InSight

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Z is For Zeta

Bharath Krishna S

Interview With
Prof. Govind Agrawal

Debanuj Chatterjee

The IISER Kolkata Science Magazine

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A Word From Our Editors

Shriparna Chattopadhyay
Editor, InScight, IISER Kolkata

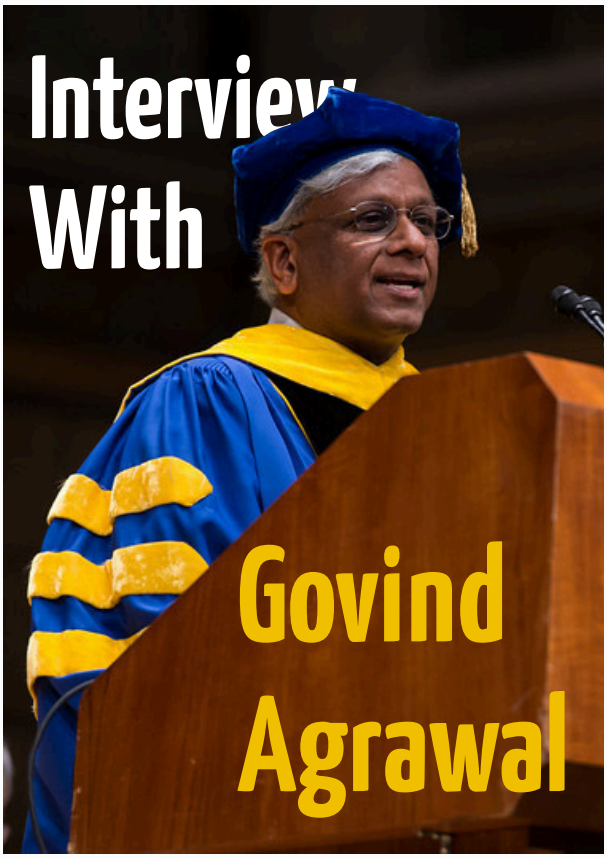
What do a financial crisis, a tangled rope, an infinite sum, and the swirling arms of a galaxy have in common?

Well! At first glance, perhaps very little. Yet each reveals something that science deals with: to question what seems quite obvious,to look beyond what is immediately visible, and to dig deeper into the reason behind things.

And so the tenth issue of *InScight* takes us across mathematics, statistics, astronomy, and optics, exploring how familiar ideas can lead to unexpectedly profound questions. We begin with a question that matters far beyond the world of economy: how do we measure risk? The risk to estimate potential financial losses. In "The Risk Beyond the Cutoff: Limitations of VaR and the Need for Coherent Measures", Dr. Suparna Biswas of IISER Tirupati examines VaR (Value at Risk), a traditional tool for estimating potential financial losses. But what happens when the losses extend beyond the threshold that VaR predicts? The article exposes the limitations of this measure in capturing the severity of extreme losses through an illustrative portfolio example. It then introduces coherent risk measures and mathematical principles which can offer a more reliable way of thinking about financial uncertainty.

Following this we dive into the enigmatic world of mathematics, where Bharath Krishna S. of IISER Thiruvananthapuram takes us into the fascinating world of the Riemann zeta function and the subtleties of infinite series.explores the fascinating mathematical expression $1 + 2 + 3 + \ldots = -1/12$, unpacking the ideas behind this seemingly paradoxical result. Most importantly, it clarifies that this is not the ordinary sum of the positive integers, but a value obtained through a different mathematical framework.From an infinite series, we move to something we can hold in our hands: a knotted rope. In "Mathematics Behind Knots and Its Fascinating Real-World Applications," Rama Mishra introduces us to knot theory, where the seemingly simple act of tying a rope opens up a rich mathematical landscape. By representing knots as closed curves in three-dimensional space, mathematicians can study when two knots are genuinely different and develop tools such as knot invariants to distinguish them. But the story does not end with abstract geometry. Knot theory finds applications in understanding DNA and protein structures, and even in the development of quantum computing.

Well! The next article by Dr. Arunima Banerjee from IISER Tirupati leads us to the entire galaxy. She explores a puzzle that used to trouble astronomers: if stars closer to a galaxy's centre orbit faster than those farther away, why haven't spiral arms been wound into increasingly tight coils over billions of years? Here, the density-wave theory offers an elegant explanation: spiral arms are not permanent collections of stars, but long-lived regions of enhanced density through which stars and gas continually move. The article brings this idea to life through a common life analogy- a traffic jam, which can persist even if the individual cars enter and leave it.The discussion reaches modern astronomy, where convolutional neural networks are used to classify spiral galaxies as grand-design or flocculent, demonstrating how



"Chance plays a big role in achieving success, but you have to be ready there to catch it." From a daunting first day in Paris to writing the definitive textbook... **Read More**

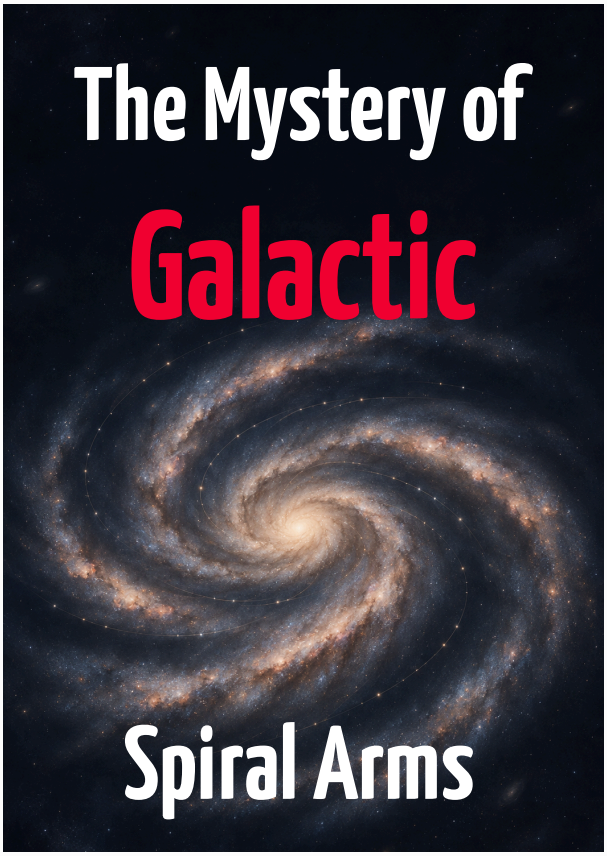
machine learning can help us make sense of vast astronomical datasets.

While these articles take us through different scientific ideas,our conversation with Prof. Govind Agarwal offers a glimpse into the people and experiences behind such progress. In this interview he looks back on his journey from his primeval interest in science to his work in nonlinear optics. The writing of the book 'Nonlinear Fiber Optics' by Prof. Agarwal reflects on the role of chance, perseverance, and being prepared to seize unexpected opportunities. His experiences also underline the importance of taking initiative, learning from failures, building professional connections, and communicating ideas effectively. His story reminds us that scientific discovery is not only about asking the right questions, but also about remaining open to the paths where those questions may lead us.

Inscight not only delivers long research articles but short summaries of hot press research papers in the 'Digest' section of the magazine. In this 10th issue of *Inscight* we have a summary of a research done on our attention span and how it is affected by our emotional states written by Shriparna Chattopadhyay of IISER Kolkata, followed by Abhirup Mukherjee of IISER Kolkata who wrote regarding 'entanglement': a distinguishing feature of quantum mechanics. We also have a section of comics and a quiz to nurture the curiosity in the readers.

The diverse contributions in this issue remind us that science is not confined to any discipline or equation, it is a way of looking at the world with endless curiosity and questions. Whether it is understanding the financial uncertainty, untangling mathematical mysteries, exploring distant galaxies, or learning from the journeys of inspiring scientists, every story offers a new perspective.

We hope this issue of *InScight* sparks interest in Science, encourages questions, and inspires you to keep exploring. After all, every great discovery begins with a curious mind! Happy reading!



An exploration of galactic spiral arms, their role in galactic evolution, the winding dilemma, and the density-wave theory that explains their persistence. Read More

Foreword For The Tenth Issue of *InScight*

Neelanjana Sengupta
Dept. of Biological Sciences, IISER Kolkata

It is a pleasure to introduce the tenth issue of InScight. In our first issue, in February 2025, I wrote about the 2024 Nobel Prizes and how they placed artificial intelligence (AI) at the centre of science. Ten issues later, the magazine has grown into a place where students and researchers from across the IISERs write for each other with care.

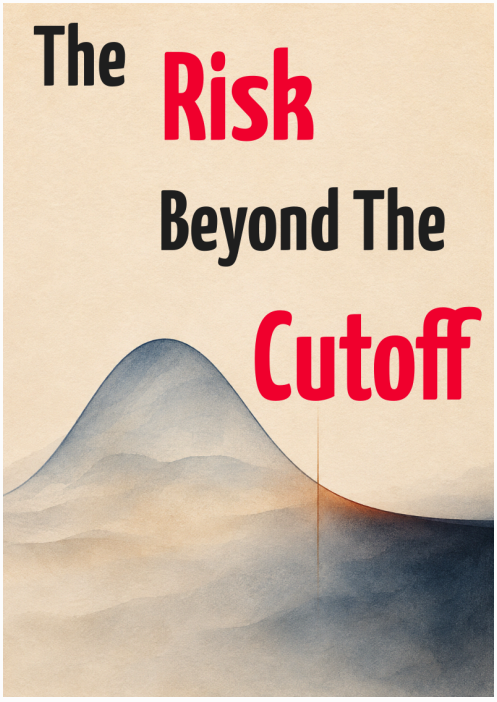
In his article *Z is for Zeta*, Bharath Krishna S from IISER Thiruvananthapuram explains why (1 + 2 + 3 + ···) does not really equal -1/12, and where that number actually comes from: the Riemann zeta function. Prof. Rama Mishra (IISER Pune) introduces the mathematics of knots and shows how it helps us understand knotted DNA, protein structure and even quantum computing. In an interview with Debanuj Chatterjee, Prof. Govind Agrawal (University of Rochester) shares his journey from a small town to the top of optics, with honest advice about failure, luck and hard work.

Dr. Arunima Banerjee (IISER Tirupati and IUCAA) explains why the spiral arms of galaxies do not wind up and disappear, and how machine learning now helps classify galaxies. Dr. Suparna Biswas (IISER Tirupati) shows where a common measure of financial risk falls short, and what better measures look like. The *Insight Digest* covers spin entanglement measured with X-rays, and how our daydreams affect attention. A crossword, a quiz and a puzzle round off the issue.

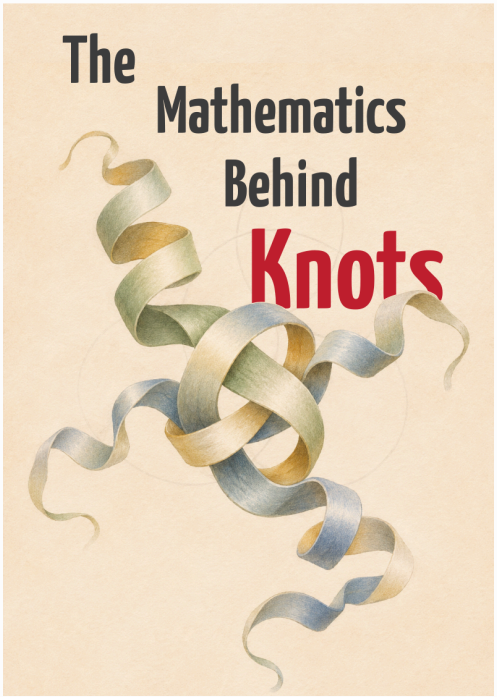
The rapid progress of AI in the last two years has not belonged to any one subject, and this issue showcases that paradigm. Knot theory describes what enzymes do to DNA, while an idea from statistics detects entanglement in a material. Similar ideas from information theory are now used, in my own group among others, to understand how biomolecules behave. Biology is where these connections may matter most, because living systems produce huge amounts of data whose rules we only partly understand. Real progress will necessitate biologists who are comfortable with mathematics and code, and computational scientists who respect real experiments.

The issue offers two good lessons. The zeta article reminds us that a convincing answer is not always a correct one, and the same holds for AI. The article on financial risk shows that a model can work well in ordinary cases and fail at the extremes. *A prediction is not an explanation, and every AI-designed system must still be tested in the lab or in the real world.* My advice to students is to build strong foundations: the tools will change every year, but the basic science behind them will not. As Prof. Agrawal's story shows, chance favours those who are ready for it. I hope our readers will be ready to cross the boundaries between subjects.

My congratulations to the authors, reviewers and the student editorial team on an excellent tenth issue.



This article discusses the fundamental principles of quantifying financial risk based on mathematical frameworks, the shortcomings of traditional value at risk (VaR) and clarifying how coherent risk measures address these... Read More



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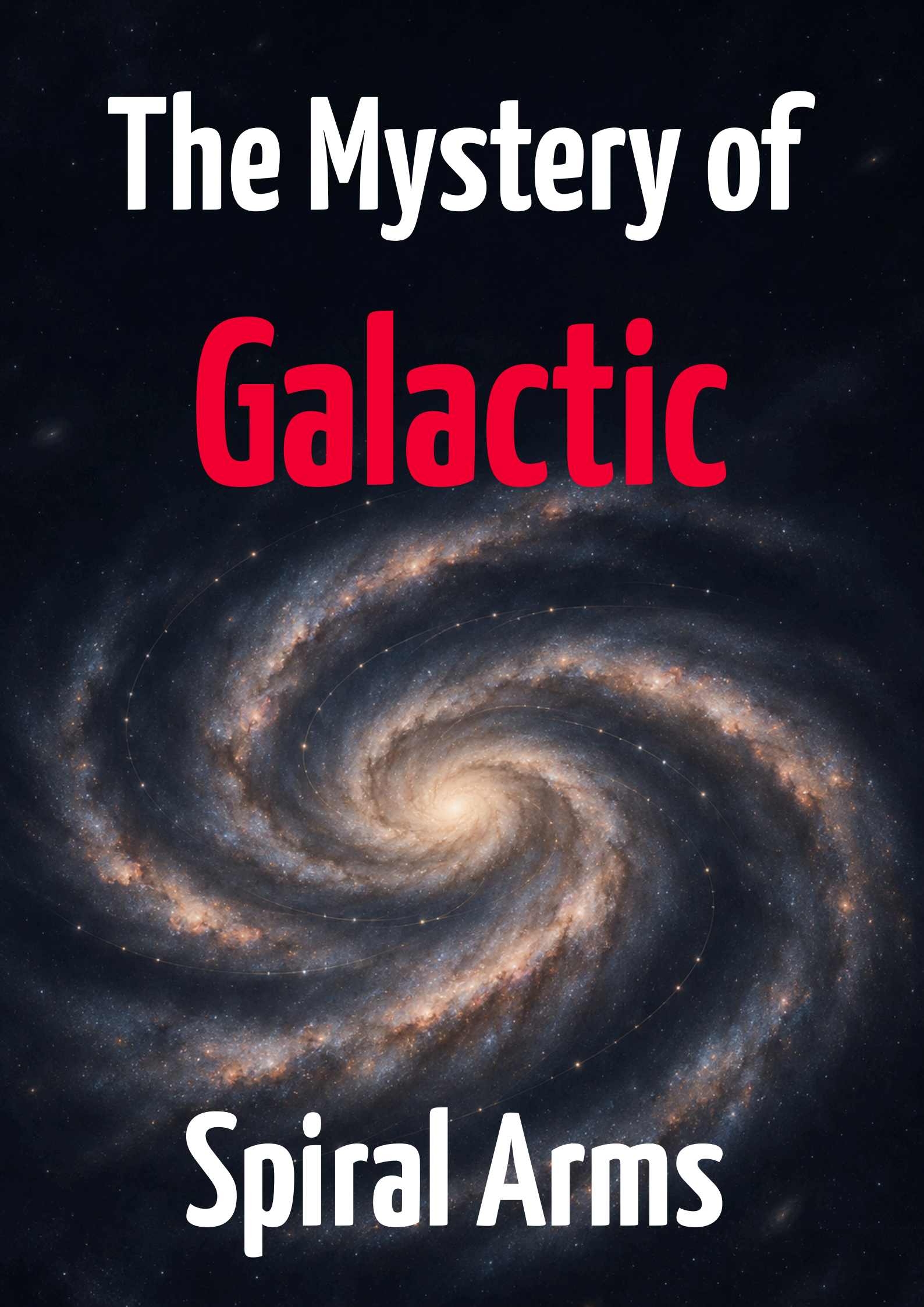
The Mystery of Galactic Spiral Arms



The Mystery of Galactic Spiral Arms: From the Winding Dilemma to Density Waves

Dr. Arunima Banerjee *Indian Institute of Science Education and Research Tirupati*
IUCAA, Pune

Jul 04, 2026

Reviewed By: Swarnendu Saha

An exploration of galactic spiral arms, their role in galactic evolution, the winding dilemma, and the density-wave theory that explains their persistence.



Dr. Arunima Banerjee is a theoretical astrophysicist, specializing in the dynamics of galaxies and the application of machine learning in galactic astronomy. She is currently an Associate Professor and the Chair at the Department of Physics, at the Indian Institute of Science Education and Research Tirupati, and a Visiting Associate of IUCAA, Pune. She is also an Editorial Board Member of the Journal of Astronomy & Astrophysics of the Indian Academy of Sciences. Music, painting, and reading are her favorite pastimes.

According to Edwin Hubble's celebrated tuning-fork classification, galaxies are broadly divided into two principal morphological classes: elliptical galaxies and disc galaxies. In elliptical galaxies, stars are supported against gravitational collapse primarily by their random motions, whereas in disc galaxies they are supported by their ordered rotation about the galactic centre.

The most striking and visually captivating features of many disc galaxies are their spiral arms, which distinguish them from other classes of galaxies. Observations show that these arms rotate approximately as a rigid pattern with a nearly constant angular velocity, known as the pattern speed, even though the individual stars and gas clouds within the disc orbit the galactic centre at different angular speeds.

Spiral Arms and the Secular (Slow) Evolution of Galaxies

Spiral arms are far more than visually striking structures; they play a pivotal role in shaping the long-term evolution of disc galaxies. As non-axisymmetric features in the galactic disc, spiral arms introduce a quadrupole component into the galaxy's gravitational potential. The resulting gravitational torques redistribute angular momentum between different parts of the disc and the surrounding dark matter halo, thereby driving the galaxy's secular evolution.

Spiral arms also profoundly influence the distribution of interstellar gas. Through gravitational torques, they induce gas inflow from the outer regions of the galaxy towards its centre, where the accumulated gas can fuel active galactic nuclei and contribute to the growth of the central supermassive black hole.

In addition, spiral arms are overdense regions of the galactic disc. As gas clouds pass through these regions, they are compressed by the enhanced gravitational field, triggering the collapse of molecular clouds and the formation of new stars. Consequently, spiral arms are the principal sites of star formation in disc galaxies.

The Winding Dilemma

Until the early 1960s, the origin and long-term persistence of spiral arms remained one of the greatest unsolved problems in galactic astronomy. Astronomers knew that galaxies exhibit differential rotation, with stars in the inner regions completing an orbit around the galactic centre much faster than those farther out. Consequently, over their lifetimes, most galaxies have completed roughly 20–100 rotations.

If spiral arms consisted of the same stars and gas clouds throughout their lifetime—that is, if they were material arms—differential rotation would progressively wind them into increasingly tighter spirals. After 20–100 galactic rotations, such arms would have wrapped around the galactic centre many times, producing tightly coiled spiral patterns.

Observations, however, tell a very different story. The spiral arms of most galaxies are wrapped only once or twice around the galactic centre and remain relatively open, even in systems that are billions of years old. This apparent contradiction became known as the winding dilemma.

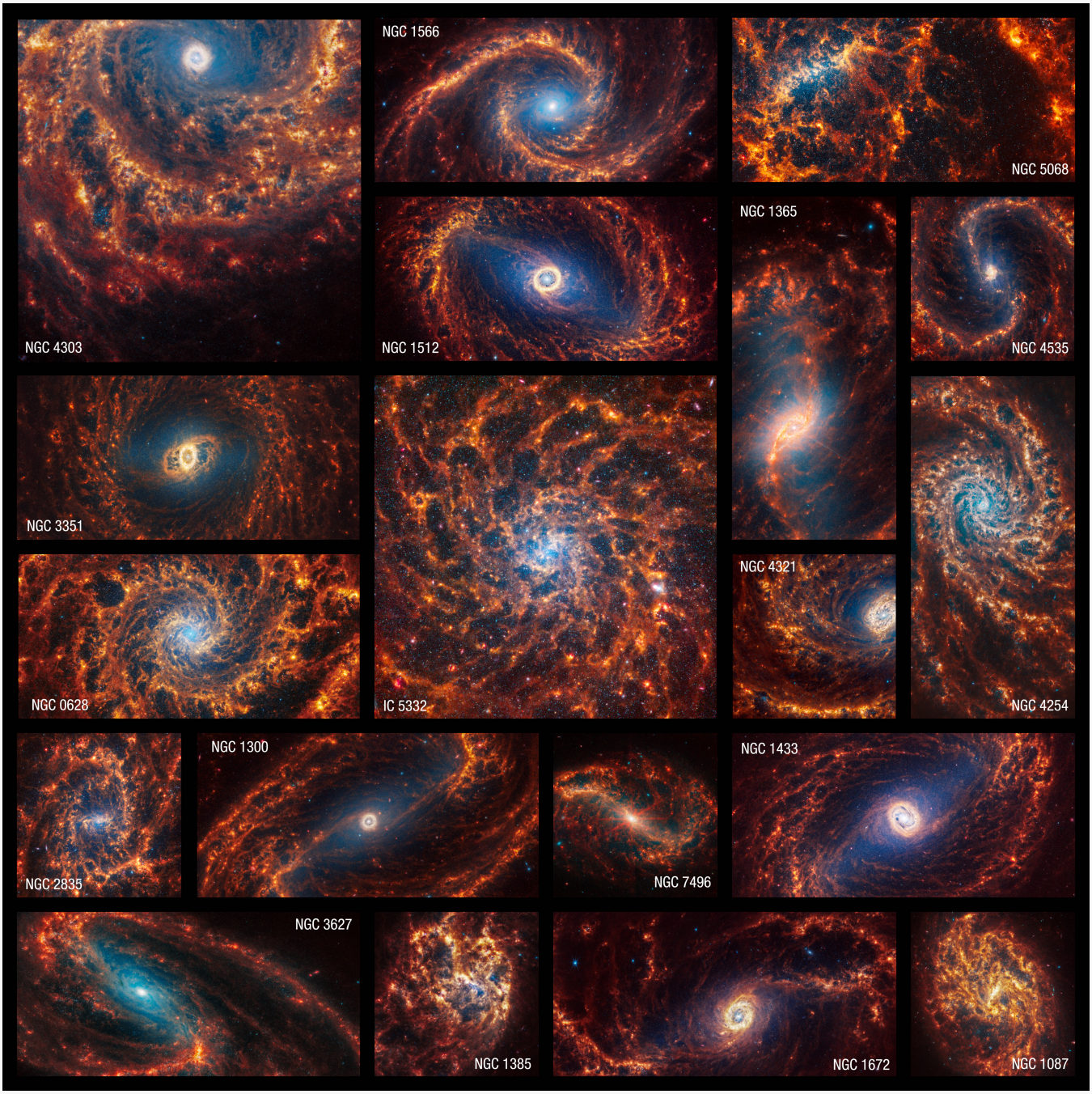


FIG 1 : *A mosaic of spiral galaxies from the NASA/ESA/CSA James Webb Space Telescope in near- and mid-infrared light.*

The winding dilemma led astronomers to an important conclusion. Either spiral arms are transient features that repeatedly form and dissolve on timescales much shorter than the age of a galaxy, or they are not material structures at all. The latter possibility ultimately gave rise to the density-wave theory, according to which spiral arms are long-lived waves of enhanced stellar and gas density rather than permanent collections of the same stars and gas clouds.

Traffic Jams, Density Waves, and Galactic Spiral Arms

How can spiral arms persist for billions of years without being composed of the same stars? An elegant answer is provided by an analogy familiar to anyone who has experienced a traffic jam.

