed within my walls.

· Many of my scientists became Nobel laureates.

Which national research laboratory am I?

· I was created in India after independence to support scientific research.

· I function under a national research council.

· I focus on fundamental and applied chemical sciences.

· I am located in Pune.

Which university am I?

· I emerged from medieval Europe and shaped the modern idea of a university.

· My name is linked to a process that harmonized higher education systems.

· I have taught students for nearly a millennium.

· I am located in Italy.

Which group of institutions am I?

· I am a network of institutions in India dedicated to research in natural sciences.

· I emphasize undergraduate research and interdisciplinary learning.

· I follow a model inspired by the Indian Institute of Science.

· I have multiple campuses across India.

Linked List — General Science Edition

Linked List is a general science-based word game. The rules are straightforward:

1. The goal is to guess eleven words that have been drawn from science.
2. The first word (the seed) will be provided to you, and hints and number of letters will be provided for the remaining words.
3. You are also informed that the first letter of any word is the last letter of the previous word. So the first letter of the second word will be the last letter of the seed word, the first letter of the third word is the last letter of the second word, and so on.
4. This property goes all the way, so that the last letter of the last (eleventh) word is also the first letter of the seed word.

Find all the words!

Today's seed: **NUCLEOPHILIC**

1. A fundamental property of matter that gives rise to electric and magnetic interactions. (6)

C					

2. The capacity to do work or produce change, appearing in forms such as kinetic, thermal, and electromagnetic energy. (6)

3. The nutrient-rich core of an egg that provides energy and raw materials for early biological development. (4)

4. The branch of physics that describes motion in terms of position, velocity, and acceleration, without considering forces. (10)

5. Collections of interacting components whose overall behavior emerges from their mutual relationships. (7)

6. A property of physical or mathematical systems that remain invariant under transformations such as rotation or reflection. (8)

7. A positively charged ion of the element yttrium, commonly encountered in materials science and solid-state physics. (7)

8. Interconnected structures used to represent relationships in fields ranging from biology and physics to computer science. (8)

9. Mathematical techniques used to analyze data, quantify uncertainty, and identify patterns in complex systems. (10)
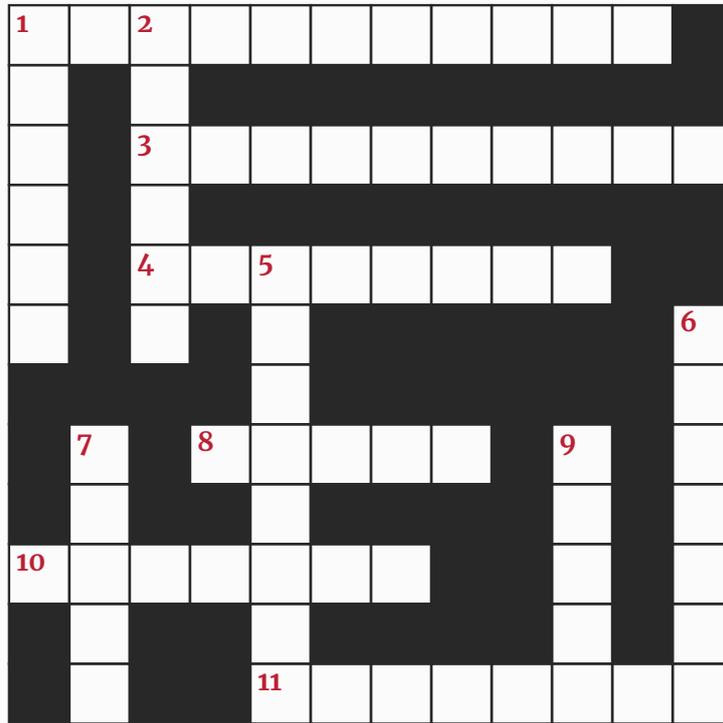

10. The use of computational or mathematical models to imitate real-world processes and study their behavior. (10)

								N	

*Solution can be found at the end of the issue. For an interactive version of this game, check out our **website**.*

Themed Crossword — Nobel prizes

This issue's crossword is based on Nobel prize trivia.