Imagine a slow-moving procession travelling along a highway. As faster-moving vehicles approach the procession, they are forced to slow down, forming a region of congestion. After passing the procession, the vehicles gradually accelerate and resume their normal speed.

Now consider this situation from the perspectives of two observers. The first observer watches from an aeroplane and notices a compact region of high traffic density moving slowly along the highway with the procession. To this observer, the traffic jam appears to be a coherent structure travelling down the road.

The second observer stands beside the highway. This observer notices something quite different. The traffic jam never consists of the same vehicles for very long. Cars continually enter the congested region from behind, remain there briefly while moving slowly, and then leave

it after accelerating. The vehicles are constantly replaced by others, even though the traffic jam itself persists.

A traffic jam is therefore not a collection of specific vehicles; rather, it is a wave of enhanced vehicle density propagating through the traffic. In other words, it is a density wave.

Galactic spiral arms behave in much the same way. They are not composed of a permanent set of stars and gas clouds. Instead, stars and interstellar gas continuously move into a spiral arm, slow down temporarily as they descend into the enhanced gravitational potential associated with the overdense region, and later accelerate as they climb out of the potential well. They then continue their orbital motion around the galactic centre until they encounter the spiral pattern again.

Thus, the stars themselves are continually moving through the spiral arms, while the spiral pattern persists as a long-lived region of enhanced stellar and gaseous density. At any given time, the spiral arm contains different stars and gas clouds than it did earlier.

An important feature common to both traffic jams and spiral arms is that the wave propagates independently of the individual constituents of the medium. Cars move through a traffic jam; stars move through a spiral arm. The pattern and the particles therefore have distinct motions.

Another remarkable consequence of this picture is that the wave can survive even after the disturbance that originally generated it has disappeared. A traffic jam may persist long after the procession has left the highway because the disturbance continues to propagate through the flow of vehicles. Likewise, a spiral density wave can continue to exist long after the perturbing influence that excited it has vanished.

This simple traffic-jam analogy captures the central idea of the density-wave theory: spiral arms are not material structures but long-lived waves of enhanced density through which stars and gas continually pass.

Density Waves as Normal Modes of Oscillation

The central idea behind the density-wave theory is that spiral arms represent normal modes of oscillation of the galactic disc, much like the standing-wave patterns observed on a vibrating string or the normal modes of a drumhead.

The theory seeks a self-consistent perturbation of the disc's surface density that can propagate through the galaxy without continuously winding up under differential rotation. In such a mode, the enhanced density of stars and gas generates a perturbation in the galaxy's gravitational potential. This perturbed gravitational field, in turn, induces stellar motions that reproduce and sustain the same density enhancement. In other words, the density perturbation and the gravitational potential perturbation reinforce one another in a self-consistent manner.

This mutual reinforcement is the hallmark of a normal mode. The gravitational field produced by the perturbed mass distribution is precisely the field required to maintain the stellar motions that created the perturbation in the first place. As a result, the spiral pattern behaves as a coherent, self-sustaining wave rather than rapidly dispersing under differential rotation.

From a mathematical standpoint, the problem therefore reduces to finding eigenfunctions and eigenfrequencies of the galactic disc that satisfy the equations governing the coupled dynamics of stars and their self-gravitational field. Each eigenmode corresponds to a possible global spiral pattern with a well-defined pattern speed. The grand-design spiral arms observed in many galaxies are believed to correspond to one or more of these global normal modes.

Energy Exchange Between Spiral Waves and Disc Stars

Because galactic discs undergo differential rotation, spiral density waves continually exchange angular momentum and energy with the stars in the disc. This interaction is not uniform throughout the galaxy; instead, it occurs most efficiently at special locations known as Lindblad resonances.

A star moving around the galactic centre periodically encounters the crests and troughs of the spiral gravitational potential. Most of the time, these encounters are out of phase and the gravitational perturbations largely average out over successive orbits. At the Lindblad resonances, however, the situation changes dramatically. Here, the frequency at which a star encounters the spiral pattern either matches the star's natural epicyclic oscillation frequency or, in the limiting case of corotation, becomes zero, so that the star remains nearly in phase with the spiral potential. Under these resonant conditions, the cumulative gravitational interaction becomes much more efficient, leading to a substantial exchange of angular momentum and energy between the stars and the spiral wave.

The Inner Lindblad Resonance (ILR) generally acts as a sink for spiral waves. As the wave transfers energy and angular momentum to the stellar disc, its amplitude decreases, causing the wave to be damped. By contrast, near the Outer Lindblad Resonance (OLR), the wave propagation changes character, and under suitable conditions the wave may be reflected back towards the inner regions of the galaxy. These resonant interactions therefore play a crucial role in determining both the propagation and the longevity of spiral density waves.

The existence of these resonances is central to modern theories of spiral structure. They govern how spiral waves transport angular momentum across the galactic disc and determine whether a spiral pattern is amplified, damped, or reflected. Consequently, the long-term evolution of spiral galaxies is intimately linked to the dynamics of these resonant wave–matter interactions.

Standing Spiral Waves and the Q-Barrier

If an inward-propagating spiral density wave reaches the Inner Lindblad Resonance (ILR), it transfers its energy and angular momentum to the stellar disc and is consequently damped. In this case, the spiral pattern gradually loses its strength and cannot survive for long times.

A much more interesting situation arises if the wave is reflected before reaching the ILR. Under these circumstances, the reflected wave can interfere with the outgoing wave to form a quasi-stationary standing wave in the radial direction. Such a standing-wave pattern can persist over many galactic rotations and provides a natural explanation for the longevity of grand-design spiral arms.

The reflection occurs because certain regions of the galactic disc inhibit the propagation of spiral density waves. These regions, commonly referred to as Q-barriers, arise where the local dynamical properties of the disc—particularly the stability parameter (Q)—prevent the wave from propagating further. Instead of penetrating these forbidden regions, the wave is reflected, much like a light wave reflected from a boundary between two media.

The repeated propagation and reflection of spiral waves between the Q-barrier and the outer disc can establish self-consistent global standing-wave patterns, often referred to as spiral wave modes. These global modes constitute one of the central ideas of the density-wave theory, explaining how spiral structure can remain coherent for timescales far longer than would be possible for material arms.

Identifying new spirals from astronomical data sets using Machine Learning

Spiral galaxies are broadly classified into grand-design and flocculent systems based on the morphology of their spiral arms. Grand-design spirals exhibit prominent, continuous, high-contrast spiral arms and are generally associated with large-scale, quasi-stationary density waves. In contrast, flocculent spirals display fragmented, patchy, and low-contrast spiral features that are thought to arise primarily from local gravitational instabilities and stochastic star formation.

We develop a deep convolutional neural network (CNN) to automatically classify spiral galaxies into these two morphological classes. The trained network is subsequently applied to classify a sample of 1,354 spiral galaxies from the Sloan Digital Sky Survey (SDSS). Of these, 721 are identified as flocculent galaxies and the remaining 633 as grand-design spirals.

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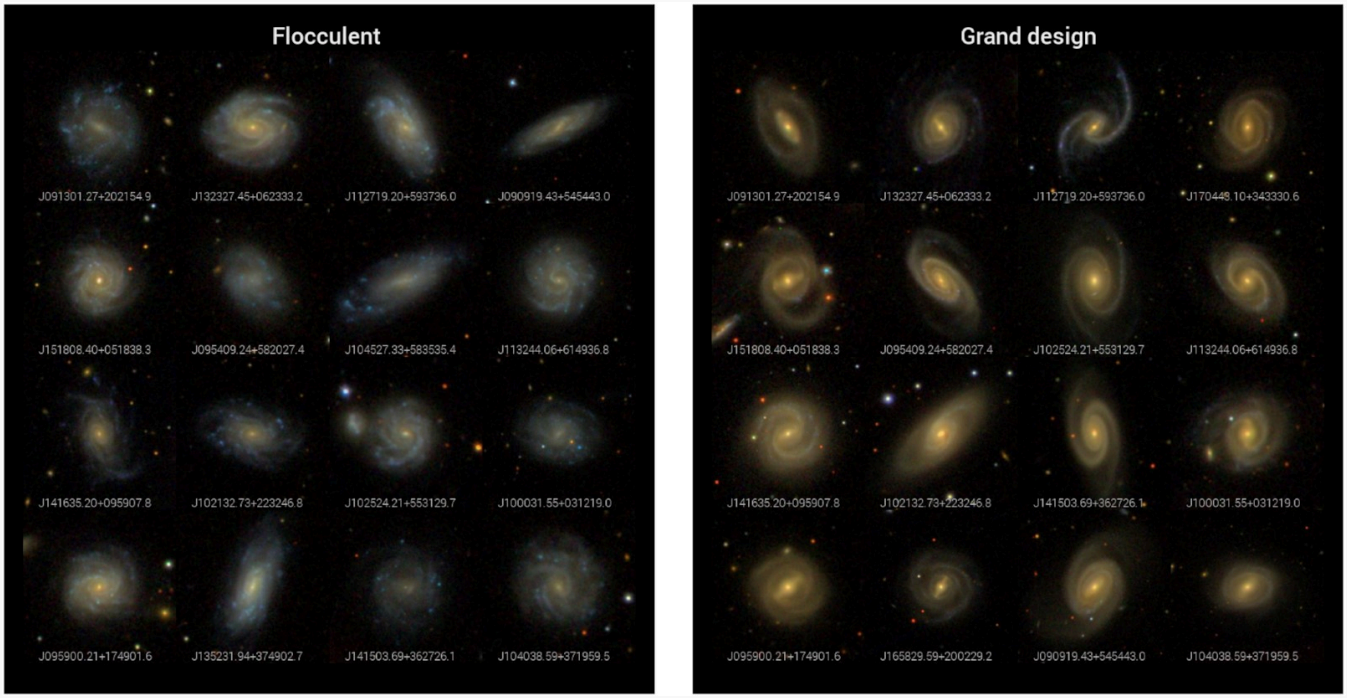
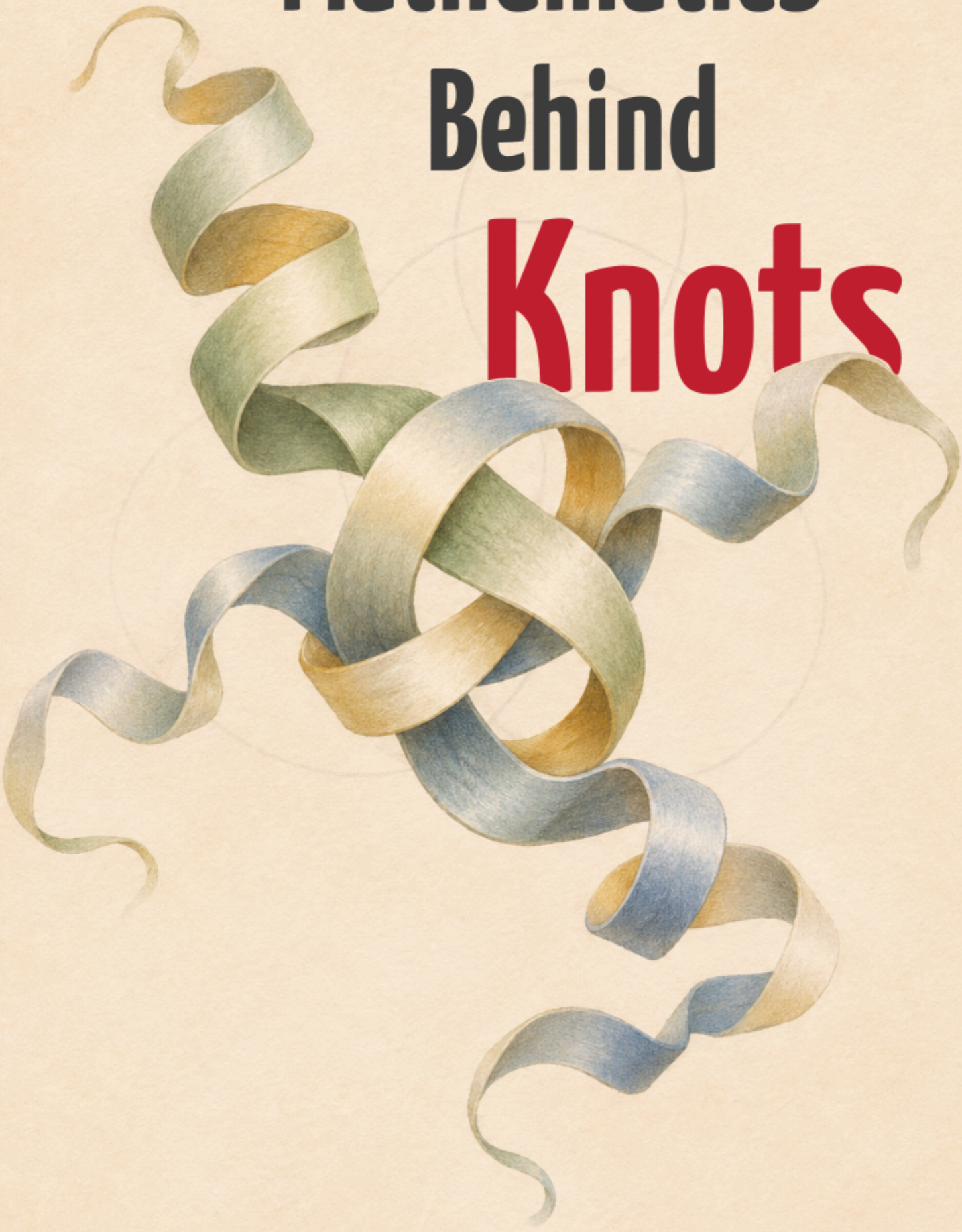


FIG 2 : *A subset of the new galaxies with Flocculent (Left) and Grand-design (Right) spiral arms as determined from an astronomical dataset by our machine learning model. (Figure Courtesy: Sarkar et al. 2023, MNRAS, 518, 1)*

The Mathematics Behind Knots



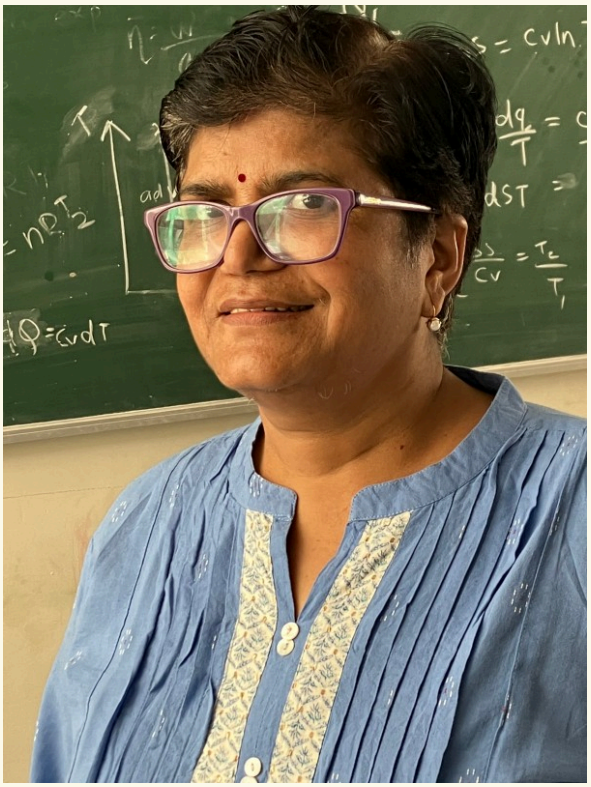
The Mathematics Behind Knots and Its Fascinating Real World Applications

Ram Mishra *IISER Pune*

Jun 22, 2026

Reviewed By: *Puppala Ainesh Kumar, Debanuj Chatterjee*

A knot may look simple, but proving whether it can be untangled—or telling one knot from another—can take sophisticated mathematics. This article introduces knot theory's tools for answering those questions, then follows their surprising reach from the twisting of DNA and protein structure to the braided particles proposed for quantum computing.



Ram Mishra is a professor in mathematics at IISER Pune. She obtained her doctoral degree from IIT Bombay in 1994. She works in knot theory. She has served in IIT Delhi and IIT Kharagpur before joining IISER Pune. Her research interests lie in low dimensional topology, real algebraic geometry and applications of knot theory in other areas of science.

Knots have a rich and long history and have been used for various activities like fishing, hunting and building. To understand knots we pick a rope and tie it as in **FIG** 1.

Now if we want to describe it mathematically, we model the rope by the real line $\mathbb{R}$ and the process of tying a knot as embedding $\mathbb{R}$ in $\mathbb{R}^3$ [6]. We can obtain several different knots. Some of them are shown in **FIG** 2.

We may simplify a knot to make sure if it is truly knotted. However there may be a mistake in drawing an inference while simplifying the knot.

To overcome this confusion, we join the ends of the rope and hence it is no longer an embedding of $\mathbb{R}$ in $\mathbb{R}^3$ but now it is embedding of S^1 (circle) in $\mathbb{R}^3$. Since S^3 is a one point compactification of $\mathbb{R}^3$ [1], we can consider a knot as embeddings of S^1 in S^3 to avail the advantage of compactness. Mathematicians take this as definition of a knot! Thus, a knot is the image of a smooth embedding of S^1 in $\mathbb{R}^3$ or in S^3 and it will be a simple closed curve in 3 dimensional space. This is also referred to as a classical knot. If the simple closed curve is drawn without a strand passing under another strand, it is called a *trivial knot* or an unknot. Mathematicians have developed a beautiful subject called ***knot theory*** that has found applications in several real life problems. **FIG** 3 shows some examples of classical knots.

Knots generalize to links and we define a link with r components to be a smooth embedding of $S^1 \cup S^1 \cup \cdots \cup S^1$ (r copies) in $\mathbb{R}^3$. Thus a knot is a link with 1 component. In **FIG** 4 we can see links with 2 and 3 components.

A brief history of knot theory

Knot theory [3] was first introduced in 1771 by **Vandermonde** who noted the significance of topology in his discussion regarding the properties of knots in relation to the geometry of position. In the 19th century, studies led to **Peter Guthrie Tait** created the first knot table. In 1885, the Scottish mathematician and physicist showed fellow mathematician Sir **William Thomson** (later to become Lord Kelvin) his device that generated smoke rings. Kelvin did develop a theory which theorized that atoms might consist of knotted vortex tubes of the



FIG 1 : *Tying a knot*

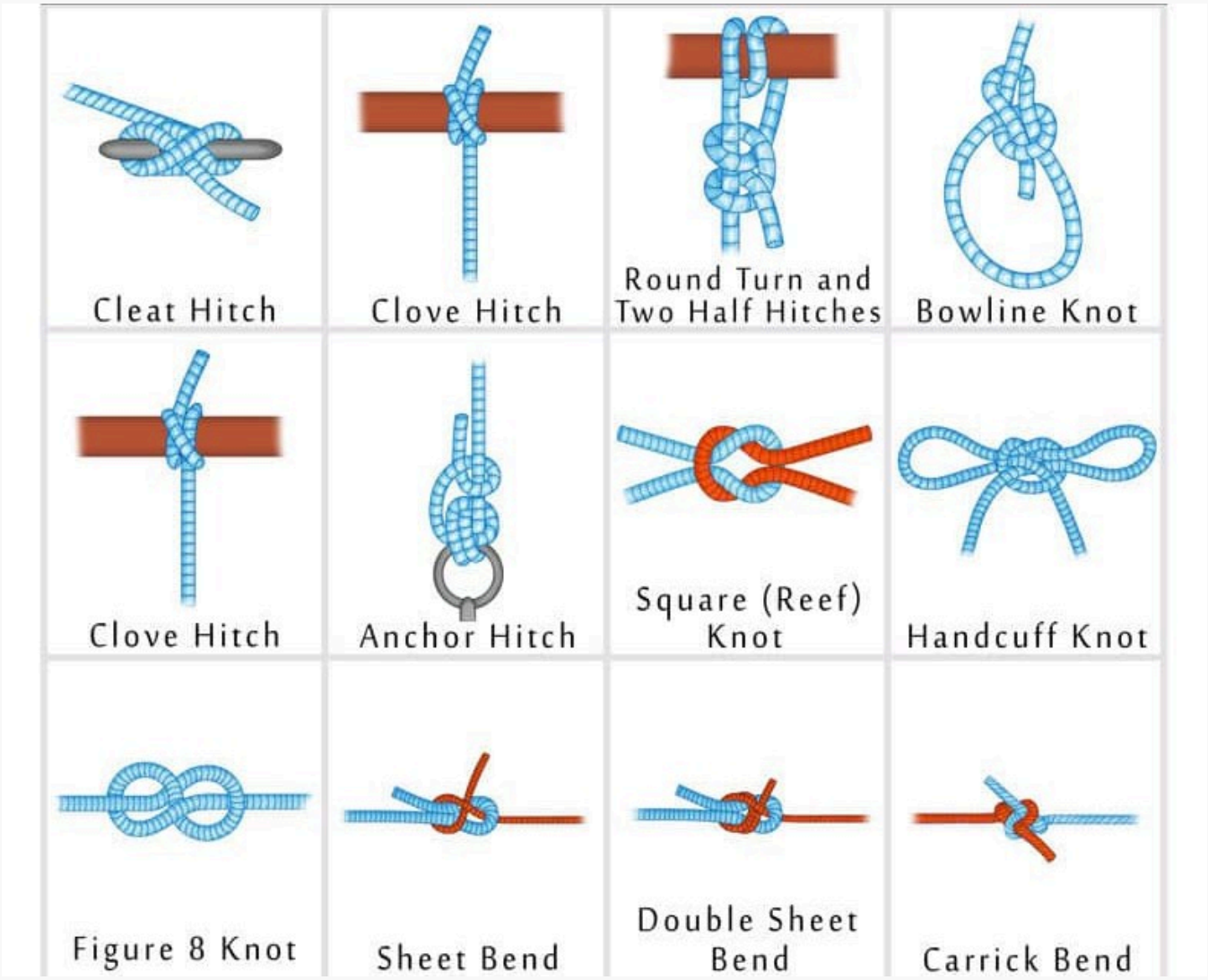


FIG 2 : *Different knots with a rope*

ether, but this theory was later rejected. Table of knots prepared by Tait using his conjectures motivated the early theorists of knot theory.

Equivalence between knots and links

In non-technical terms, if we can change a knot into another knot without cutting or tearing it, then we say that these two knots are equivalent. For examples, these two knots drawn in **FIG** 5 are equivalent and it is easy to prove it.

One can try hard but cannot convert the knot A into the knot B in **FIG** 6. However, it is difficult to prove this.

A formal definition of knot equivalence is as follows: A knot k_1, is said to be ambient isotopic to knot k_2 if there exists an orientation preserving diffeomorphism

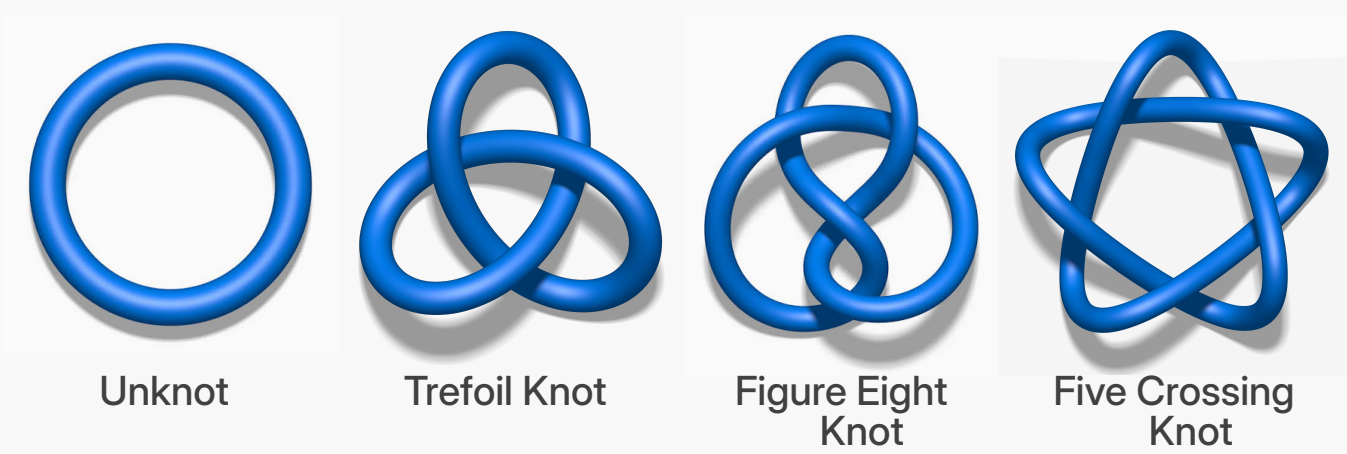


FIG 3 : *Examples of classical knots*

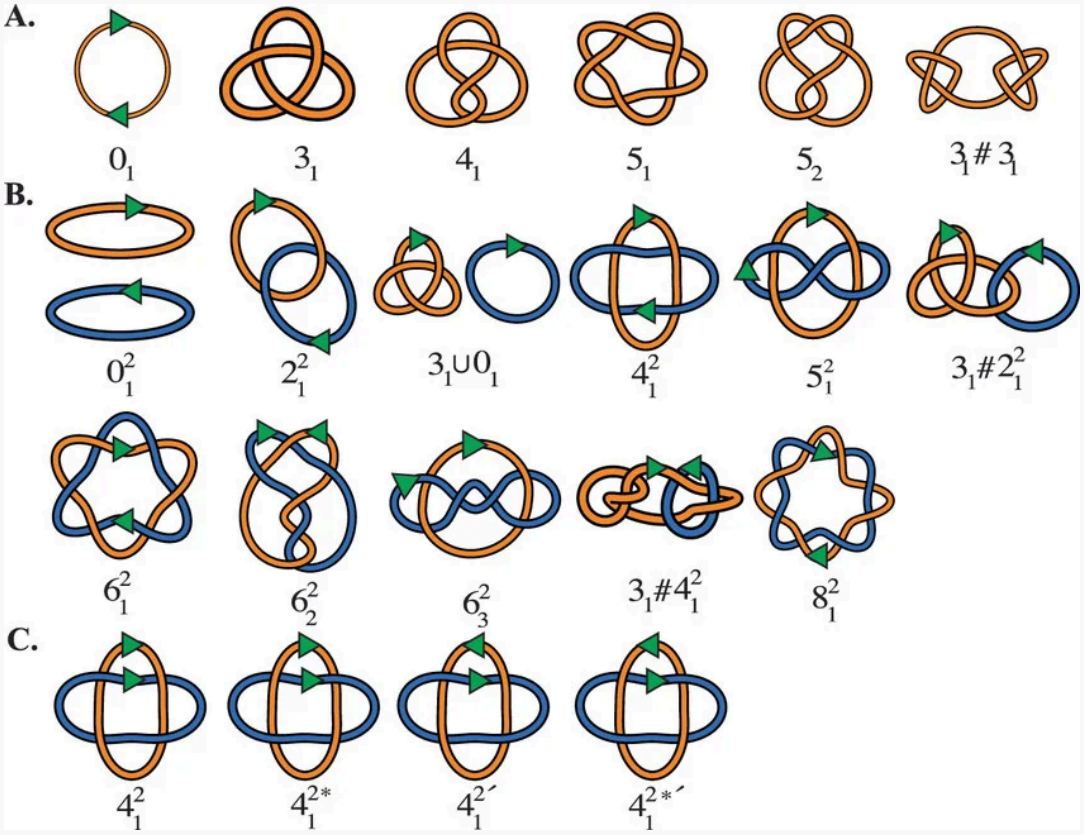


FIG 4 : *Examples of links with 2 and 3 components. [Sci Rep 7, 12420 (2017)]*

$h : S^3 \to S^3$ such that $h(k_1) = k_2$. This definition can be extended for links as well. We shall only use the term knot that will mean knots and links both. This matches with our intuitive idea of knot equivalence. However, it is very difficult to prove the knot equivalence using this definition. Thus, we need to find some properties or the structures or quantities that are preserved under this equivalence. These are known as ***knot invariants***. The main objective in knot theory is to invent effective knot invariants. The simplest questions one might ask are: *given a knot can we detect if it is NOT un-knot?*, *Can a knot be equivalent to its mirror image?*, *given two knots k_1 and k_2 can we detect if k_2 is neither equivalent to k_1 nor to the mirror image of k_1?* Till date none of these questions are answered completely. There is no invariant that is complete and can be computed easily. The most popular method to define knot invariants is by representing knots by a diagram [7]. For that we choose to project our knot into a plane such that the projected image is a *generic immersion* [5]. It consists of only transverse double points as in **FIG** 7 (Diagram A). It should not have multiple points as in **FIG** 7 (B), tangential intersections as in **FIG** 7 (C) and cusps as in **FIG** 7 (D). Such a projection always exists. This type of projection is known as a regular projection.

After obtaining a regular projection, at each double point we provide a choice of over/under crossing information and the resulting picture is called a *knot diagram* as shown in **FIG** 8.

Knot diagrams are a sufficient data to tell us about the knot, except that there may be several knot diagrams for a particular knot. We need an equivalence at the diagram level! There are moves (shown in the figure below), known as Reidemeister moves [7] that capture the isotopy at the diagram level and we have the following:

Theorem: *Two knot diagrams represent isotopic knots if and only if one can be transformed into the other by a sequence of finitely many Reidemeister Moves*, mentioned in **FIG** 9.

When we define properties/ quantities/structures associated to a knot diagram, we need to check that it does not change when we perform any of the



FIG 5 : *Two equivalent knots*

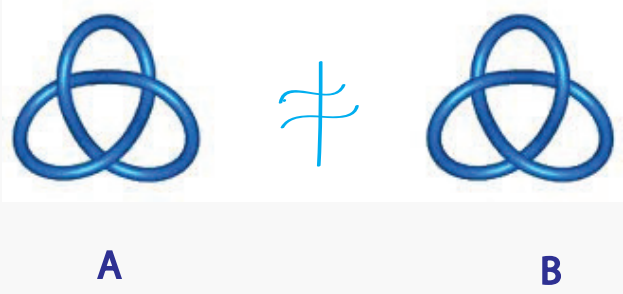


FIG 6 : *Two non-equivalent knots*

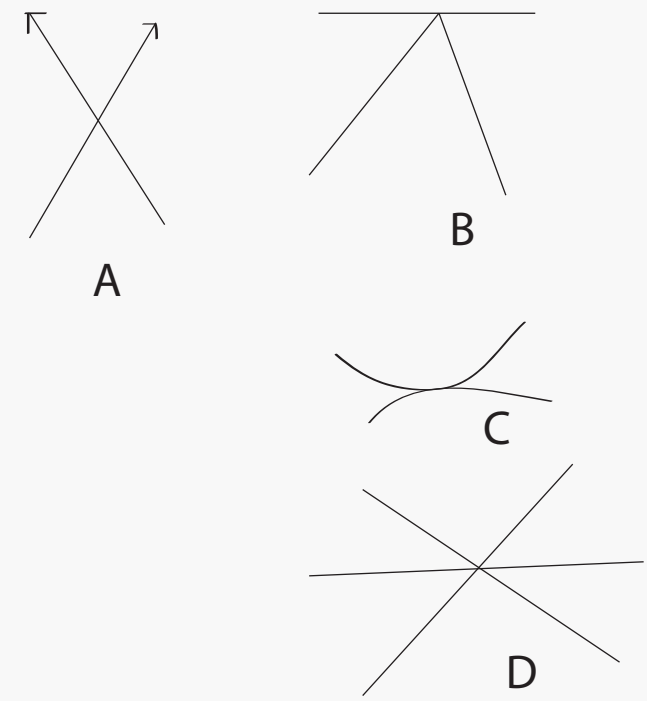


FIG 7 : *Regular and non-regular projections*

Reidemeister move. Then, we can say that the associated object is an invariant for knots and links.

One of the easiest invariants that can be defined using a knot diagram is *tri-colorability* [4]. A knot diagram is tri-colorable (sometimes called simply colorable) if each arc can be colored with one of three colors such that: at least two colors are used in the diagram; and at every crossing, either: all three colors are used; or only one color is used, as depicted in **FIG** 10.

It is easy to check that tri-colorability is a knot invariant by verifying its invariance under all Reidemeister moves (shown in **FIG** 11). Using this we can prove that the trefoil knot is actually knotted (**FIG** 12). Also the trefoil knot and the figure eight knot are not equivalent (**FIG** 13). Tri-colorability has been generalized to *n- colorability* [7] and there is an invariant called *coloring number* that is useful in distinguishing between few knots.

There are plethora of invariants defined using diagrams. Some of them are numerical invariants such as *bridge number* and *unknotting number* [7]. Some invariants are algebraic. Many of them are polynomials. Alexander polynomial [7] is the most understood invariant but it cannot differentiate a knot from the unknot and also a knot from its mirror image.

Another successful approach towards defining invariants is using the connection of links with braids [7]. Jones polynomial [7] is one of the celebrated invariants that was discovered in 1984 using representation theory of braid groups and it revolutionized the entire community. Many unresolved conjectures were proved with the help of Jones polynomial and it fetched the most prestigious award "Fields medal" to Vaughan Jones who discovered it. The surprising thing is that the Jones polynomial can be explained using the diagram in a very simple manner skipping the deep mathematics behind it. Around the same time when Vaughan Jones came up with his polynomial, **Louis Kauffman** [7] defined a polynomial invariant purely from diagram which turned out to be same as the Jones polynomial upon a change of variable! Thus the Jones polynomial was re-invented.

We need to incorporate an orientation into a diagram as shown in **FIG** 14 and provide a weight +1 or −1 at each crossing as per the convention shown in **FIG** 15. The writhe $w(D)$ of a knot diagram D is defined to be the sum of the weights at all the crossings of knot diagram as in **FIG** 14. Note that writhe is not an invariant since it is not an invariant under Reidemeister Move I, even though it is preserved under Moves II and III.

Note that in a given knot diagram flipping one crossing from over to under (or vice-versa) reduces the complexity of the knot. The reason being, every knot can be converted into an unknot if we perform this change a certain number of times. Also a knot/link with higher number of times it crosses is considered as more complex. Thus in an oriented diagram, if we skip the crossing and follow the orientation of the knot, we are reducing the complexity. This is called smoothing. Some invariants for a given knot can be computed if one knows their values for knots obtained after performing above two operations at a fixed crossing. This may not be possible for every invariant. Invariants possessing this property come under a class known as *skein invariants*

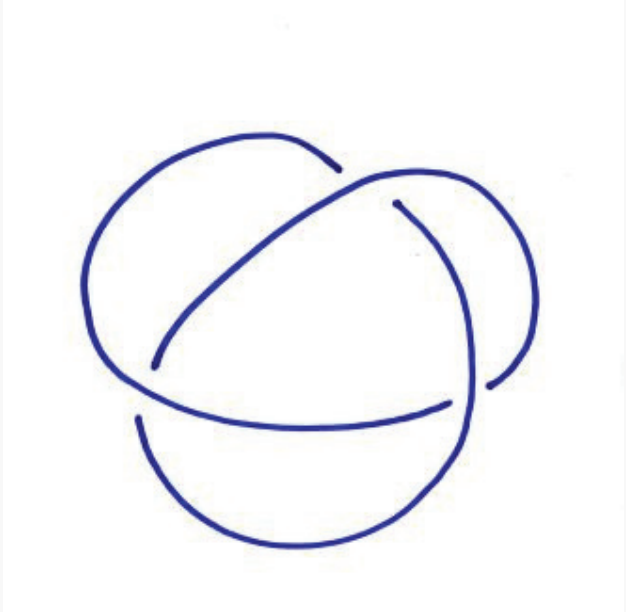


FIG 8 : *A typical knot diagram*

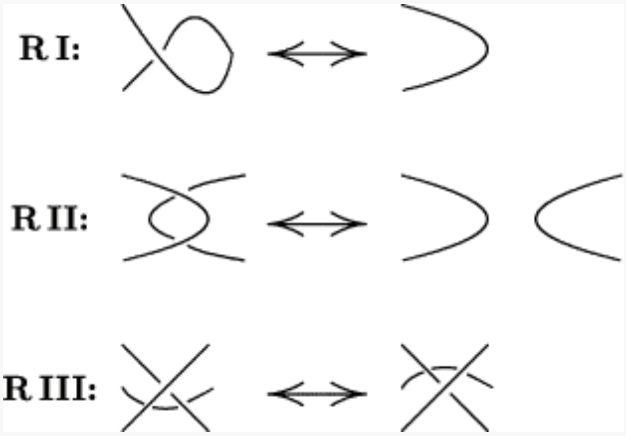


FIG 9 : *Reidemeister Moves*

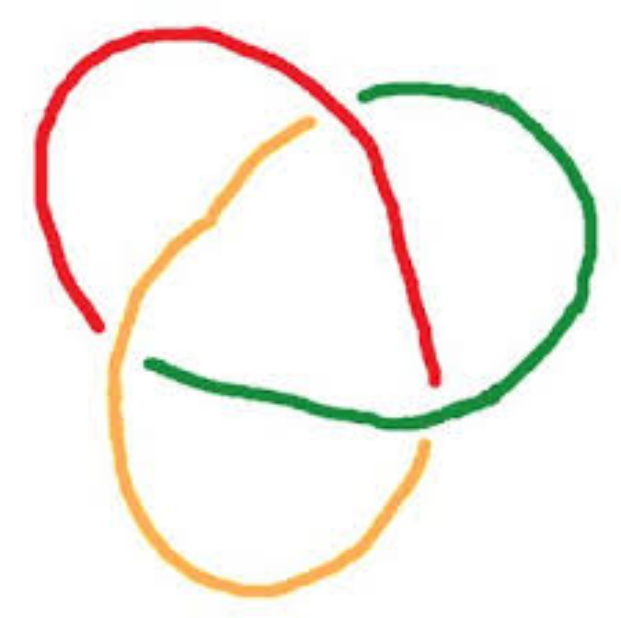


FIG 10 : *A tri-colorable diagram*

[7]. To understand them, we first define skein related diagrams. Three oriented diagrams are said to be skein related if they are identical everywhere except at one crossing and at that they look as shown in the **FIG** 16.

Suppose $\mathcal{I}$ is an invariant for knots and links that takes values in a ring R. We say that $\mathcal{I}$ is a **linear skein invariant** if (i) $\mathcal{I}(O) = 1$ and (ii) there exist three invertible elements a_+, a_- and a_0 in R such that whenever we have three skein related diagrams $\mathcal{L}_+$, $\mathcal{L}_-$ and $\mathcal{L}_0$ the invariants evaluated at these knots satisfy the linear relation:

$$a_+\mathcal{I}(\mathcal{L}_+) + a_-\mathcal{I}(\mathcal{L}_-) + a_0\mathcal{I}(\mathcal{L}_0) = 0.$$

In fact, these coefficients are unique for an invariant except up to a change of variable.

Alexander polynomial and Jones polynomial both are skein invariants. They take values in the ring $\mathbb{Z}[t, t^{-1}]$. For a knot K its Alexander polynomial $\Delta(K)$ satisfies the skein relation

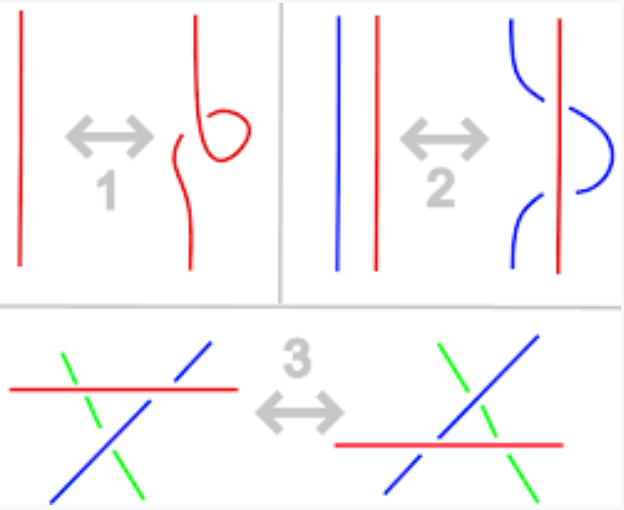


FIG 11 : *Try-colorability is invariant under Reidemeister Moves*

$$\Delta(L_+) - \Delta(L_-) - (t^{\frac{1}{2}} - t^{-\frac{1}{2}})\Delta(L_0) = 0$$

and its Jones polynomial $J(K)$ satisfies the skein relation

$$tJ(L_+) - t^{-1}J(L_-) - (t^{\frac{1}{2}} - t^{-\frac{1}{2}})J(L_0) = 0.$$

Jones polynomial helped in solving many intrinsic questions in knot theory but it still has pitfalls. There are non-equivalent knots that shared the same Jones polynomial. This has been resolved by creating more refined invariants such as Khovanov homology [12] which categorifies the Jones polynomial. There are programs written which help in computing these invariants. One can find the information about these invariants on Knot atlas (https://katlas.org). In nutshell, researchers in knot theory have been developing tools that help in checking if the given knot is truly knotted as well as differentiating between two given knots.

Now comes the question how could knot theory be useful in the real world problems. Imagine a bunch of wires kept together in a box and the box is undergoing a continuous shaking process. Do you think the wires will remain as it is? No, there is a high chance that individual wires may get knotted and also two different wires may get linked. This is the situation that arises in nature. There are places where the structures have the potential to get entangled and knotted. The most profound application of knot theory is in the study of DNA [8] that leads to the discovery of several drugs. The double helix structure of DNA (shown in **FIG** 17) enables it to get knotted under the influence of certain enzymes, called *topoisomerase*. Topoisomerases act by cutting a given number of DNA strands and recombining them in a new manner (eg., see **FIG** 2). The number of strands that are cut and recombined is related to the unknotting number of that knot. In fact these enzymes perform operations on DNA strands similar to Reidemeister moves to be able to simplify the knotted DNA.

It is known that Cancer is a disease caused by abnormalities in the DNA of a cell. A tumor is formed when these abnormal cells accumulate in one area. Developing drugs that can fight tumors with minimal side effect has been a long term goal in the cancer treatment research. In place of using radiation and chemotherapy, knot theory can be utilized in manipulating DNA replication through enzymes. This might stop the damaged cells to multiply and eventually the tumors may shrink and hence could be an effective treatment for cancer [9].

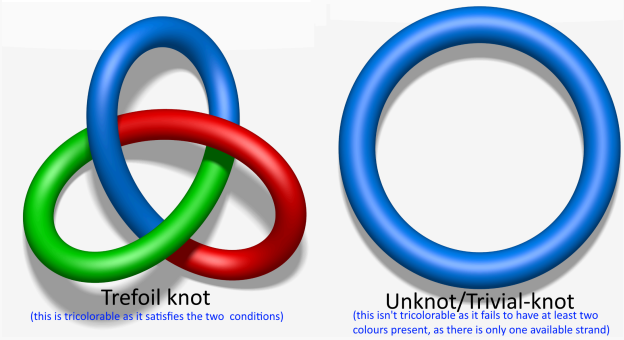


FIG 12 : *Proof that the trefoil knot is truly knotted*

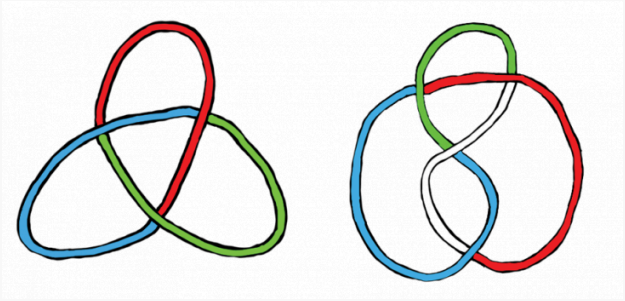


FIG 13 : *Trefoil and Figure eight knots are not equivalent*

Another situation where knot theory has been found useful is in the understanding of protein structures [10]. Proteins are complex molecules and do most work in a cell. Collagen, Keratin, hemoglobin and insulin are some of the examples of proteins in our body. Proteins are continuous linear molecules appearing as linear segments embedded in 3-space. Protein structure for a single protein is formed on three different levels (Primary, Secondary and Tertiary) as shown in **FIG** 19.

In the primary structure of a protein amino acids are arranged in a single line- there are no branches. When looking at a chain of amino acids it is always helpful to first orient yourself by finding this **backbone pattern** starting from the N terminus to the C terminus. The sequence and the number of amino acids ultimately determine the protein's shape, size, and function. Each amino acid is attached to another amino acid by a covalent bond, known as a peptide bond. Every amino acid also has another atom or group of atoms bonded to the alpha carbon known alternately as the R group, the variable group or the side-chain. The process by which the final 3-D structure of a protein is attained is known as *Protein folding*. Classifying proteins on the basis of their structure is an important problem for biologists as structure and function are very closely related. From the mathematical perspective, a protein structure is represented as a collection of atoms together with their coordinates in three-dimensional space, $\mathbb{R}^3$. Thus, if N atoms are listed with coordinates we have a vector in $(\mathbb{R}^3)^N$. *Two protein structures are said to be the same if one can be transformed into the other by an orientation-preserving Euclidean motion.*

It is convenient to think of a protein as a collection of piecewise linear (PL) curves formed by a sequence of points $p_0, p_1, \ldots, p_n$ in three-dimensional space joined

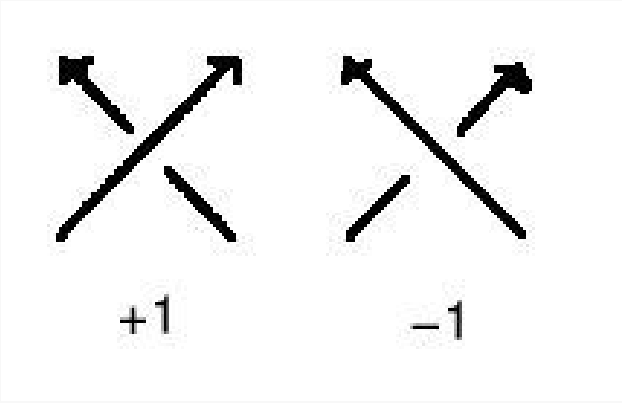


FIG 15 : *Sign convention at a crossing*

by line segments. Thus the backbone of a protein is a PL curve, with no self intersections and consists of a sequence of points representing the various C_a atoms proceeding from the N –terminus to the C – terminus. By putting the atoms in sequential order, a side-chain can also be represented as a PL curve. Since the Backbone of a protein can be regarded as a PL curve, whose one end represents amide end (N terminus) and the other end represents the carboxyl end (C terminus). This is technically an open oriented knot. It is of interest to understand the "knottedness" of this chain. Final structure of protein after folding will largely depend on the topology of its backbone, in the sense if backbones of two different proteins are differently knotted their final structures will have different shape and hence their functions will be different. Relevant questions to ask are: How to detect if the backbone knot is trivial?, If the backbone knot is non-trivial, can we determine its knot-type? Can knot theory be used in the classification of proteins by their 3-D structure? Many scientists have tried to answer the above questions. Broadly speaking, there are two different ways of studying the conformations of "knotted" protein chains: Hold the ends fixed and deform the backbone. And then interpret the final configuration. Start with an open knot and complete it to a topological knot and then identify the knot type. It is noted that both these methods have a certain degree of uncertainty. As the backbone knot is regarded as a PL knot we use the elementary moves to simplify it. William R. Taylor developed an algorithm for detecting knots in protein backbones and this was further refined and is now know as **KMT algorithm**. If the backbone gets straightened to the straight line joining

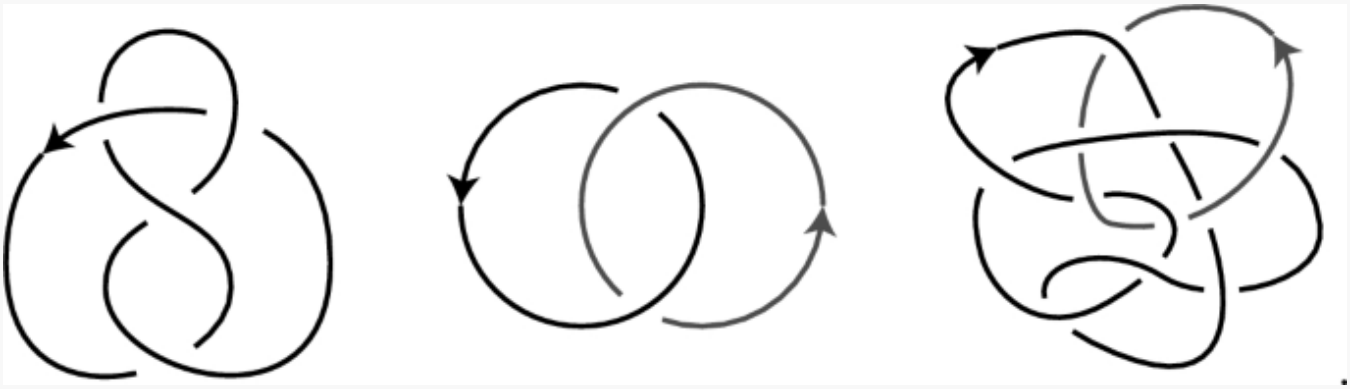


FIG 14 : *An oriented knot diagram*

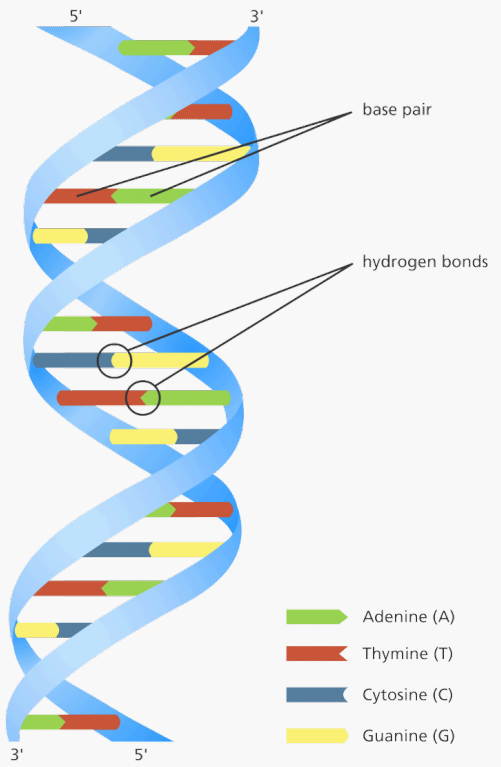


FIG 17 : *Structure of a DNA*

the end points, then the backbone does not contain a knot. Thus this algorithm answers the first question. Once it is detected that the backbone has a non trivial knot, with the help of simplifying the diagram we identify the knot types (**FIG** 20) . Knot invariants are useful in this.