Across

1. Youngest Nobel Prize laureate ever (Peace Prize, age 17). (6)
2. First Asian to receive the Nobel Prize in Literature. (6)
5. Alfred Nobel's invention whose commercial success funded the Nobel Prizes. (8)
6. Physicist who won the Nobel Prize twice (transistor and superconductivity). (7)
7. Nobel Prize category awarded in Oslo instead of Stockholm. (5)
9. Only person to win Nobel Prizes in two different sciences (Physics & Chemistry). (5)

Down

1. Subject for which no Nobel Prize exists. (11)
3. Oldest Nobel laureate ever (Chemistry, age 97). (10)
4. Organization awarded the Nobel Peace Prize three times. (8)
8. First scientist from India to be awarded a Nobel Prize in a scientific category. (5)
10. Only person to receive two unshared Nobel Prizes. (7)
11. Scientist awarded the Nobel Prize for the photoelectric effect. (8)

Solution can be found at the end of the issue. For an interactive version of the crossword, check out our website.

Academic Listings: Internships, PhDs, Post-docs

Position titles are hyperlinked

INTERNSHIPS

Summer Student Program 2026 at HZDR   🕐 Deadline: 2026-02-22

DoS/ISRO Internship & Student Project Trainee Schemes   🕐 Deadline: —

Visiting Student Research Program at KAUST   🕐 Deadline: —

Long Term Visiting Students Program   🕐 Deadline: —

Undergraduate Student Internships Max Planck Institute for Gravitatinal Waves   🕐 Deadline: —

Undergraduate Student Internships Max Planck Institute for Gravitatinal Waves   🕐 Deadline: —

Undergraduate Student Internships Max Planck Institute for Gravitatinal Waves   🕐 Deadline: —

Undergraduate Student Internships Max Planck Institute for Gravitatinal Waves   🕐 Deadline: —

University of Tokyo UTRIP   🕐 Deadline: —

2026 Summer Student Programme IISER Pune   🕐 Deadline: —

PHD POSITIONS

Max Planck Institute for Informatics - PhD Applications   🕐 Deadline: —

Exoplanet Characterisation Predictions via Gravitational Microlensing: Competition Funded PhD Project   🕐 Deadline: —

University of Chicago Graduate School Programme   🕐 Deadline: —

University of Texas at Austin Graduate School Programme   🕐 Deadline: —

University of Utah Graduate School Programme   🕐 Deadline: —

Louisiana State University PhD Programme   🕐 Deadline: —

PhD Position - EPFL   🕐 Deadline: —

Doctoral (PhD) and postdoctoral position (m/f/d) in Computational Biology | Biomathematics | Biosystems Engineering   🕐 Deadline: —

Doctoral (PhD) and postdoctoral position (m/f/d) in Computational Biology | Biomathematics | Biosystems Engineering   🕐 Deadline: —

Doctoral (PhD) position (m/f/d) in Glia Biology | Neurobiology at the Dioscuri Centre for Chromatin Biology and Epigenomics at the Nencki Institute for Experimental biology (Poland)   ⏰ Deadline: —

PHD in geospatial analytics   ⏰ Deadline: —

POSTDOCTORAL AND OTHER POSITIONS

Cosmological frontiers in fundamental phsyics 2026   ⏰ Deadline: 2026-02-16

Nasa postdoc fellowships   ⏰ Deadline: 2026-05-01

HRDG- Nehru Science Postdoctoral research Fellowship   ⏰ Deadline: —

Postdoc position (m/f/d) | Development and Application of an Ultra-Bright Coherent Soft X-Ray Beamline   ⏰ Deadline: —

Doctoral (PhD) and postdoctoral position (m/f/d) in Computational Biology | Biomathematics | Biosystems Engineering   ⏰ Deadline: —

Postdoc/Research Scientist in the Reproductive Ageing Research Group (m/f/d)   ⏰ Deadline: —

Reasearch fellow in geophysics   ⏰ Deadline: —

Geospatial AI scientist   ⏰ Deadline: —

Join the Conversation

Contribute

Contributions are welcome from students and faculty members across all academic institutions, on a rolling basis. Submission portals for all types of content can be found on our website. More detailed guidelines are also available there, so please refer to the linked page if you're interested in contributing. For any queries, feel free to contact us at *scicomm@iiserkol.ac.in*.