Similar to the question of detecting two knots being ambient isotopic, scientists may *ask can two proteins be rearranged (without cutting) to be exactly alike?* In this direction Chen and Dill used knot theory in modeling proteins by associating a knot with every protein so that if two protein Structures are equivalent then the associated knots are equivalent. This is done as follows: To each chain they associate a planar signed graph known as its *Polymer graph*. From this graph using the *medial construction* they obtain a link diagram. Then knot theoretic tools could be used to determine the isotopy classes of these links. Application of knot theory and its tools to understanding proteins is still in a nascent stage. Some of the unanswered questions are stated below. Can the knotted protein backbones be unknotted *in vivo*? If they can be unknotted *in vivo* then what is the mechanism for unknotting and knotting? Is there a suitable model explaining the occurrence of existing knots and predicting the kinds of knots that might occur

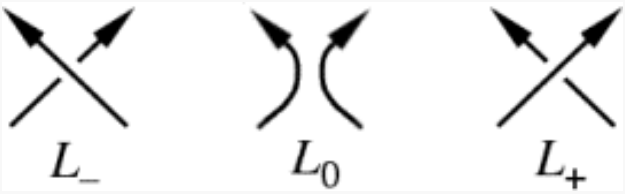


FIG 16 : *Skein-related diagrams*

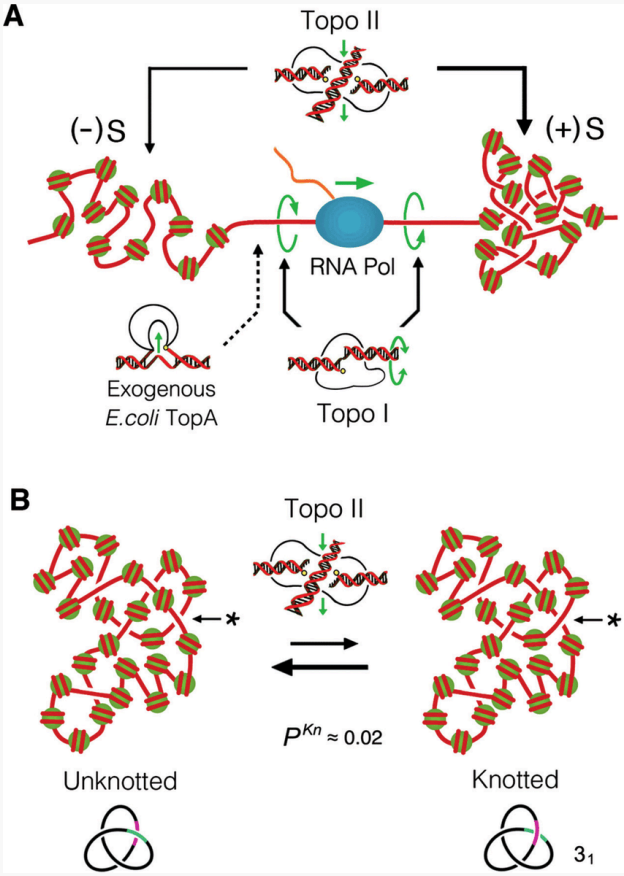


FIG 18 : *A knotted DNA*

or evolve? Given a knot K, can one construct a protein molecule whose backbone has the knot K?

Apart from biological sciences , knot theory is used in chemistry to synthesize new molecules and create new kinds of medicines. In physics, it is used to study the behavior of particles and fields in three-dimensional space. In Art, it is used to study patterns and symmetries in textile design and weaving. In the current era, the most fascinating application of knot theory has come to notice is in building *quantum computers* [11]. The main idea of using knot theory in quantum computing is to encode and process information using the topological 'braiding' of exotic quasiparticles called *anyons*. Because these knots depend on global tangling rather than local state, they provide fault-tolerant, noise-resistant qubits that are immune to most environmental disturbances.

I would like to conclude with a statement of Collin Adams, '*Knot theory began with sailors' ropes and mathematicians' curiosities. Today, it's helping decode the fundamental processes of life itself. This marriage of pure mathematics and molecular biology shows how abstract ideas can yield concrete, even life-saving, applications*'.

An earlier version of this article was published [13] in the Exhibit A science newsletter of IISER TVM. It appeared in the July - December 2024 edition of the newsletter under the same title. There have been some minor edits made in this version of the article - one paragraph has been included and some other changes in phrasing have been made. The author would like to thank the editors of the article at Exhibit A for their comments and suggestions regarding the article.

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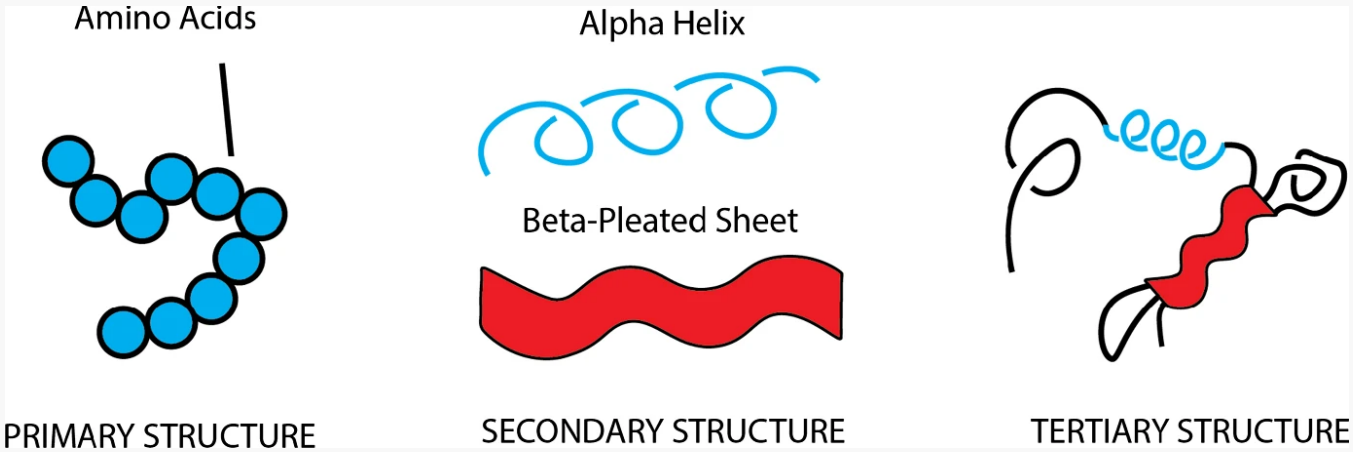

FIG 19 : *Protein structure at different level*

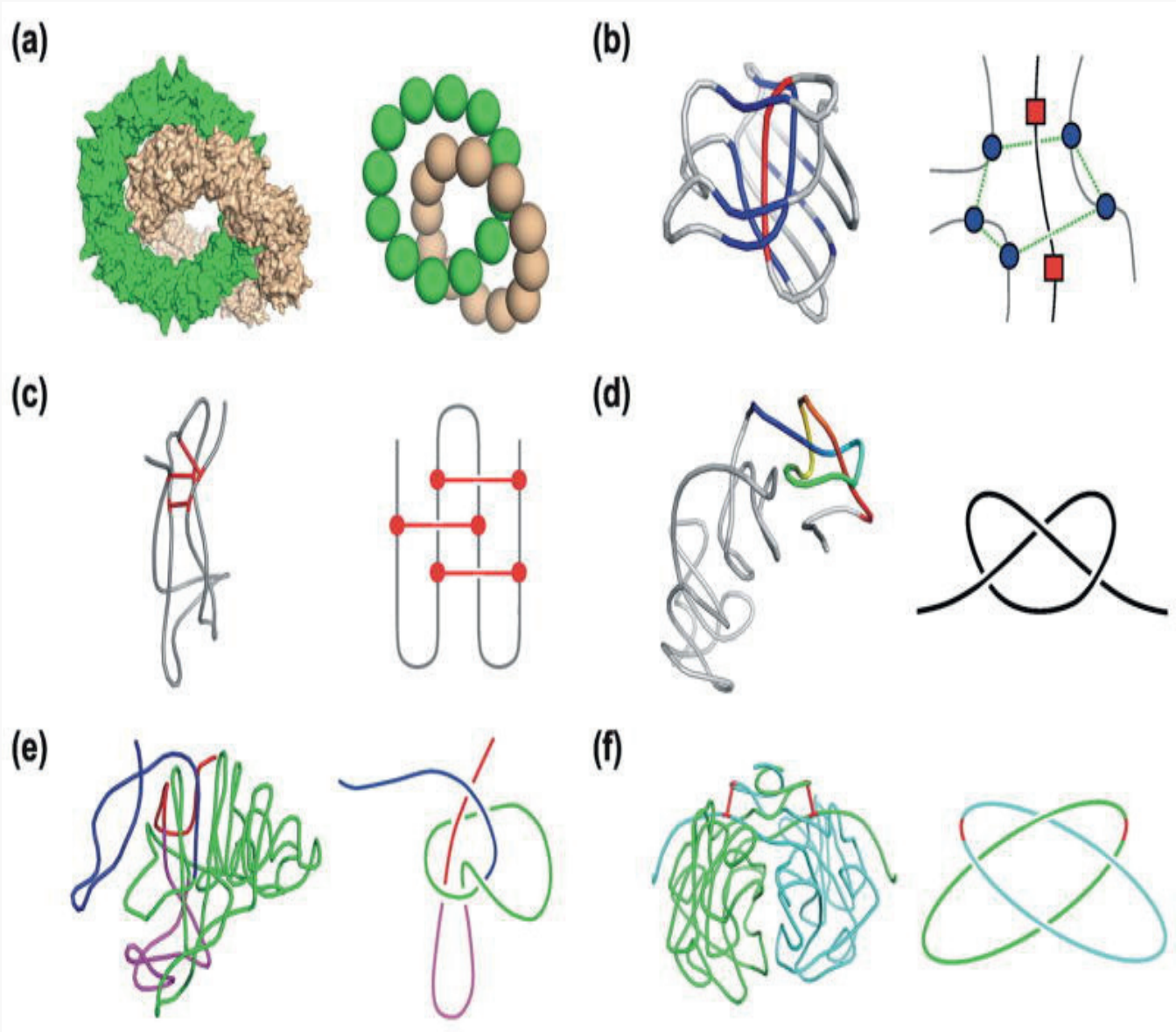

FIG 20 : *Knot types detected in protein*

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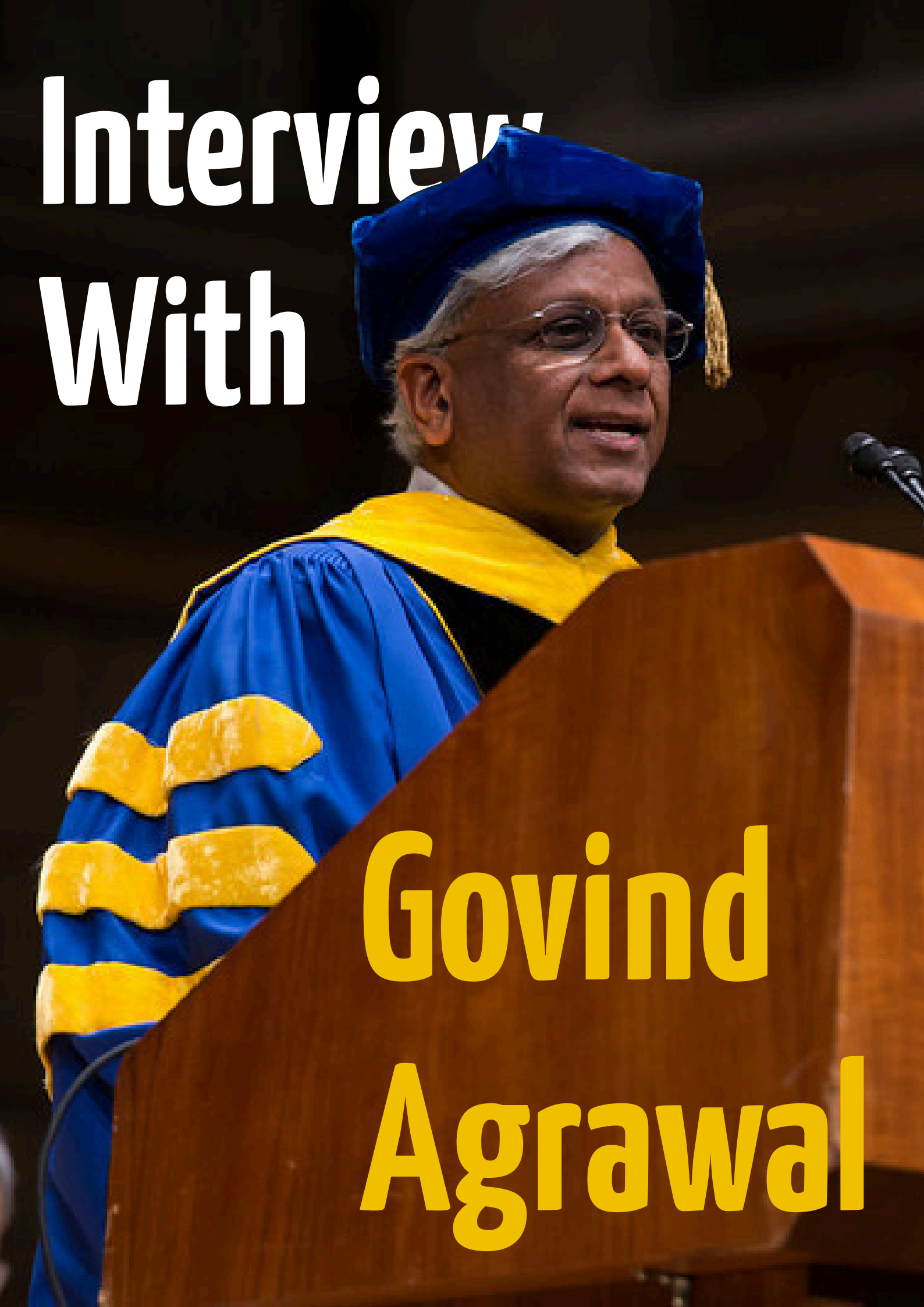
Interview
With
Govind
Agrawal

Interview With Prof. Govind Agrawal: A Pioneer In Nonlinear Optics

interview by

Debanuj Chatterjee *University of Lille, France*

July 21, 2026

"Chance plays a big role in achieving success, but you have to be ready there to catch it." From a daunting first day in Paris to writing the definitive textbook on fiber optics, **Prof. Govind Agrawal** reflects on a career defined by embracing the unexpected.



Professor Govind Agrawal is the James C. Wyant professor of Optics at the Institute of Optics, University of Rochester, USA. He is famous for his book Nonlinear Fiber Optics, which is widely accepted as the standard reference in the field, and is cited over 30,000 times. He is also an extremely accomplished researcher in optics and photonics, with more than 500 papers in peer-reviewed scientific journals. He is the recipient of several prestigious awards including the Optica Max Born Award for outstanding contributions in physical optics. Here is an interview with him, conducted by Debanuj Chatterjee.

DC: First I would like to ask about your childhood a bit. Where did you grow up and how was your childhood days and how did you get inspired to become a scientist?

GA: So, I grew up in a small town called Kashipur. At that time it used to be in Uttar Pradesh but nowadays it is part of Uttarakhand. So at that time the population was only about 30,000 people. I come from a business family. We were a very lower-middle-class kind of family. So childhood was pretty simple, nothing very special. My father wanted me to take over the shop, work with him, I mean later on. But I was not interested in that. I hated business and so I wanted to really get out of that place and I thought education was the only way for me to do that. So it's very early on that I discovered that I was good in math and I started getting good grades. In the primary schools and throughout I was number one every time. So my father realized that it would be better for me to probably go out to study and that was because he wanted to do that when he was young. He tried to go to Allahabad for a degree but he came from a very poor family and his health gave up. I think he was very happy that I could do the education.

Now how did I come towards science? So when I came to high school in the 9th grade I was only 12 years old because I was two years early everywhere. In those days you had to make a choice between science track, liberal arts and commerce track in 9th grade and I had no idea what to do at that point. So I inquired with lots of people and everybody simply said if you want to live a decent life with a good salary then choose the science track. So without knowing anything I picked the science track. And it turns out I was good at science. I started liking physics. I was very good at math. So for that reason I just stayed with science and then for my bachelor's (BSc) I went to Lucknow which is the capital of Uttar Pradesh. And again when I went there normally people took physics, chemistry and mathematics. But I went with the intention of topping the university and several people told me there don't take chemistry, take statistics because that is like math and you might do better with that. So I ended up doing statistics and maths. And then when I graduated from there, someone told me the best place for you to go is ISI in Kolkata because that's the best institute in statistics. I was good at statistics. So I applied for ISI and it turns out I made the interview. I got admission in ISI. However, at that time I was very young. When I did my bachelor's I was 18 years old. In those days Kolkata had lots of Naxalite problems. So my family did not want me to go to Kolkata in that situation. So I ended up coming to IIT Delhi for a master's in physics.

Then again when I was doing master's there the question was what subject you want to do PhD in? Everybody told me to do solid state physics, or condensed matter physics. I said okay, so I didn't take any optics class at all, thinking that I would focus on condensed matter. Then I asked what is the best place to do a PhD and the advice was go to TIFR in Bombay. I said okay let me apply for TIFR and let's see what happens. But unfortunately it so happened that on the day I was supposed to go to Bombay for an interview on the same day IIT Delhi also had their PhD interview. PhD at IIT Delhi was actually my backup plan, if something didn't work out at TIFR. So I came to see the department head and I told him that I'm the top student here, so why is this interview necessary? I mean perhaps you can take me without an interview. He was very unhappy and didn't want to agree to that. But eventually he said if any professor in the department is willing to take you without an interview then he can agree to it. So before leaving for Bombay at 5:00 pm I was looking for some faculty member in the department, and only one room's light was lit. That was a professor

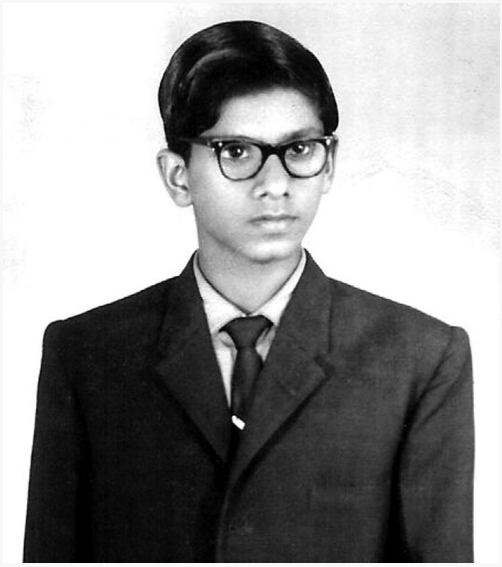

FIG 1 : ***Prof. Agrawal*** *when he was in high school, in 1965*

who had come from Rochester recently, his name was Chandra Lal Mehta. He had no students at that time, so I went to see him and asked him if he was willing to take me as a student. He said "you are a top student, I have no one, so sure I'll take you" and that's what brought me to optics basically. So again, it was not my desire, it's just chance that I didn't make a TIFR interview. I came back to Delhi, did a PhD there and that's how I came to optics.

DC: So, it was the start of nonlinear optics as well in your PhD days.

GA: I can tell you a little bit there because my advisor Mehta was a student of Emil Wolf in Rochester. So, his field was quantum optics basically. I mean this was a relatively old field and I was a little concerned that if I did a PhD in that area I would not make much progress. So I wanted to do some work in some other area. So I went to the library and started looking and I discovered that nonlinear optics is a new field. So I went to my advisor and said "I want to work in nonlinear optics" and he said "I don't know anything about nonlinear optics. I've not worked in this area so how can I advise you in this area?". So I suggested to him that I will go to the library, read the material and I will give weekly seminars in the group on nonlinear optics. Based on that perhaps I can do some work in nonlinear optics. That's why I ended up doing that. So I worked in that area which he was interested in but I also did some nonlinear optics work and that brought me to Paris. So I came to Paris for a postdoc in 1974. That was in nonlinear optics. That's how I became part of nonlinear optics.

DC: Was there any particular moment in your career which significantly shaped your scientific career or let's say you started from a well-known path, checking all the boxes, and then some incident happened which took you on a tangent?

GA: Right. Right. Yeah. Yeah. I think there are a couple of things I can think of. One was in high school, I think, where I was not shooting high enough. I mean, I didn't think I could do very well. So one of the teachers told me, "you can try to become number one", and I said "but no, I mean I'm not like that. I can't do that, that is impossible for me..", so he told me "but why? You have two eyes, two ears, everything like everybody else, so why do you think you can't do it? You have to change your mindset and think that everything is possible", and I think that struck me and gave me a kind of confidence. My ambitions became much much higher after that. And the second person I can give credit to when I left Paris for a second postdoc in New York, I had a professor, Melvin Lax, he actually did a lot for me from all points of view. I mean he helped me in every respect. I think whatever I became is because of him in some way. Even my immigration to the US was because of his help. My job finding was because of his help. So he was very helpful in many respects and so the second person would be this person here.

DC: He was your postdoc supervisor?

GA: Yeah. He was my postdoc supervisor. I did end up with two postdocs. I mean so the story was, I tried to do a PhD in the United States. So after master's I applied to the US for PhD and then I got admission in three four places. But my problem was that my TOEFL score was not good. So none of them wanted to give me a research assistantship or teaching opportunity. So they admitted me and they said that I can come for a semester and if I do a semester then they can give me support afterwards. But I come from such a low income family there was no way for me to come there for a semester. So that was a kind of big disappointment for me. But then I thought it's not a bad thing after all because if I went to the US it would have taken me six years to do a PhD, but if I stayed in India I could finish a PhD in three years. So because of that in July 1974, I got my PhD and I was 23 years old at that time. Then I moved to Paris as a postdoc. So I did a two year postdoc in Paris and then I questioned myself, what do I do now? So I started applying for postdocs in Europe, in the United States, wherever, and everytime just ended up with rejections. Absolutely nothing. So I

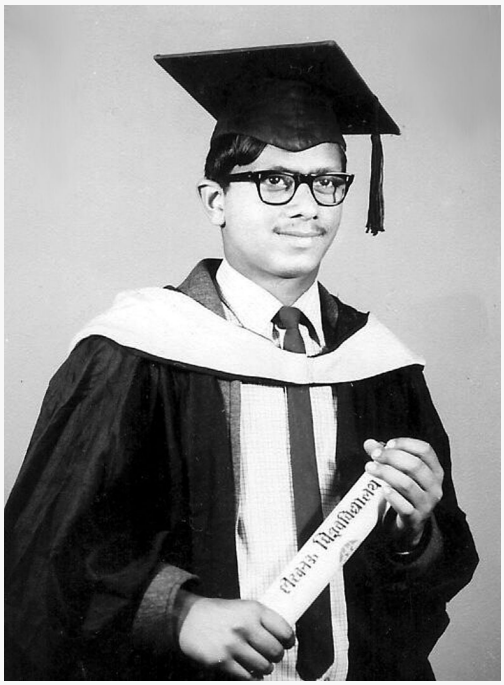

FIG 2 : ***Prof. Agrawal*** *at his graduation from Lucknow University*



FIG 3 : ***Prof. Govind Agrawal*** *(middle) in the Eiffel Tower, Paris*

thought, okay, I'll have to go back to India. And then one day, something interesting happened. And people should know that doing hard work is not enough, you have to have luck or chance or whatever you call it. I went to the library in France, and I saw the magazine "physical review" and I took it and opened it and suddenly I saw a paper and the author's name was M. Lax. That name suddenly sounded familiar. How do I know this name? And I could not figure out why I knew this name. But then eventually I remembered that when I got admission (in US) three years before when I was applying for a PhD, at the City University of New York, they told me something like "we recently hired a new professor Melvin Lax, who is very well known and if you change your mind and want to come next year, you can apply again and you can work with him". So that's why I remember that name. And because of this relationship, I said to myself, "what do I have to lose? I'm just going to send my CV to this guy". I mean in those days you just mail them because there was no internet. So I just sent my CV to him with a request for a post and I was not even working in that area. I was working in a totally different area. So I didn't have high hopes, but believe it or not in 10 days I got a reply back saying "I'm interested, send me your letters of recommendations". And letters went and I got the offer!

So that was very surprising to me, how someone I didn't expect at all gave me a job and I still don't know fully why it worked out, because I got 20 rejections and suddenly at the last minute when I completely lost hope I got this. That happened to be the best thing for my life. I mean this person was the best person who did everything for me. And here's another story. So I reached New York to work with this professor and there many Indian students were there and I was talking to them and they were saying "you came to the wrong group, you should not have come here". They said "Lax does not do physics at all anymore, he's totally on the computer. His students and postdocs have not published a paper in several years, nobody has been able to publish with him and so you will have problems". So I got very scared. In fact it turned out to be the truth to some extent because when I went to see my professor for the first time, rather than talking about physics or my problem he told me "I have a magnetic tape that I got from Los Alamos, that I cannot read on the computer here. So your job is to write a computer program that will be able to read this tape here." And I have never used computers before. I had absolutely no idea what he was talking about!

DC: This was in which year?

GA: 1977.

DC: 1977. Okay. Computers were just beginning to be a thing at that time...

GA: Yeah, Yeah. So I was working mostly with analytic things but I didn't use computers before. Even in France I was not using computers. And this guy comes and says "I want to write a program in C language or whatever" and this and that, and says "here is the tape, do something, so that I can read it". And he's not talking about physics at all. So I was really scared. I didn't know how to do it, but I could not say no to him. After all, he is my postdoc advisor. So I said "but how would I?". He said "there's a terminal here. You will create an account and log into this and then you do it". I said "but how?" He said "use the editor" and I said "but who is the editor?". And then he started laughing because I have no idea the editor could be a program and not a person! He said "looks like you've not used computers much". So anyway, he said "okay I give you a month and you do this". And I did it for him. Basically I learned everything very quickly. So he became impressed to see that within a month I could do his job starting from scratch. And that kind of helped me out later on somehow.

DC: I think we'll come back to this because one important thing here is to learn very quickly something new.

GA: Yes. Yes.

DC: I'm just curious to know that after doing a PhD in India, why did you choose France? Because France has a big language problem.

GA: Yes. Yes.

GA: So that's a good point. Let me tell you what happened. When I applied for a postdoc after PhD, I got only rejections from the United States. But I got three positive replies. One was from New Zealand, one was from Italy and one was from France. And I was inclined towards New Zealand. Because New Zealand had no France-level language issue, and I guess I had no idea which one would be best. I mean because all the professors were pretty good. When these offers came to me I was at Raman Research Institute in Bangalore. I was visiting them for a month because I had applied there and they said why don't you come and spend a month here before we make you an offer. So there I made an appointment with the director of the Raman Institute and I went to see him. I said that I have several postdoc offers. I'm thinking of going for a postdoc. I can come back to you later on if you want. And right now I have these three offers and I'm wondering where to go. And it turns out that the wife of this director was a French woman. So he knew about French culture and about France. So he looked at my offers and he told me that "out of these, to me it looks like the French one is the best. Because financially and the kind of lab you're going to, they make much better sense", and he also told me "if you go to New Zealand you



FIG 4 : *In front of the Maison de l'Inde at the Cité internationale universitaire de Paris.*

will kind of get lost there, because it's so isolated". So he basically pushed me towards France.

So I said, okay I'll try to go to France then. But when I came to France immediately I had a huge issue. So let me tell you my experience of the first day in France. I landed at 8:00 am in the morning at the Charles de Gaulle airport. The airport was very new, just opened a few months ago. In those days they would not give me a lot of foreign currency because India did not have a lot of foreign currency reserves. So all they gave me was 40 francs. I had 40 Franks in my pocket and that's all I had. And there was absolutely no other money of any kind. And I had written to get a room in a place called the Maison de l'Inde (India House) which is a part of Cité Universitaire (a residential area in Paris, for student expats). I had also written a letter to my advisor saying that I'm coming and I will join you soon. So after I landed in Paris, I inquired and they said if you want to go to Cité Universitaire, you have to take the Air France bus. After paying for the bus I was left with 10 Franks. Then I had to take a metro. I didn't know the French language but somehow I managed to arrive at the Cité Universitaire and went to the Maison de l'Inde. I told them that I have written a letter and I would like to get the room there. They told me there's been a postal strike for the last two months here and there's no delivery so we have no idea. There's nothing for you here. There's no room here because we didn't get your request. So that was my first bombshell. It happened to me at around maybe 10:30 am in the morning. I said okay, if that's the case then maybe I should go to see my professor. So somehow I figured out how to get there by talking to people. This was in Orsay, so I had to take a train. I had just enough money to buy the roundtrip train ticket and I had nothing left in my pocket. I went to Orsay and I walked over to the street of Institut d'Optique. Over there I talked to the secretary in English and she didn't know English very well. I showed my letter of appointment. I showed her the name of the professor and the lab I'm supposed to work at and she said "but this professor does not work here". I said, "but this letter shows you that he should be here!" She said, "I don't understand what you're saying, but I can guarantee you there's nobody with this name here". I looked around, tried to talk to some other people there, but nobody responded. So, I didn't know what to do. I simply came back to the Maison de l'Inde. By now, it's about 4 pm. I've not eaten the whole day. I have no money in my pocket. I have no place to sleep. So I asked the guy at the reception, "what do I do? This is my situation". He said that one option is that he can call the Indian consulate. They will probably send me back to India. They will come and pick me up and send me back to India and ask my parents to pay for the plane fare. I said "I won't do that yet". So I was just sitting in the reception area, feeling very sad and rejected and it was dark outside (by 4:00 pm in France it is dark in December). And here is the amazing thing about being in Maison de l'Inde. Somebody noticed me. He came to me and he said "you look very sad, what is the problem?". So I explained all this to him and he said "okay let me think if anybody from IIT Delhi is around here or not". So after some time he said there's a Sikh guy who lives in Maison du Portugal (Portugal House) nearby. He said maybe you can try to go and find him.

So I went and found the room and his wife opened the door and I told her that I want to see her husband and she said that he's still working. So I had to wait there for a couple of hours. He came back around 7:00 pm and here is a very interesting part which is sort of a surprise to hear. This guy was working at the Institut d'Optique, the same institute where I am supposed to work and where I went a couple of hours ago. And look, here's luck! So when I explained to him and I told the professor's name he said yeah he used to be there but he moved to another institute called Ecole Polytechnique a few months ago. So it's true that he's no longer there. So now everything made sense to me suddenly. So this guy again I would say is one of the guys whom I should credit, because he did everything for me. He said "have you eaten anything?" I said "no". So he gave me food. Then he took me to a hotel and booked a room for me (he spoke French). Then he found someone who worked with the same professor I was supposed to work with, and the next day I could visit my professor. My professor was happy to see me, but he didn't know I was coming. He said "tell me something, do you need money?" And I said "yes I do". So he took me to some place and he asked someone to give me my first month's salary. So once I got the money my problem was solved, I didn't have to worry about anything.

DC: My next question is about your book: Nonlinear Fiber Optics. It's a very well-known and very well-cited book. What is the story behind this book? What motivated you to write a book? Did you anticipate the impact that it has?



FIG 5 : *The **Institut d'Optique** in Orsay, where Prof. Agrawal arrived only to discover his professor no longer worked there*

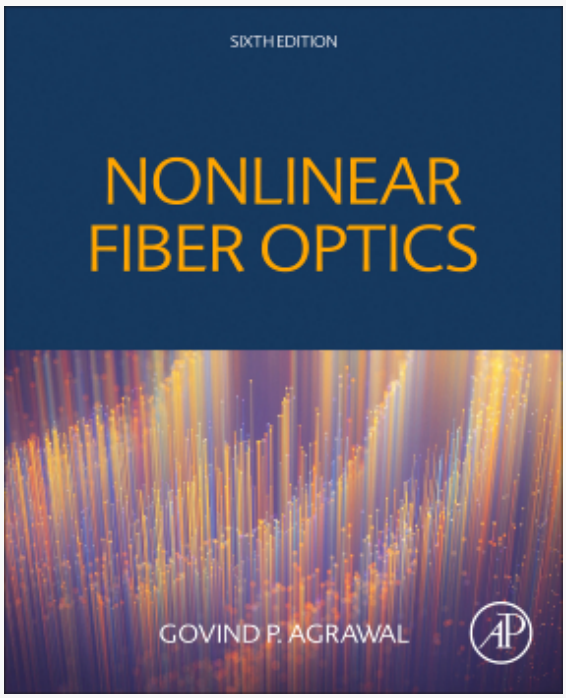

FIG 6 : *Prof. Govind Agrawal's* ***Nonlinear Fiber Optics****, is a well-known and well-cited book in the field*

GA: So first of all, let me tell you that until the time I joined Bell Labs, in New Jersey, I had problems publishing papers because my English was not so good. But after I reached Bell Labs, after a few months I wrote an internal report and my American boss there came to me and said, Govind, what you write here is 100 times better than what my American friends write here. None of them can write so clearly. He said "I'm very impressed how you explain these things". So I was very surprised. I said, "Wow, okay, I mean, I have become a much better writer now, looks like". So I thought, let's take advantage of that and I will write a book. I was working with semiconductor lasers at that time. So I thought that the first book I will write will be about semiconductors. I teamed up with a guy called Niloy Dutta who was also there, a Bengali guy. We wrote a book. He was an experimentalist. I was modeling the theory. Together we wrote our first book on semiconductor lasers and this book was published in 1985. At that time I was not working in fibers. Fibers were far from my mind and then one day in 1985, a woman walked into my office and she said "I'm working on optical solitons and I would like to talk to you. Perhaps you can help me". I asked her "Who sent you here? Because I do not work in this area. I don't know what solitons are. I have no work with pulses. I only work with beams. I have no idea what you're talking about!" She said, "it's okay but let's just talk". So I started talking to her, she was telling me about this soliton thing, this and that and she said "I need someone who can develop some code for this, so that we can do both analytical and numerical work on this". Initially I was very reluctant but then suddenly it hit me that I have written these codes for relaxation of a laser which are basically diffraction of beams but what she's saying is diffraction or dispersion they're actually the same thing, so I can just use my existing code. So I agreed to work with her. So she's the one who really brought me to the fiber optics area. And for a year we worked together, published three papers on solitons and pulse propagation in a fiber. I started learning the language a little bit and here we're talking about 1985, 1986.

DC: So at that time Erbium doped fibers were already there?

GA: No they were not there yet. So I started learning and then I started seeing all this interesting work being done in Bell Labs at different groups. We started learning about different things. I said maybe I want to write a second book on my own. I can write a book in this area which I'm just learning and that will force me to learn better than I can do otherwise. So I didn't think that this book would become a popular book. I just thought of writing because I wanted to learn this area and because I want to learn this area I ended up writing in such a way that it is easier for people to follow. So the book was written to teach myself fiber optics. The success of the book was just completely unexpected for me. So in my life a lot of things happened by chance, that's kind of one theme I find throughout my life. Chance plays a big role in achieving success, but you have to be ready there to catch it.

DC: So according to you what are a few unanswered exciting questions in the field of let's say nonlinear optics or you can say optics in general that in your opinion might have a very big impact on the future of science and technology.

GA: So when I think about it I can think of two things which could be interesting in the future. One is what people call quantum nonlinear optics right, so to me it's sort of strange. When you think about quantum, it generally means a single or few photons and on the other hand nonlinear optics always requires high intensities. So to me how can we observe nonlinear effects using very low intensities, is very interesting. So I think this area may develop in the future. I don't know how much, but it will require some very different materials, probably because after all you need something which has a very high nonlinear coefficient. And the second direction which I think is ultra-intense light-matter interaction. People are going towards very high energy pulses and in that regime you have this breakdown and all those kinds of things. That's another direction where a lot of new things may happen because electric fields are very huge and atoms are breaking apart. There's plasma and there is much more to come in the future I think.

DC: Do you think there are few skills which undergraduate physics students should develop for a bright scientific career in the future?

GA: So the one skill which I tell my students as well is to do things early, not wait to finish anything. I mean when I give homeworks to my students in my class they typically ask to give them a week's time to finish their homework, but they don't start it the day before the deadline. They're always running at the last minute to ask for it. So what's the point of giving a week to do it if you're always going to work on it on the last day? So it's sort of very strange to me. My philosophy has always been to do things right away, and I'm known for it in my department. When I get an email and it asks me something and it gives me a deadline I actually do it then and there. So I do things as they come in. So do things in advance, early on, so that you're never under pressure.Your life will be much much

simpler if you don't wait for the deadline. Most people don't do that. So that's one skill.

I can give you a little tangent to tell you about something, that before I moved to Rochester I said if I move to Rochester I should probably know someone there. So I should try to make contact with someone there. And that's the second advice I wanted to give to students. Don't be afraid of making connections. So I was saying that I needed to know someone there who can vouch for me, otherwise if nobody knows me how do you get a job? So I thought my grand advisor (advisor's advisor) Emil Wolf was there. And then I found out that he was actually publishing a book series called Progress in Optics. So I sent a letter to him. I told him I was his academic grandson (student's student). I would like to propose to write a book chapter. I gave him a deadline. So he consulted and he asked me to write it. I worked on it immediately and sent it to him before the deadline. He sent me a letter saying that nobody in his entire career had ever delivered the chapter on the first deadline. I sent him two weeks before the deadline! He was so impressed he immediately wrote me a letter saying this has never happened. I was surprised. Why not? Because my habit is if I say something I do it in advance. So that struck him and when my application came in, I'm sure he really supported me for that. So second advice in that topic is don't hesitate in making contact and even senior people who are high up, you may think that they don't care about you but if you talk to them they will be very nice to you. I was impressed how nice Wolf was to me whenever I talked to him. And third I think communication skills are very important for everyone. So I think the three points you were asking me are basically : do things in time or early, don't hesitate talking to senior people, and then develop communication skills. Those are very important.

DC: If you could have dinner with any scientist from history, who would it be and why would you choose that person?

GA: Probably everybody thinks of Einstein. But when I think about it the person who comes to my mind is Niels Bohr, and the reason he came to my mind was because he was doing quantum mechanics before quantum mechanics was developed. No one had the tools and he still could quantize the hydrogen atom and tell you what energy levels are. He was using classical physics. He did classical physics and basically based on that he came up with the idea of having the energy state. To me that is amazing.

DC: Do you have any final message for young scientists especially in India?

GA: I guess the message from my life story I told you simply is that failures are always part of the game. You don't expect to succeed at everything you want to do. Certain things will not work. You have to learn to take the failures without feeling depressed, and still keep doing your part and eventually things will work out. There will always be rough times. Nobody succeeds without a rough time. And luck plays a big part. As I have seen, just being hardworking and being intelligent is not enough. I mean I succeeded but by luck partly, because there's hundreds of people in India like me who don't succeed even though they are as good as I am.

DC: So yeah, that was awesome. I really enjoyed discussing all these things with you. Thank you very much for this interview!

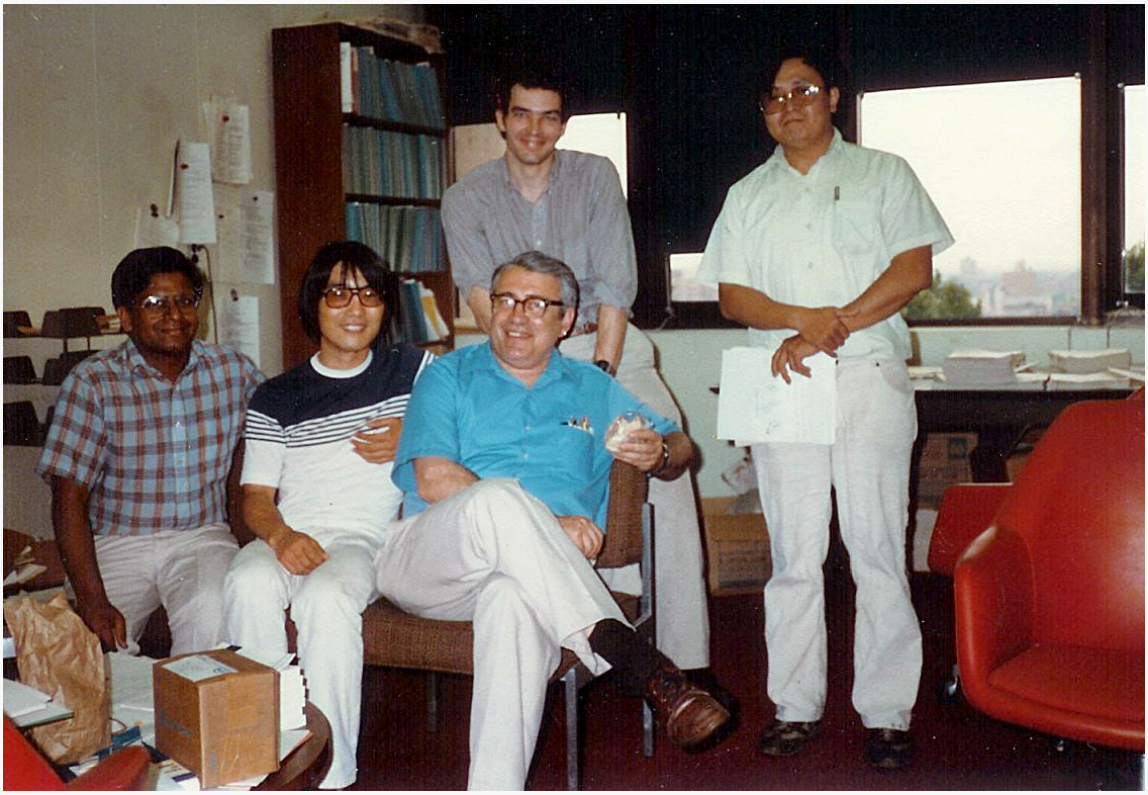

FIG 7 : ***Prof. Govind Agrawal*** *(left) with his Postdoc superviser,* ***Prof. Melvin Lax*** *(middle), who was a professor at the City University of New York*

Z Is For



Z is for Zeta

Bharath Krishna S *IISER Thiruvananthapuram*

Reviewed By: Sobitri Sen, Chitradeep Saha

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The claim that $1 + 2 + 3 + \ldots = -\frac{1}{12}$ is intriguing–but what does it really mean to "sum" infinitely many numbers? This article introduces the ordinary definition of an infinite sum, explores how other summation methods can assign values to some divergent series, and explains why $1 + 2 + 3 + c\ldots$ still does not equal $-\frac{1}{12}$ in the usual sense. The number $-\frac{1}{12}$ instead comes from extending the Riemann zeta function beyond the region where its defining series converges.



Bharath Krishna S is an Integrated PhD student from IISER Thiruvananthapuram working in Complex dynamics and Ergodic theory. Apart from his research, he is interested in studying the history of mathematics and writes expository math content where he simplifies concepts from higher math for a general audience.

Leopold Kronecker is famously known for his quote,

God created natural numbers:
all else is the work of man

Right from the usual counting that we do on a daily basis, the set of natural numbers - given a nice notation $\mathbb{N}$ by mathematicians, is ubiquitous in daily life. One rather mysterious and highly misunderstood notion regarding the set of natural numbers is the statement

$$1 + 2 + 3 + \ldots = -\frac{1}{12}.$$

The buzz around this *"identity"* has been exemplified in math popular culture due to the scores of online content - some of which unfortunately serve as clickbait without conveying the whole mathematical idea properly. This article is an attempt to dispel some of the myths around this so-called mysterious *"identity"* and see whether this statement is true. This article shall take a mathematical perspective and hence shall not address things related to this concept in physics or any other world other than math. The beauty of mathematics lies in unlocking these *"mysteries"* through rigor and logic - a trait that makes math the **Queen of all sciences**. In fact, a thorough mathematical understanding of these concepts actually helps in concluding that there is no mystery here but just the sheer beauty of math.

If one were to see the expression $1 + 2 + 3 + \ldots = -1/12$, the first thing that probably crosses their mind is possibly that, how can the "sum" of positive numbers be a negative number? But hang on, the first question should be, **How does one find the sum of infinitely many terms,** ain't it? A more interesting question is whether such a number as the sum of the given infinitely many numbers even exists. Here is where the mathematical formulation of a series comes in. An illustrative example to explain the notion of convergence or sum of what we call an infinite series (a category to which the sum in question - namely $1 + 2 + 3 + \ldots$ belongs) was given in [1]. We revisit that example here, but before that let's shed some light on sequences, which are, in some sense, the building blocks of series.

A sequence of real (resp. complex) numbers is a function from $\mathbb{N}$ to $\mathbb{R}$ (resp. $\mathbb{C}$), that is, to each natural number there is a unique real (resp. complex) number associated. In short, we can represent a sequence by $(a_n)_{n\in\mathbb{N}}$, which means n is mapped to a_n. Given a sequence, an interesting question is whether we can associate one single number that depicts the behaviour of the whole sequence. Let us consider the sequence $a_n = 1 \quad \forall n \in \mathbb{N}$. Since all the terms in the sequence are 1, it makes sense to designate 1 as that number which depicts this sequence. Ok, this was easy! What if the sequence is given by

$$a_1 = 1, \quad a_2 = 2, \quad a_3 = 3 \text{ and } a_n = 0 \quad \forall n \geq 4.$$

Now it looks like a good candidate for the number we are looking for is 0. This is nothing particular to this case - where every term except the first three terms are identical. In fact if we define the sequence $a_n = n$ for all $n \leq 10,000$ and $a_n = 0$ for all $n > 10,000$ then too, it makes sense to take 0 as the candidate as "more and more" terms of the sequence take the value 0.