Science Articles

We publish engaging articles on scientific ideas, provided they are accessible to a broad audience and not limited to experts. Submit your articles **here**.

Quizzes and Games

We feature science-themed games such as crosswords, quizzes, and word-link challenges. If you have content for these or ideas for new games, please email us at *scicomm@iiserkol.ac.in*.

Short Summaries

To showcase cutting-edge research, we publish short summaries (350–400 words) of recently published scientific papers. The summary should broadly outline the research questions and highlight the key findings.. Submit your research stories **here**.

Interview Recommendations

If you would like us to interview a particular scientific personality, or if you have an interview you'd like us to consider for publication, please reach out to us at *scicomm@iiserkol.ac.in*.

More importantly, we would love to hear any feedback that you might have about this endeavour, so please send us your comments at *scicomm@iiserkol.ac.in*.

Join our team

We are currently looking to expand our team. If you are interested in what we do and are confident that you will be able to devote time to this, please reach out to us at *scicomm@iiserkol.ac.in*. We will be happy to discuss possible roles for you depending on your skills.

Chief Editor

Swarnendu Saha

Advisors

Prof. Somnath Basu
Prof. Anindita Bhadra
Prof. Subhajit Bandyopadhyay

Editors

Aranya Mukherjee
Archita Sarkar
Ayan Biswas
Budhaditya Banerjee
Debanuj Chatterjee
Dishari Dasgupta
Ishita Bardhan
Janusri D
Medha Roy Choudhury
Puppala Ainesh Kumar
Sharanya Chatterjee
Shibaraj Sahu
Suddhajit Bishyee
Sukalyan Deb
Suman Halder

Design & Website

Abhirup Mukherjee
Madhura Theng
Mayank Acharya
Meghithdharshan R
Shiikhar Chandra
Thushiyantheswaran

Social Media & Games

Alekhya Kundu
Sayan Saha

The Last Page

Crossword

Across

1. MALALA
2. TAGORE
5. DYNAMITE
6. BARDEEN
7. PEACE
9. CURIE

Down

1. MATHEMATICS
3. GOODENOUGH
4. REDCROSS
8. RAMAN
10. PAULING
11. EINSTEIN

Quiz

1. Edward Lorenz
2. Grigori Perelman
3. Goldbach's Conjecture
4. Milton Sirotta
5. The Golden Ratio
6. Calculus
7. Digit 9, Feynman Point
8. Leonhard Euler
9. Euclid
10. Love
11. General Relativity
12. Mobius strip

Who Am I?

1. Bell Telephone Laboratories (Bell Labs)
2. National Chemical Laboratory (NCL)
3. University of Bologna
4. Indian Institutes of Science Education and Research (IISERs)

Linked List

1. CHARGE
2. ENERGY
3. YOLK
4. KINEMATICS
5. SYSTEMS
6. SYMMETRY
7. YTTRION
8. NETWORKS
9. STATISTICS
10. SIMULATION

You made it to the end! While we cook up the next issue, here's a random photo dump.

Champions, Again

At IICM 2025, hosted by IISER Behrampur, IISER Kolkata clinched the Overall Championship for the fourth consecutive time, a testament to sustained excellence, teamwork, and determination.
Credit: IISER Kolkata

77th Republic Day

As principles meet practice at IISER Kolkata, the tricolour stood tall, mirroring a republic shaped by reason, diversity, and shared purpose. *Credit: IISER Kolkata*

Swift to the summit

At IISM 2025 in NISER Bhubaneswar, IISER Kolkata's Men's Kho Kho team struck gold through lightning reflexes, sharp tactics, and unwavering resolve.
Credit: Khokho Club of IISER-K