Now what if there is no repetition of terms? Can we find a candidate number to represent the sequence? On that note, let us look at the sequence

$$a_n = \frac{1}{n} \quad \text{for } n \in \mathbb{N}.$$



FIG 1 : ***Leopold Kronecker***, *the German mathematician who worked in number theory, abstract algebra and logic. The popular quote "God created natural numbers: all else is the work of man", is famously attributed to him.*
Source: [10]

The terms in the sequence are $1, \frac{1}{2}, \frac{1}{3}, \cdots$. Now, there is no term that is repeating, and it looks like there is no end in sight in terms of repetitions. But it does look like the terms eventually go closer and closer to 0. How do we make sense of it? If you had observed in the above examples, it turns out that, in defining our candidate number, all that mattered was a section of the sequence that lay beyond a specific n. We call this the tail of the sequence. So, does it mean that for any given sequence we can associate a candidate to represent it? It turns out that the answer is, no! Let us look at the sequence $a_n = (-1)^n$. This sequence is $-1, 1, -1, 1, \cdots$ (i.e.) -1 and 1 appear alternatively. It is clear that we can't get a proper "tail" in sight as beyond any given n, both 1 and -1 keep occurring alternatively.

We call a number a to be the limit (the candidate number that we were talking about in the examples) of the sequence a_n, no matter how "close" we get to a, we can find a tail of the sequence that lies in that proximity. Such sequences are called convergent sequences and by $a_n \to a$ we mean that a_n converges to a (i.e.) the limit of the sequence a_n is a. We list a few properties of convergent sequences.

1. Suppose $a_n \to a$ and $b_n \to b$, then $a_n + b_n \to a + b$. (sum of two convergent sequences converges)
2. Let $\alpha \in \mathbb{C}$ and $a_n \to a$, then $\alpha a_n \to \alpha a$ (multiplying the terms of a convergent sequence by a scalar does not affect the convergence of a sequence and in fact the limit of the resultant sequence is scaled by the appropriate scalar)
3. Its a non-trivial and interesting result that if $a_n \to a$ then the sequence

$$b_n = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

converges to a. (the sequence of averages upto n terms of a convergent sequence converges to the same limit as that of the original sequence)

We now try to explore infinite sums and see if we can assign particular values to them. We shall try to explore a few summation "techniques" on the way and try to compare the different "definitions".

A series is typically an infinite sum $\sum_{n=0}^{\infty} a_n$ where each a_n is a real or complex number. **Now, how do we make sense of this sum? Can we associate a limit for a series and call it the value of this infinite sum?** To achieve this, we use the help of sequences. At the moment even as we don't know a proper way to find the sum of an infinite series, we do know how to add finitely many terms. We use that knowledge to define sum of the series and find that it has all the nice properties. We now view the above series as a sequence in which each term is actually a sum of finitely many terms. More precisely, the first term is just one number, a_1, second term is a sum of the first two numbers, $a_1 + a_2$, third is a sum of three numbers, $a_1 + a_2 + a_3$ and so on. We can represent the kth term of this sequence by $S_k = \sum_{n=1}^{k} a_n$ and this is called the kth - partial sum.

D1: We define the sum of the series to be the limit of the sequence S_k (if it exists).

A series for which this limit exists is called **convergent**, and one for which it does not exist is called a **divergent series**. This definition gives all the nice properties to series. For example, using this definition, it follows that the sum of two convergent series is convergent, and its limit is the sum of the individual limits. In fact, this conclusion is a corollary of a similar result on sequences, and it follows from the properties of convergent sequences given above. With some effort, it can be observed that this definition efficiently facilitates the addition of two convergent series, multiplying elements of a convergent series by a scalar, removing and adding finitely many terms to a convergent series and still not altering the sum of the series. It also clearly distinguishes when the rearranging of terms is possible in a series and also takes into account different ways of grouping terms while adding and thus minimizes ambiguity, thereby ending up with a unique value of the sum of an infinite series, provided it exists.

One of the basic examples of a convergent series is the geometric series $\sum_{n=0}^{\infty} x^n$ where $|x| < 1$. Another good class of examples of convergent series is $\sum \frac{1}{n^k}$ where $k > 1$. While we saw that the sequence $\frac{1}{n}$ converges, with a little effort, it can be shown that the series $\sum \frac{1}{n}$ does not converge. From the definition, it is straightforward to see that $\sum_{n=0}^{\infty} 1$ is not convergent as the sequence of partial sums $S_n = n$ diverges. There are various tests to

determine the convergence of a series of positive terms and ones with terms having terms whose sign alternate - which we call an **alternating series**. One example is $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ and it can be shown using one of the above-mentioned tests that this series converges. Speaking of alternating series, it is time to revisit our example from [1], namely $\sum_{n=0}^{\infty}(-1)^n$. Clearly, by definition, this series diverges as the sequence of partial sums $(1, 0, 1, 0, \cdots)$ does not converge.

This example leads us to see if we *make* this series converge in a different world. A natural question that might arise in this regard is the need to do this in the first place. We know that our definition classifies this series as divergent, so why bother much? It turns out that having a notion of "limit" for certain divergent series helps in some branches of mathematics - one example of which is **Fejer's theorem** in Fourier analysis. Another argument in favour of trying to do this is related to the property (3) that we saw above for convergent sequences. We have seen that any convergent series follows that property, but the converse is not true. Thus, it makes sense to have a universe where we can try to see if the converse is true. While we will not go into it in more detail, it is worth mentioning that a few results exist in this direction. We will generalise our definition of sum of a series by using property (3) to give the notion of limit to a few more series which missed out on this benefit using the previous definition. But when we do this, we should make sure that the series that are already convergent using the previous definition shall continue to converge to the same limit using the new definition as well - which is exactly why we use a property (3) to give the new definition of the limit of a series.

D2: Suppose $\sum_{n=0}^{\infty} a_n$ is a series whose sequence of partial sums is denoted by S_n. Define the sequence $T_k = \frac{1}{k}\sum_{n=0}^{k} S_n$. We say a is the limit of the series $\sum_{n=0}^{\infty} a_n$, if the sequence T_k converges to a.

So far, we have seen two different notions of the limit of a series. The previous definition, marked as **D1** denotes Definition 1 and the above notation **D2** denotes Definition 2. Suppose a series converges using D1, it clearly converges using D2 as well - by simply using the property (3) for its sequence of partial sums. Also, note that the series $1 - 1 + 1 - 1 + \cdots$ does not converge using D1, but it converges using D2. This is because the sequence T_k in this case is $1, \frac{1}{2}, \frac{2}{3}, \frac{2}{4}, \frac{3}{5}, \cdots$ can be shown to converge to $\frac{1}{2}$. Many such alternating series that diverge using D1 can be given a limit using this technique. D2 is called the **Cesaro summation method**. Many other summation techniques exist and are studied in the context of divergent series.

Now, let us look at how the series $1 + 2 + 3 + \cdots$ fares under our definitions. Clearly, it does not converge under D1 because the sequence of partial sums $1, 3, 6, \cdots$ does not converge. As a side note, we hope that an avid reader would have noted that the sequence of partial sums is actually the sequence of triangular numbers (i.e.) the sequence $\frac{n(n+1)}{2}$. Also observe that this series does not converge under D2 as well since the sequence T_k in our case turns out to be $1, 2, \frac{10}{3}, 5, \cdots$, which also diverges. In fact, this definition has been manipulated in multiple places on the internet without rigorous explanation to "prove" that the series $1 + 2 + 3 + \cdots$ has limit $\frac{-1}{12}$ - which is actually false according to our mathematical setup. One of those online resources actually came from a highly popular YouTube channel. The interested reader may check out [6].

The following arguments are put forward keeping in mind the set up we have developed in this article. One big fallacy in the above-mentioned video is this. They add $1 + 1 - 1 + 1 - \cdots$ in two different ways namely $(1 - 1) + (1 - 1) + \cdots$ and $1 - (1 - 1) - (1 - 1) - \cdots$ and say that we get limits 0 and 1 respectively - which "clearly" gives the limit to be $\frac{(0+1)}{2} = \frac{1}{2}$. Firstly to start with, **how did we know that the series has a limit? What definition was used? How can the grouping of terms be justified?** Another popular "explanation" for the limit of the series $1 - 1 + 1 - 1 + \cdots$ being $\frac{1}{2}$ is that we can take $s = 1 - 1 + 1 - 1 + \cdots = 1 - (1 - 1 + 1 - 1 + \cdots) = 1 - s$, which gives $s = \frac{1}{2}$. But hang on! How do we know that such an s exists? This cannot be assumed. This explains the importance of a rigorous definition. While there's no stopping anyone from defining their own notion of convergence such that their definition allows for the limit of the series $1 - 1 + 1 - 1 + \cdots$ to be $\frac{1}{2}$, the rigors of mathematics do not give us the illogical luxury of beginning with a belief that such a limit exists in the first place. There is a fine line between

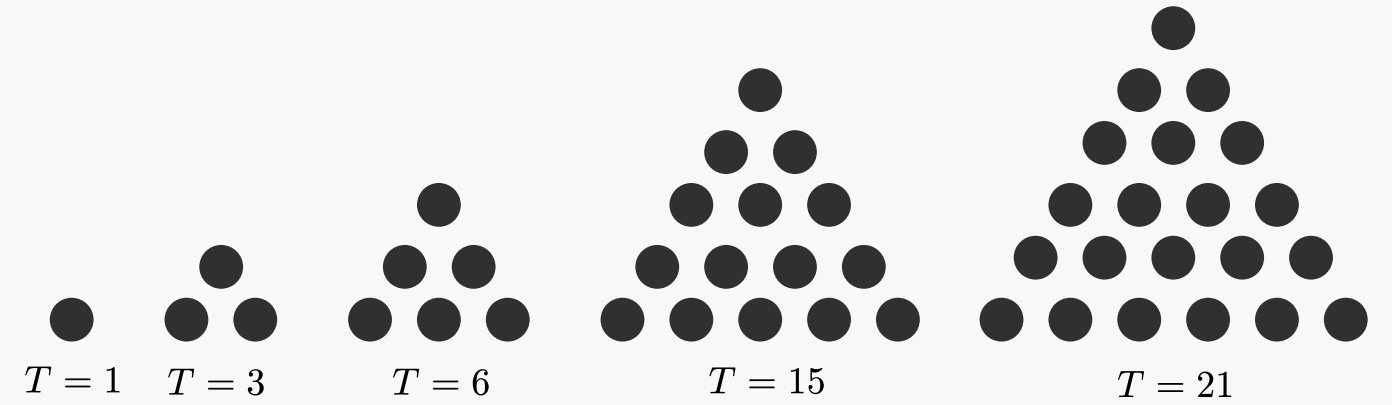

FIG 2 : ***Triangular numbers***, *an interesting sequence of numbers whose nth term gives the sum of first n natural numbers - the picture illustrates the idea. The points representing the natural numbers can be arranged in the form of a triangle as shown in the image, hence the name. Source [11]*

these two things. As an aside, it is worth mentioning that the video [6] invited sharp reactions from various math communities online, one of them being a nice mathematical exposition, which can be seen in [7].

While we have seen summation methods and have observed that our protagonist $1+2+3+\cdots$ does not converge, it makes sense to ask how an unlikely constant like $\frac{-1}{12}$ began to be associated with the above series. This is where we go on to the next stage of our exposition. For those who were wondering when the titular character of the article will appear in the article, here is the answer. Well, after hundreds of words into the article, it's time to unveil the special entity who was waiting to be revealed right from the beginning. Enter the **Riemann Zeta function** - which is popularly known for its association with the million-dollar open problem of the Riemann hypothesis. So, what does it have to do with $1+2+3+\cdots$? Well, the Riemann zeta function is a function which gives a series as an output for a complex number that is given as input. It is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

Guess what! Putting $s = -1$ gives the series

$$\sum_{n=1}^{\infty} n,$$

which is precisely our protagonist series. But the only catch here is that $\zeta(s)$ converges (this convergence is viewed using D1) for complex numbers with real part greater than 1. Thus, substituting $s = -1$ and "evaluating the value" of the series at this point does not make sense mathematically. So, what can we try to do further?

For a "nice" function like ζ, it would be great to have it defined for all complex numbers. In what follows, we shall actually try to extend its definition to the whole of $\mathbb{C}$. Now, does this idea ring a bell? Let us consider the function

$$f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$$
$$f(x) = \frac{\sin x}{x}.$$

Clearly, this function is not defined at $x = 0$ but is well-defined and is continuous at all other points. Thus, it would be nice to make it continuous on all of $\mathbb{R}$. **Why should $x = 0$ alone be deprived of the nice property of continuity?** It can be justified by using definitions of limits and continuity that this can be achieved by defining $f(0) = 0$. Here comes the subtle but important difference that needs to be noted. We have defined that the function f takes the value 0 at the point $x = 0$ and not that the value of $\frac{\sin x}{x}$ is 0 at $x = 0$. This is the process of making a discontinuous function continuous by removing the discontinuity at a point. With regards to the zeta function, we do something similar to extend its definition to the whole of $\mathbb{C}$. The method by which it is done is called an **analytic continuation**, which in some sense is slightly analogous (only in principle, but not in technique) to the motivation for the removal of a discontinuity given above.

While differentiability is an important characteristic of functions on $\mathbb{R}$, when we move on to $\mathbb{C}$, there is a general notion called **analyticity** (which, in fact, can be spoken about in the case of $\mathbb{R}$ too), which encompasses the notion of differentiability too. From a naive standpoint, analytic functions on $\mathbb{C}$ are functions that are differentiable infinitely many times. Typical

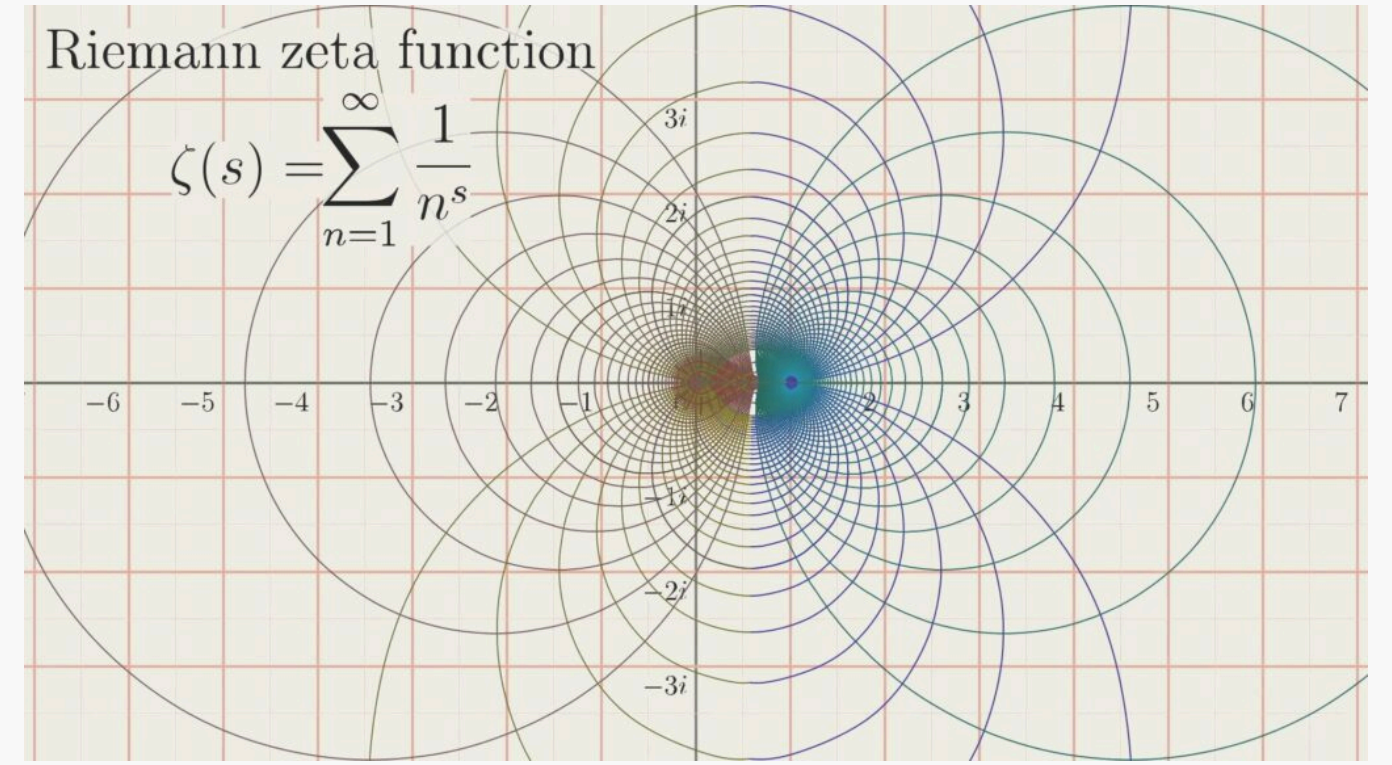

FIG 3 : *Visualizing the **Riemann zeta function**. This has been beautifully explained in the video [14]. Image Source:[12], which in turn was from a content courtesy 3Blue1Brown.*

examples of such functions are polynomials. However, the most important feature of an analytic function is the ability of it to be represented as a special kind of series called "power series" in the appropriate domain. The Riemann zeta function ζ is analytic for all $s \in \mathbb{C}$ with real part greater than 1 and the series fails to converge for all other $s \in \mathbb{C}$, and hence we can't speak of its analyticity there. So, we now try to extend the definition of ζ for all other values. It should also be noted that the point $s = 1$ behaves a bit differently even after extension. This is a highly non-trivial result and involves various mathematical concepts. It involves the well-known **gamma function** - which again can be considered as an extension of the notion of factorial to complex numbers. The above-mentioned analytic continuation of ζ is achieved by what is called the **functional equation of** ζ. It can be proved that for $s \in \mathbb{C}$, it follows that

$$\pi^{-s/2}\Gamma(s/2)\zeta(s) = \pi^{-(1-s)/2}\Gamma((1-s)/2)\zeta(1-s).$$

Here $\Gamma(s) = \int_0^\infty e^{-x}x^{s-1}dx$ if real part of s is greater than 1. Otherwise, the gamma function has its own analytic continuation, which is defined using $\Gamma(s+1) = s\Gamma(s)$ for $s \in \mathbb{C}$. Now by substituting $s = -1$ in the equation above, we get

$$\sqrt{\pi}\Gamma(-1/2)\zeta(-1) = \frac{1}{\pi}\Gamma(1)\zeta(2).$$

The famous **Basel problem** solved by Euler (which is yet another non-trivial result) tells us that $\zeta(2) = \frac{\pi^2}{6}$ while $\Gamma(1) = 1$ and $\Gamma(-1/2) = -2\Gamma(1/2) = -2\sqrt{\pi}$. By using all these values, we end up with $\zeta(-1) = \frac{-1}{12}$. This magic number doesn't come out of nowhere, and this is the proper reasoning behind the phenomenon.

The moral of the story is that $\zeta(-1) = \frac{-1}{12}$ and that $1 + 2 + 3 + \cdots$ is *not equal* to $\frac{-1}{12}$. Unfortunately, the wrong idea that substituting $s = -1$ in $\sum_{n=1}^{\infty} \frac{1}{n^s}$ "gives" the series $1 + 2 + 3 + \cdots$ is associated with most of the clickbait contents on the internet that claim the value of this sum is $\frac{-1}{12}$. But as we just saw, this substitution itself is not possible as the series does not converge at $s = -1$. The key takeaway is that, $\zeta(-1)$ is not equal to $1 + 2 + 3 + \cdots$, but courtesy the analytic continuation of the function ζ, $\zeta(-1)$ gets assigned the value $\frac{-1}{12}$. It is analogous to the situation that we started with. The series $1 - 1 + 1 - 1 + \cdots$ does not converge using the usual definition (D1) of a limit of a series, however it gets assigned the value $\frac{1}{2}$ via definition D2. So it all depends on what universe we are working in and what definitions we have in our hands. As long as one can construct a theory and prove results that are consistent with the rules of the game that hold in the universe they're working in, it does not have any flaws. However, the perspective has to be conveyed clearly without creating any unnecessary confusion. Since the main aim of this article was to convey the whole picture of this misunderstood concept in an expository manner, we did not spend much time on a lot of details and verifications. The interested reader can go through the following resources to get a taste of them. For sequences and series - any first-course text in real analysis, for example, [8] should work. The details on the analytic continuation of the zeta function can be found in an Analytic number theory textbook, such as , [9].

We conclude this article with a short note on zeta functions, for those who are interested in further exploration. The idea of Riemann zeta function has now been extended to various branches of mathematics - ranging from dynamical systems, geometry and group theory. The main principle behind zeta functions is that they serve as a type of generating function to count something interesting. In the case of number theory the Riemann zeta function is found to be connected to the prime counting function - which counts the number of prime numbers that are less than or equal to a given real number. Further generalisation of the zeta function are found within number theory as well. Dynamicists have studied zeta functions related to counting the number of orbits (of the function that they iterate) of period less than or equal to a given real number. Whatever be the avatar in which the zeta function appears, its analytic properties - especially the region of analyticity and the notion of convergence, etc., remain the main focal points that drive the theory in that setting. In fact, one could write a separate exposition on each of the above concepts - but that is a topic for another day!

An earlier version of this article was published [13] in the Exhibit A science newsletter of IISER TVM. It appeared in the July - December 2024 edition of the newsletter under the same title. There have been some minor edits made in this version of the article - one paragraph has been included and some other changes in phrasing have been made. The author would like to thank the editors of the article at Exhibit A for their comments and suggestions regarding the article.

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The Risk Beyond The Cutoff



The Risk Beyond the Cutoff: Limitations of VaR and the Need for Coherent Measures

Dr. Suparna Biswas *IISER Tirupati*

Reviewed By: Shriparna Chattopadhyay

Sep 05, 2026

This article discusses the fundamental principles of quantifying financial risk based on mathematical frameworks, the shortcomings of traditional value at risk (VaR) and clarifying how coherent risk measures address these issues. Four axioms of coherence are explored, describing how diversification reduces risk. A brief overview of the statistical estimation of VaR is given, explaining the use of empirical data to assess risk in situations where the underlying distribution remains unknown. An illustrative portfolio example which focuses VaR's shortcomings in tackling the severity of tail risk, emphasizing the need for coherent risk measures. The article concludes by highlighting the fundamental research problem in the estimation of financial risk.



Suparna Biswas is an assistant professor in the Department of Mathematics at the Indian Institute of Science Education and Research in Tirupati, India. Her research interests encompass mathematical finance and statistical finance, with a particular emphasis on the development and application of mathematical and statistical techniques for the modelling of financial risk.

Each substantial financial choice entails a degree of uncertainty. A person preparing for retirement cannot foresee future market performance. A company that invests in emerging technologies cannot predict the project's success. Financial institutions that provide loans face the risk of borrower default on repayments. In every case, uncertainty presents the possibility of adverse outcomes, which we denote as **"risk"**. In finance "risk" can be defined as the variability associated with unexpected changes in the values of assets and liabilities [5].

The challenge of managing risk is not exclusive to contemporary financial markets. Throughout history, individuals have looked for methods to safeguard themselves from unpredictable occurrences. Merchants of antiquity navigating extensive trade routes encountered threats such as theft, storms, and the peril of shipwrecks. To mitigate these risks, they established frameworks that enabled the distribution of losses among numerous participants [6]. Similar concepts gradually developed into the insurance contracts and financial institutions we have today.

As economies grew more intricate, the necessity for quantitative approaches to evaluate risk became ever more significant. Investors, banks, insurance companies, and regulators all need tools that address a crucial question: **"What level of risk is being assumed?"** Although risk is frequently talked about in casual terms, successful risk management necessitates a quantitative assessment that can evaluate various financial situations and ascertain the capital required to endure possible losses.

For numerous years, professionals depended on their intuition and experience to evaluate financial risk. Nonetheless, the swift expansion of financial markets throughout the twentieth century generated a need for more thorough and methodical strategies. This prompted researchers to explore the essential properties that a suitable measure of risk should possess.

Delbaen [4] and *Artzner et al.* ([1], [2]) made major contributions to the field of financial mathematics. Their work constitutes a notable contribution in the development of quantitative theory of financial mathematics. The authors define the **"axioms of coherence"** with extraordinary financial insight, which lead them to identify a minimal set of properties that a portfolio statistics must satisfy. Intrinsic concepts contained in the four "axioms of coherence" constitute the idea of risk.

The introduction of this framework significantly influenced financial mathematics. It was revealed that certain commonly utilized measures, such as VaR, have significant shortcomings despite their widespread acceptance in both industry and regulation. Grasping these strengths and weaknesses is a key goal of contemporary risk management. "Risk measure" and "measure of risk" are used interchangeably here.

Coherence

Definition 1 (Axioms of Coherence): Let ψ denote the set of real-valued random variables. A coherent risk

measure ρ is a function from ψ to $\mathbb{R}$ satisfies the following properties:

1. monotonicity: $A, B \in \psi, B \geq A \Rightarrow \rho(B) \leq \rho(A)$.
2. positively homogeneous: $A \in \psi, \lambda > 0, \lambda A \in \psi \Rightarrow \rho(\lambda A) = \lambda \rho(A)$.
3. translational invariant: $A \in \psi, c \in \mathbb{R} \Rightarrow \rho(A + c) = \rho(A) - c$.
4. subadditivity: $A, B, A + B \in \psi \Rightarrow \rho(A + B) \leq \rho(A) + \rho(B)$.

The variables A and B denote the profit and loss of the portfolios over a certain time horizon t. The axioms have the following interpretations.

1. Monotonicity implies that an asset that always has higher return in all future states is less risky.
2. Positive homogeneity implies that if a portfolio or capital asset is, say, doubled ($\lambda = 2$), then the risk will also be doubled.
3. Translation invariance implies that if an amount of cash c (or risk free asset) is added to a portfolio, then the risk is reduced by that amount.
4. Suppose that U and V, two banks, both have sufficient capital reserves that are commensurate to the risks $\rho(\Gamma_U)$ and $\rho(\Gamma_V)$ associated with their risky portfolios Γ_U and Γ_V (portfolio means collection of assets). Imagine for a moment that the two financial institutions combine to form one larger entity, combining the risk portfolios $\Gamma_U + \Gamma_V$ with the capital reserves. The total of the company's reserves is sufficient to cover the risk $\rho(\Gamma_U + \Gamma_V)$. By ensuring that merging does not raise risk through diversity, subadditivity guarantees that risk is reduced.

VaR and its shortcomings

In 1997, these axioms revealed a major flaw in VaR, even though it has been widely used by the regulatory frameworks and financial organizations [1]. Because VaR does not adhere to the subadditivity assumption, it breaks the diversification principle. However, it satisfies the rest of the three axioms. For a mathematical formulation of VaR, we introduce the cumulative distribution function (cdf) $F_X(x)$ of the profit and loss random variable X. Let the lower and upper quantile of the distribution be $Q_p(X) = \sup\{x \mid F_X(x) < p\}$ and $Q^p(X) = \sup\{x \mid F_X(x) \leq p\}$.

Definition 2: Consider a fixed future time horizon t and $p \in (0, 1)$ the confidence level. Then the lower and upper p-VaR are defined by $\mathrm{VaR}_p(X) = -Q_p(X)$ and $\mathrm{VaR}^p(X) = -Q^p(X)$.

The lower and upper versions coincide when $F_X(x)$ is continuous. Different notations are given because VaR_p and VaR^p may have different values if $F_X(x)$ is not continuous, as illustrated in Figure 1.

Interpretation and Critique

Let's try to understand Definition 2. A 5% VaR can be interpreted as the loss incurred in the best of the 5% worst-case scenarios, or equivalently, in the worst of the 95% best-case scenarios of the probability distribution of the future profit and loss. VaR is mistakenly seen as an accurate representation of prospective losses that may occur with a 5% likelihood. Two critical flaws are:

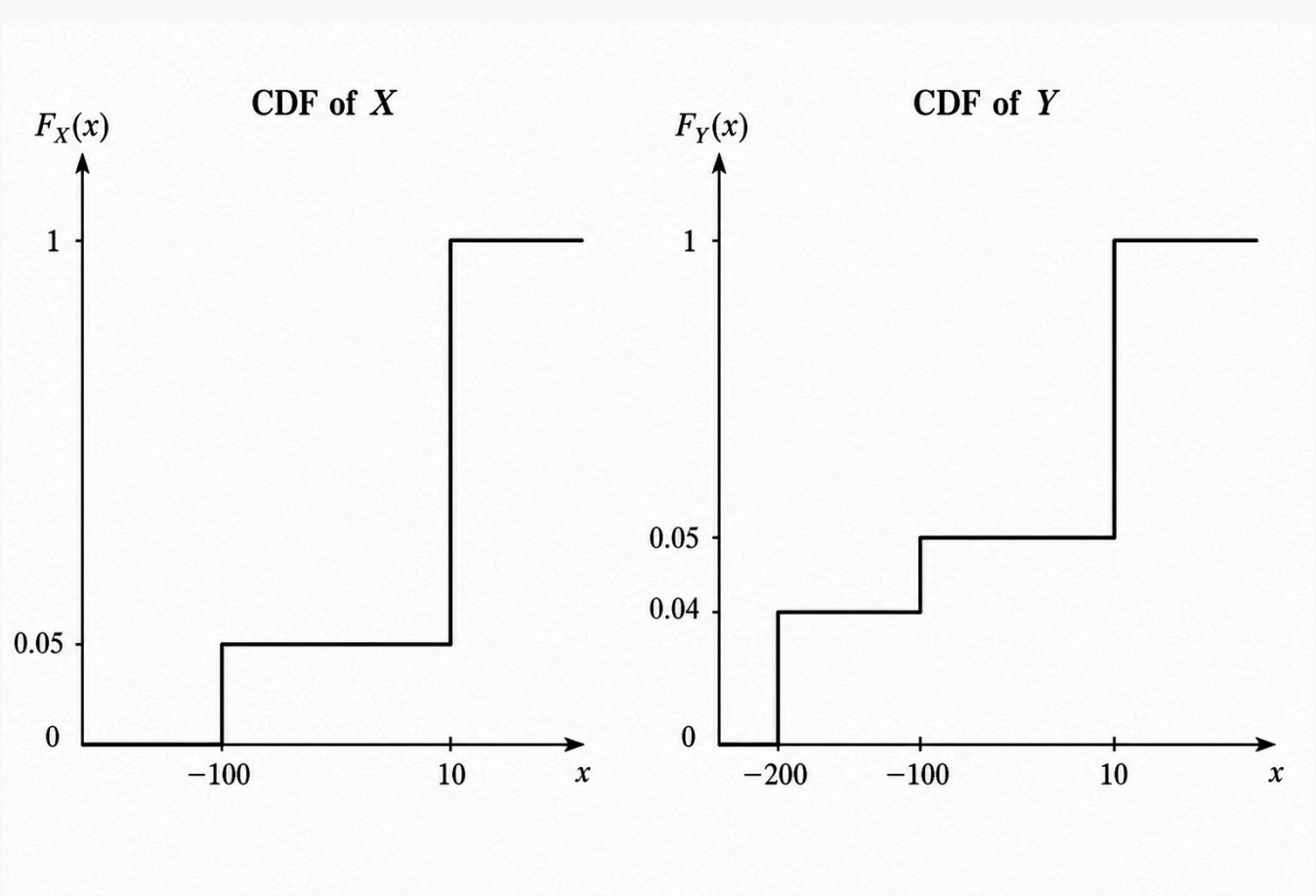

FIG 1 : *Figure showing upper and lower p-VaR.*

1. From Definition 2, we see that VaR consistently undervalues the risks of the worst-case scenarios it chooses, favoring the least dangerous one.
2. VaR exhibits a complete lack of sensitivity to the severity of the discarded worst-case scenarios, allowing for the possibility of encountering situations where two portfolios, despite having distinctly different risk profiles, can still present identical VaR values.

Concrete example

Example 1. Consider two portfolios. The first portfolio with a two-states profit and loss random variable X and the second portfolio Y with more severe losses (see Figure 2).

X($)	PROBABILITY
-100	5%
+10	95%

Y($)	PROBABILITY
-200	4%
-100	1%
+10	95%

In the above example, both portfolios have same 5% VaR,

$$\mathrm{VaR}_{0.05}(X) = \mathrm{VaR}_{0.05}(Y) = -(-\$100) = \$100,$$

even though portfolio Y is clearly riskier because it is exposed to more severe losses. Furthermore, the upper and lower 5% VaR have different values and that upper VaR turns out to be negative, i.e. is unable to detect any risk in these portfolios.

$$\mathrm{VaR}^{0.05}(X) = \mathrm{VaR}^{0.05}(Y) = -\$10$$

This is another unfavorable characteristic of VaR, as it reveals hazards obscured in events with probabilities lower than its confidence level. This example demonstrates that VaR is indifferent to the magnitude of losses in the tail beyond the designated confidence level. The ignoring of a portion of worst-case situations is the fundamental source of VaR's inadequacies. To address this, quantile-based coherence statistics have been proposed, which are sensitive to the extreme situations of the probability density function, specifically considering the leftmost segment of its tail [2].

Estimation of VaR

In practice, the probability distribution of future profits and losses is unknown. Therefore, the VaR cannot be computed directly from the true cdf F_X. Instead, it must be estimated from historical observations.

Suppose that $X_1, X_2, \ldots, X_n$ are observed profit and loss data from the past. The empirical cdf is defined by

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{\{X_i \le x\}},$$

where $I_{\{X_i \le x\}}$ is equal to 1 if $X_i \le x$ and 0 otherwise.

Replacing F_X by F_n in Definition 2 yields an estimator of VaR. Equivalently, one may sort the observations in increasing order, $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$, and estimate the p-quantile by the corresponding order statistic. For example, with $n = 100$ observations and $p = 0.05$, the 5% quantile is approximated by the fifth smallest observation. The estimated VaR is then the negative of this estimated quantile.

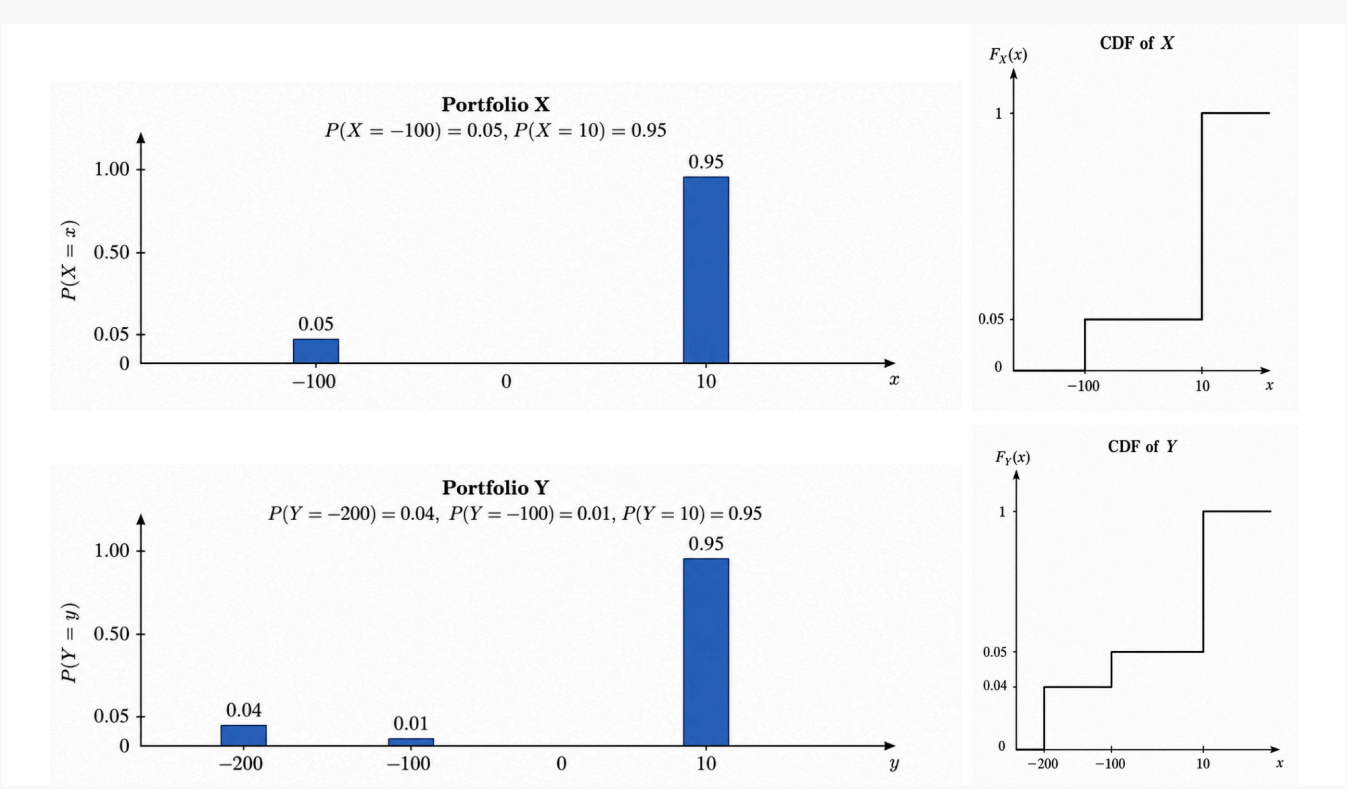

FIG 2 : *Probability distributions of portfolios X and Y.*

The empirical cdf has the properties,

$$E[F_n(x)] = F_X(x) \text{ and}$$
$$\text{VaR}(F_n(x)) = \frac{F_X(x)(1-F_X(x))}{n}$$

Thus, the variability of the estimator decreases as the sample size increases, indicating that larger samples generally provide more reliable estimates of VaR.

Research Problem

VaR remains the most widely used measure of financial risk, yet many fundamental problems remain unsolved. The main difficulty is that the probability distribution of the financial returns is unknown. While the empirical cdf provides the basic nonparametric estimator, asset return data exhibit stylized properties, such as heavy tails, nonlinear dependence, skewness, volatility clustering etc. that are not adequately captured by a basic statistical model. Hence, the most important question arises.

How to estimate the distribution function of the financial returns, and consequently estimate the VaR and the coherent risk measures, when the underlying distribution is unknown?

This question motivates an extensive research in statistical and mathematical finance. There are many approaches available in the literature which includes parametric approach, historical simulation, GARCH-type volatility models, extreme value theory, copula-based models, and recently machine learning and deep learning techniques. Each method attempts to balance the statistical accuracy, computational efficiency, and robustness under changing market conditions.

Another active area of research focuses on constructing new risk measures or coherent risk measures that remain practical for estimation of finite samples. These research problems provide insight that measuring financial risk is an active field where probability theory, statistics, and optimization interact to address the uncertainty that exists in the financial market.

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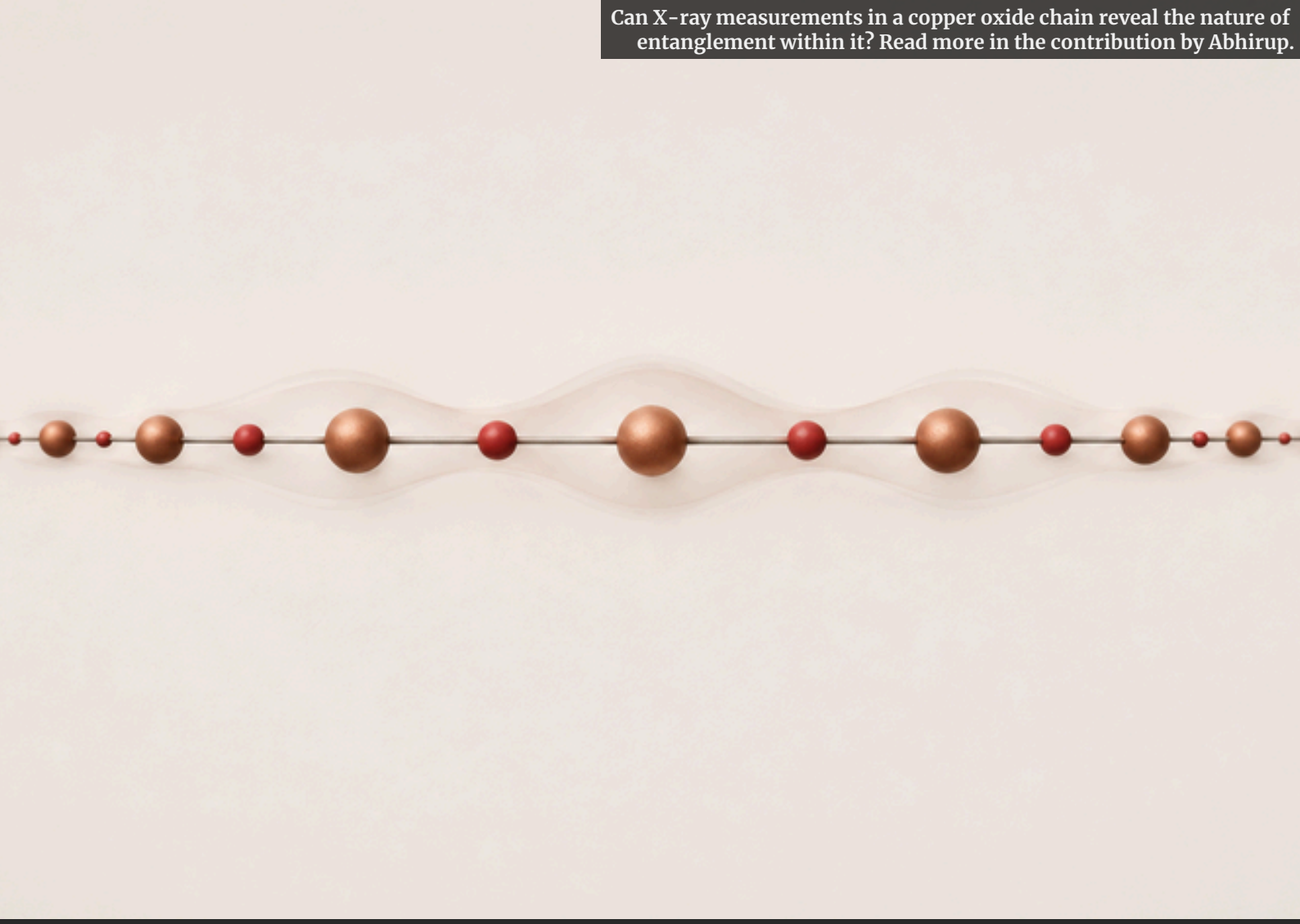
Can X-ray measurements in a copper oxide chain reveal the nature of entanglement within it? Read more in the contribution by Abhirup.



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Seeing entanglement in a cuprate spin chain

S. F. R. TenHuisen et al., arXiv:2609.08134, Sept. 08, 2026

Contributed by **Abhirup Mukherjee (Department of Physics & Astronomy, Seoul National University)**

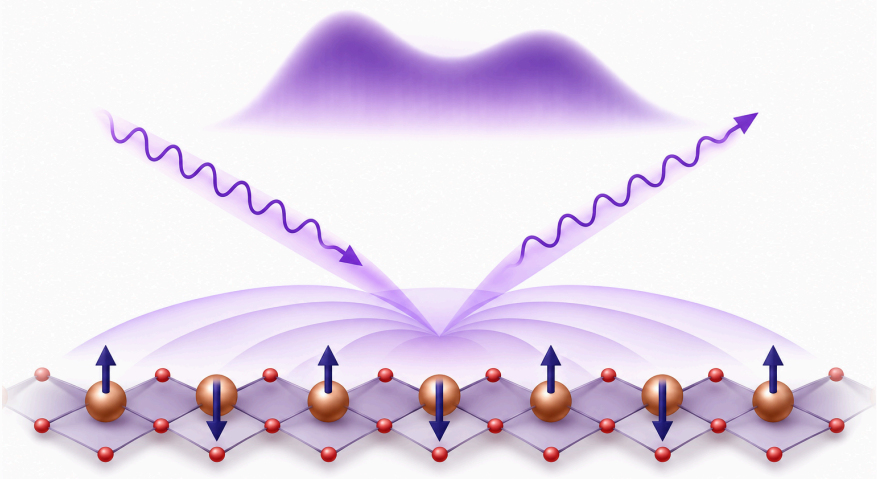
Subjects: strong-correlations, entanglement, materials-science

Entanglement is one of the distinguishing features of quantum mechanics and a core aspect of modern quantum condensed matter physics. Many of the interesting phenomena seen in the so called quantum materials bear their origin to intricate patterns of entanglement between the constituent particles. Yet, it is difficult to quantify entanglement measures in the laboratory: reconstructing a many-body quantum state requires measuring an exponentially large number of observables because of the exponential growth of the Hilbert space seen in systems of interacting particles. In this work, TenHuisen and colleagues ask whether resonant inelastic X-ray scattering (RIXS), a widely used probe of excitations in correlated materials, can instead witness entanglement through what's called an entanglement witness. Entanglement witnesses are quantities that are not as powerful as entanglement measures – they can detect whether a state has entanglement but cannot quantify the amount of entanglement. But their main advantage is that they can be experimentally measured. For example, in this work, the authors probe the **quantum Fisher information** (QFI) – an entanglement witness that can be linked to a dynamical spin structure factor. The spin structure factor roughly quantifies how likely it is that a system will be excited by a specific energy through a specific spin-flip process.

Their test case is Sr_2CuO_3, an insulator with strong magnetic interactions. It turns out that its low-energy state is quantum critical, which essentially means that the correlations within it span long distances. The low-energy excitations fractionalize into spinons, which means its quantum numbers are fractional. These long-range quantum fluctuations make the chain a promising place to find deep spin entanglement—and a well-characterized setting in which to test a new measurement protocol.

As mentioned, the authors use QFI to probe the system. For a spin-1/2 system, the QFI per spin above a value m certifies entanglement involving at least $m+1$ spins. The challenge is that RIXS intensity is not simply the spin structure factor: in an interacting many-body system, other processes always contribute. The work calculates the appropriate spin structure factor by combining the experimental data with theoretical calculations of relevant models using exact diagonalisation and density matrix renormalisation group (DMRG). At 35 K, the momentum-resolved QFI peaks at a value exceeding six, witnessing at least seven-partite spin entanglement. At 220 K, the witness remains above four near specific directions in momentum space, still certifying entanglement among at least five spins. The result shows that substantial multipartite correlations survive well above the lowest measurement temperature, because the chain's large exchange scale keeps it in the quantum regime.

This is a first demonstration of RIXS as a probe of multipartite spin entanglement in a solid. The inference depends on a material- and geometry-specific, theory-assisted mapping from RIXS intensity to the spin response, so it is not yet a model-free measure. Even so, the method opens a path toward testing entanglement in less tractable materials.



Resonant X-ray scattering probes spin entanglement in the cuprate Sr_2CuO_3. The row of copper and oxygen atoms represents the material's nearly one-dimensional chain with antiferromagnetic arrangement of spins. An incoming X-ray scatters from the chain and leaves with a changed energy, carrying information about its spin excitations. The soft haze represents the broad response arising from quantum critical excitations, while the arcs indicate correlations extending across multiple spins. The study asks whether this scattering signal can reveal how many spins are entangled. By filtering out the appropriate spin-response from the X-ray signal, the researchers infer entanglement involving at least seven spins at 35 K.

The Wandering Mind: When Our Thoughts Steal Our Attention!

Welhaf MS, Banks JB. J Exp Psychol Learn Mem Cogn. 51(2), 238–254 (2025)

Contributed by **Shriparna Chattopadhyay (PhD Scholar, DBS, IISER Kolkata)**

Subjects: attention span, task related thoughts, sustained attention-to-response task, valence, psychology

Did you ever end up planning for a holiday trip or pondering about an unpleasant scenario while completing an important task of yours? Now, that is termed as 'mind wandering': when our attention shifts from an ongoing activity towards any unrelated thoughts. This ignited some minds to ask whether the emotional nature of thoughts (negative, positive or neutral) affect our attention?

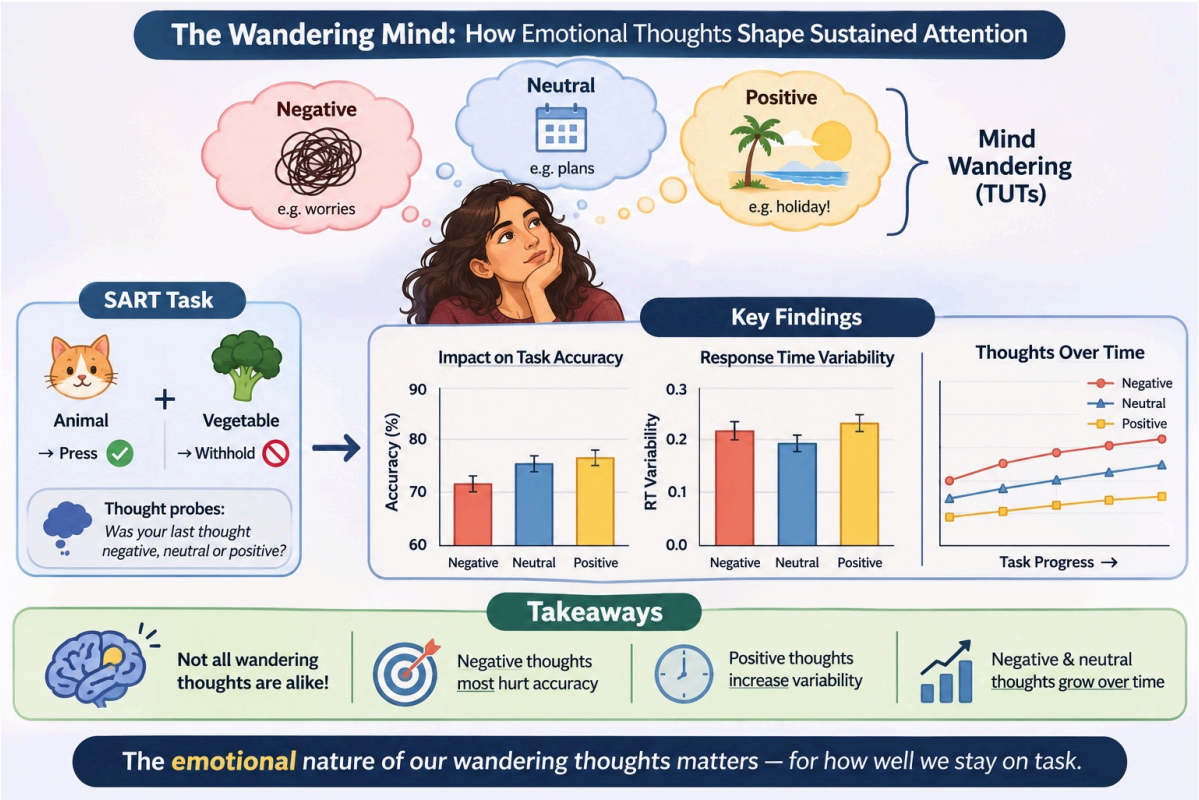
Keeping in mind that 'mind wandering' is generally associated with reduced task performance, a group of researchers carried out a simple yet interesting study to investigate whether negatively, positively and neutrally valenced task-unrelated thoughts (TUTs) have different relationships with sustained attention(Maintaining focus on a specific task over an extended period). They also noted whether these different types of thoughts become more frequent as a task progresses.

For the study the researchers pooled and analysed six datasets collected across several years involving participants aged 18-35 years, out of which most of them were university students. The participants were asked to perform a sustained attention-to-response task (SART):i.e.,'to press the spacebar for animal words, but withhold the response for rare vegetable words. And at random intervals,the participants were interrupted with thought probes asking them to categorise their immediately preceding thoughts as positive, negative or neutral. The researchers compared these reports with measures of response-time variability (refers to how much a person's reaction time fluctuates from one to another during a task or how consistently or inconsistently someone responds) and task accuracy, using meta-analysis, and examined changes in thought frequency across successive task blocks.

The findings of the study revealed something intriguing: 'mind wandering' is not a uniform phenomenon and varies with the emotional content of our thoughts.Both pleasant(positive) and unpleasant(negative) thoughts can divert our attention from an ongoing task, though their effects may manifest differently.

Negative thoughts may particularly disrupt the accuracy, while positive thoughts may contribute to fluctuations in response consistency. The researchers also found an increasing occurrence of neutral and negative thoughts over time (while doing the task).

This research concludes by focusing on the point that 'mind wandering' isn't only a matter of losing focus, the emotional nature of our wandering thoughts may also affect the consistency and accuracy of the attention we pay to do a task.



The illustration summarises the SART(Sustained Attention to Response Task) paradigm which assesses how negative, neutral, and positive task-unrelated thoughts (TUTs) relate to attentional performance. Negative thoughts are more consistently associated with reduced accuracy, while positive thoughts show the strongest association with response-time variability.

When Rivers Flow Through the Sky: Atmospheric Rivers and the Changing Face of Global Rainfall

Pradhan, S., Wasko, C. & Peel, M.C. npj Nat. Hazards 3, 68 (2026)

Contributed by **Swarnendu Saha**

Subjects: atmospheric rivers, atmospheric moisture transport, precipitation variability, extreme precipitation, climate change

Atmospheric rivers (ARs) are narrow corridors in the atmosphere that transport large amounts of water vapour towards the mid-latitudes. When they reach land, particularly in coastal and mountainous regions, they can produce heavy rainfall and flooding. Although their importance is well established, their contribution to changes in average and extreme precipitation has not been examined consistently at the global scale. This study investigates changes in AR frequency and related precipitation between 1980 and 2019.

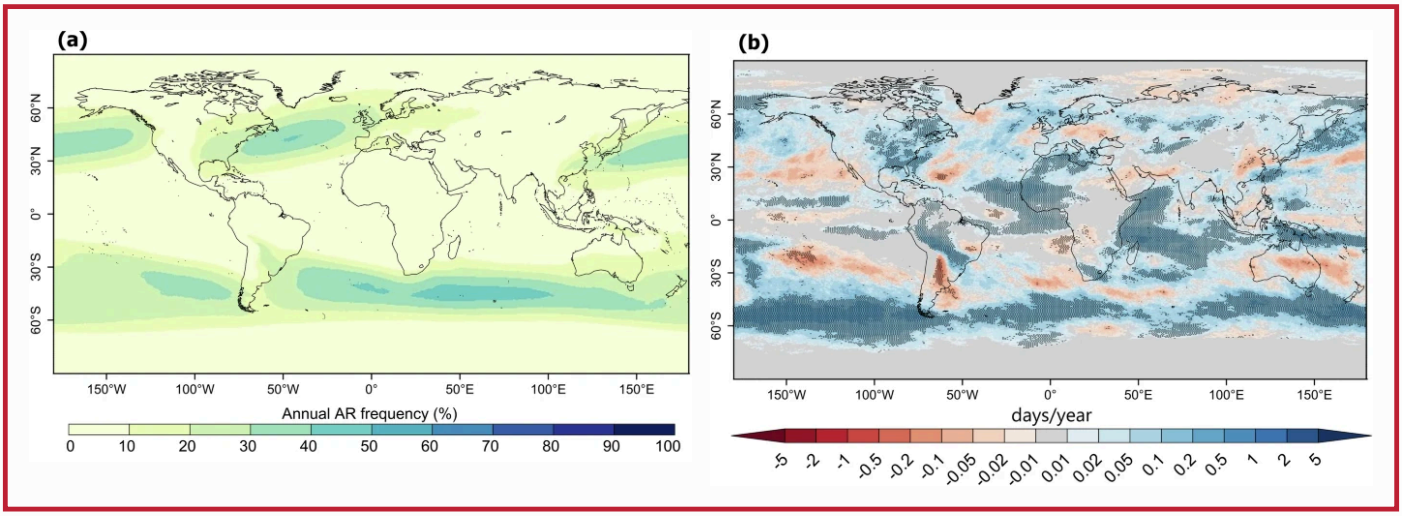
The authors combine precipitation estimates from the MSWEP V2 dataset with atmospheric river records derived from MERRA-2 reanalysis. To avoid relying on a single definition of an atmospheric river, they compare three detection methods. Rainfall occurring on a day when an AR is detected at the same grid cell is classified as AR-induced. Extreme rainfall is identified using a peaks-over-threshold approach based on the 90th percentile. Trends are estimated using Sen's slope, and their significance is assessed with the Mann–Kendall test.

The results show that atmospheric rivers are major contributors to year-to-year precipitation variability across many mid-latitude regions. In parts of North America, western Europe, southern South America, southern Africa and Australia, AR-related precipitation explains more than 70% of this variability. The study also finds an increase in AR frequency at mid-to-high latitudes, alongside decreases in some subtropical regions. The increase is particularly evident over parts of the Southern Ocean and North Atlantic.

Both the frequency and magnitude of AR-associated precipitation have increased over the study period. The rise in AR-related extreme rainfall is especially apparent near landfall regions, including parts of northeastern North America, Brazil, western Africa, the Mediterranean coast, Japan and southeastern China. Here, concentrated moisture transport can interact with terrain and weather systems to produce intense rainfall.

The authors suggest that a warmer atmosphere, which can hold more moisture, and changes in storm-track positions may contribute to these trends. However, their method identifies rainfall and ARs occurring together; this does not, by itself, prove that an AR caused the rainfall. Detection algorithms also differ in the geographical extent of trends, although the main patterns are consistent across all three.

Overall, the study indicates that atmospheric rivers are becoming more important to precipitation extremes and flood risk. The findings highlight the need for water-resource planning and flood-risk assessments to account for changing AR activity. Further research is needed to distinguish the effects of atmospheric warming, circulation changes and natural climate variability.



When a Canonical Copper Transporter Takes on The Challenge to Detoxify a Toxic Heavy Metal: Cadmium

Bhattacharya, D., Chakraborty, K., Pandey, R., Maji, B., Bhattacharjee, A. & Gupta, A. FEBS Letters (2026)

Contributed by **Debosmita Bhattacharya (Department of Biological Sciences, IISER Kolkata)**

Subjects: metal homeostasis, cadmium toxicity, atp7b, intracellular trafficking, lysosomal biology

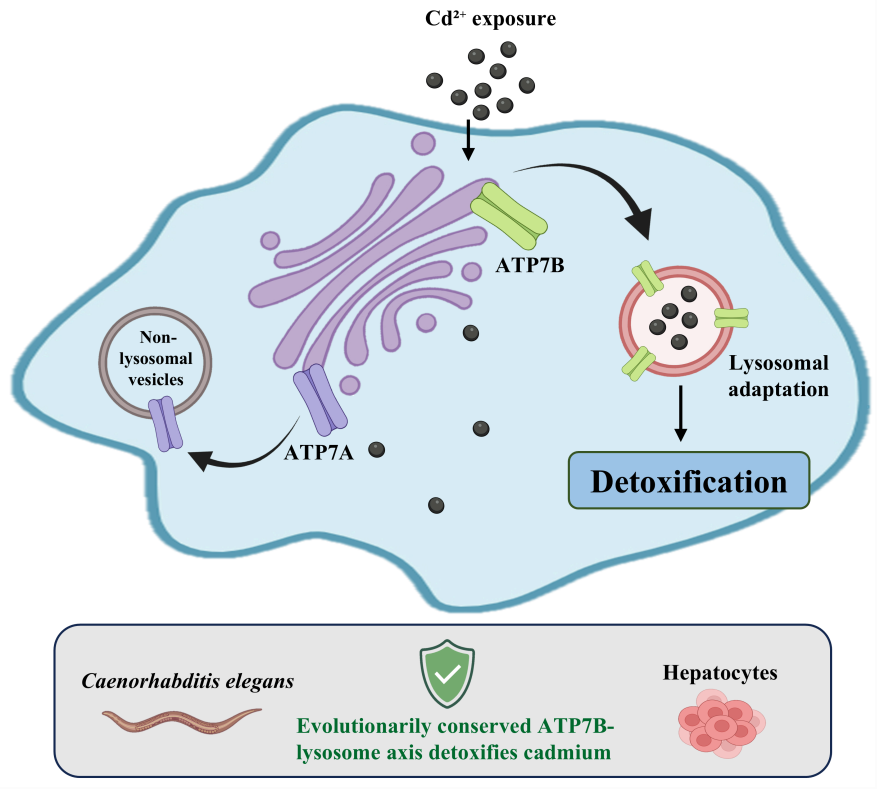
Imagine a protein whose primary responsibility is to maintain copper balance suddenly taking on the challenge of protecting cells against an entirely different toxic metal! This is precisely the unexpected cellular phenomenon explored in our recent study. Cadmium (Cd) is a highly toxic heavy metal that enters the body through environmental and occupational exposure. Unlike essential metals such as copper, cadmium has no known biological function in humans. Its strong affinity for sulfur-containing groups allows it to interact with cellular proteins, disrupt essential biological processes and cause damage to vital organs, particularly the liver and lungs. Although the toxic effects of cadmium are well recognised, how cells actively eliminate this metal remains relatively poorly understood.

Our research investigated whether two closely related copper-transporting proteins, ATP7A and ATP7B, could contribute to cellular cadmium handling. These proteins are best known for maintaining copper homeostasis by transporting excess copper across cellular compartments. Interestingly, we discovered that ATP7B has an unexpected role in cadmium detoxification. In response to cadmium exposure, ATP7B undergoes a remarkable change in its intracellular location, moving away from the trans-Golgi network (TGN), where it normally resides, towards lysosomes. Lysosomes are membrane-bound organelles traditionally recognised as the cell's recycling and waste-disposal centres. Cells do more than just move ATP7B around. They also make lysosomes more acidic. This might be the cell adapting so that the transporter and lysosomes team up for getting rid of cadmium. Cells without ATP7B were worse off when exposed to it. That part stands out because it shows how important the protein is for protection.

ATP7A was different, though. In lung cancer cells, it moved around after cadmium but stayed away from lysosomes. Maybe different cells have their own ways of handling it.

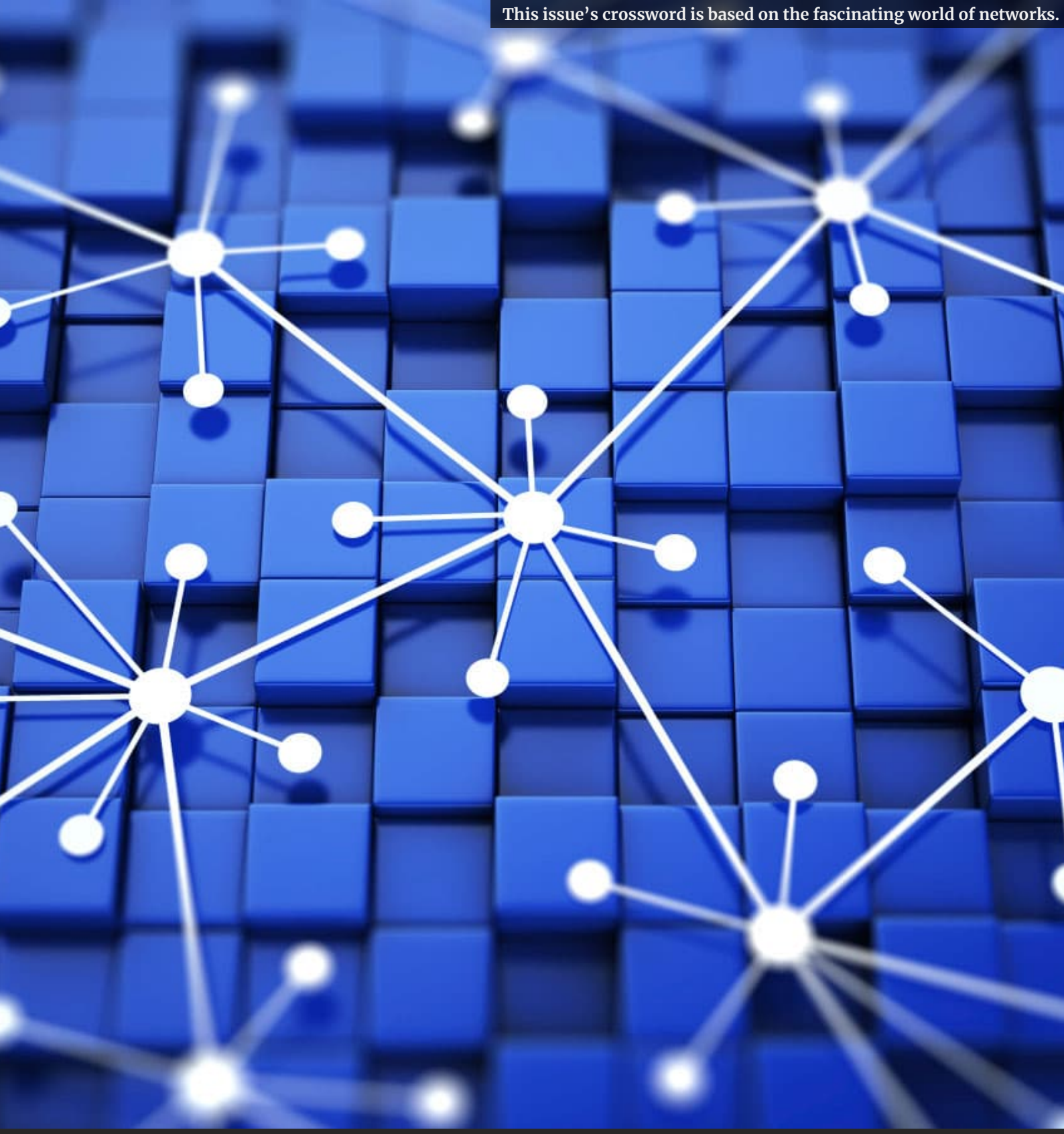
The same kind of protection showed up in worms when things went wrong with lysosomes or the ATPase. It could be an old mechanism that stuck around. There is still a lot that needs figuring out, though.

This connects metal transport to lysosomes in a way that was not seen before. It changes how people think about ATP7B since it is not just for copper anymore. Cells seem to reuse their trafficking for dealing with stress from metals.



Cadmium-induced intracellular trafficking of ATP7B. Following cadmium exposure, ATP7B relocates from the trans-Golgi network towards lysosomes through retromer-mediated trafficking. The accompanying expansion and functional adaptation of lysosomes contribute to cellular cadmium detoxification.

This issue's crossword is based on the fascinating world of networks.



Science Games

It's a Small World — Crossword on The Science of Networks

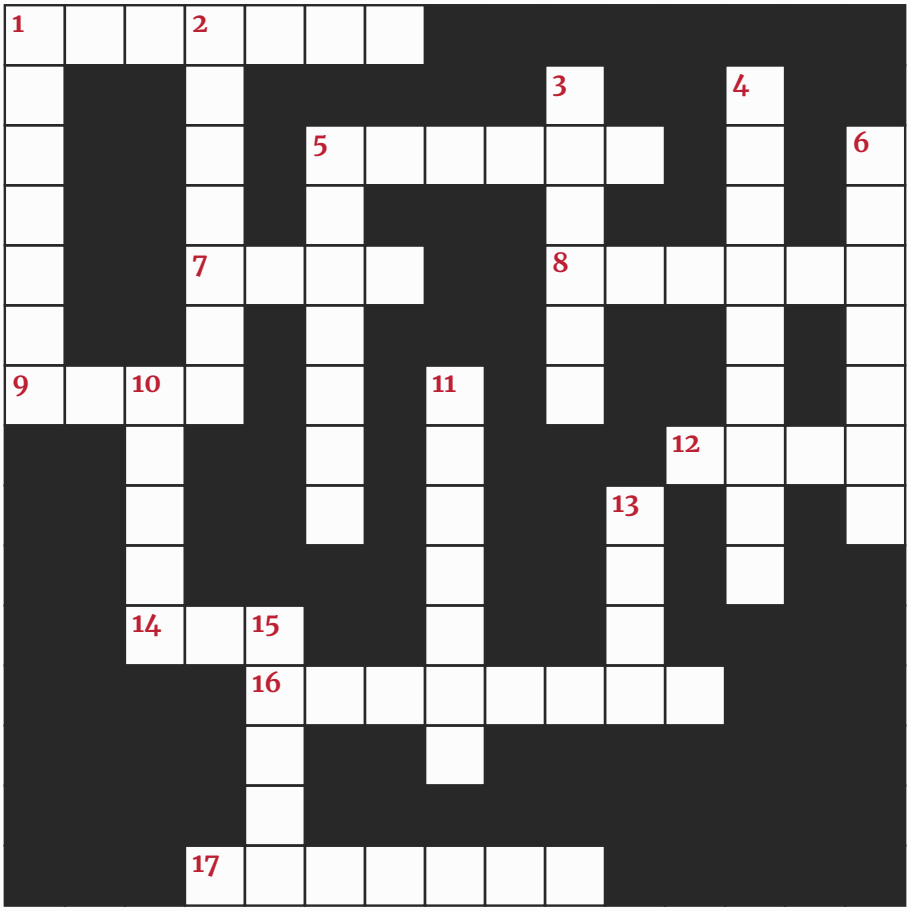
Quiz: India's Greatest Scientists

Maths x Physics — Guess the words to complete the Linked List

It's a Small World — Crossword on The Science of Networks

This issue's crossword explores how the idea of networks appears in various domains such as brain wiring, food webs, epidemics, social connections, power grids, and the internet.

Abhirup Mukherjee *[Department of Physics & Astronomy, Seoul National University]*



Across

1. A group of nodes that are more densely connected to one another than to the rest of the network (7)
5. A small unit of data sent across a computer network (6)
7. The route that connects two points in a network (4)
8. A device that directs data between computer networks (6)
9. A connection between two points in a network graph (4)
12. Wireless technology that connects devices to a local network (4)
14. A highly connected point that can link distant parts of a network (3)
16. The system formed by connected computers and networks around the world (8)
17. A system of connected points through which information, energy, or influence can travel (7)

Down

1. A chain reaction that spreads from one part of a system to another (7)
2. The junction where one neuron communicates with another (7)
3. A nerve cell; one of the basic units in the brain's communication network (6)
4. The spread of an infectious disease through a population (9)
5. A route through which a signal or process travels in a system (7)
6. A set of connected components through which electric current can flow (7)
10. A mathematical network made of points and the connections between them (5)
11. Map of who eats whom in an ecological community (7)
13. A point in a network, such as a person in a social network (4)
15. A large ecological region characterized by its climate and typical communities of organisms (5)

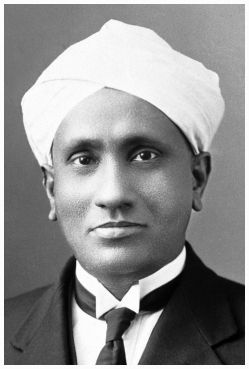
Solution can be found at the end of the issue. For an interactive version of the crossword, check out our ***website****.*

Quiz: India's Greatest Scientists

Can You Identify Their Achievements In This Quiz?

***Ravi Sharma** [IISER Kolkata]*

Q1. Scientist 1: C. V. Raman. Which statement most accurately describes his landmark scientific contribution?



I. He established that light passing through matter could undergo a measurable change in wavelength due to interactions within the material.
II. He demonstrated that the intensity of scattered light could be used to determine the thermal state of individual molecules.
III. He established that the colour of scattered light changes systematically with the density of the material through which it travels.
IV. He showed that the polarization of scattered light could provide a complete description of the internal energy structure of molecules.

Q2. Scientist 2: Homi J. Bhabha. Which statement most accurately captures his historic scientific contribution to India?

I. He developed India's first practical nuclear reactor design and personally demonstrated sustained nuclear power generation.
II. He developed the theoretical foundation for nuclear reactions and then helped create institutions through which India could pursue atomic research.
III. He transformed India's nuclear research by concentrating primarily on nuclear weapons technology rather than fundamental particle physics.
IV. He established experimental methods for controlling nuclear reactions and used them to demonstrate India's ability to generate atomic energy independently.

Q3. Scientist 3: Vikram Sarabhai. Which statement most accurately describes his contribution?

I. He demonstrated that space technology could be developed as a national tool for communication, education and development, and built institutions around that vision.
II. He concentrated India's early space programme on human spaceflight and developed the institutional framework that later produced India's first astronauts.
III. He established India's satellite programme mainly to develop independent military reconnaissance capabilities during the early Cold War.
IV. He pioneered India's rocket technology by personally developing the propulsion systems that became the direct basis of modern Indian launch vehicles.

Q4. Scientist 4: Satyendra Nath Bose. Which statement is historically accurate?



I. He established the relationship between molecular vibrations and shifted spectral lines produced when light interacts with matter.
II. He developed a statistical model in which all quantum particles were treated as distinguishable, providing the basis for classical statistical mechanics.
III. He derived a statistical explanation for the ionisation of atoms in stellar atmospheres and applied it to astronomical spectra.
IV. He developed a new statistical treatment of quantum particles whose indistinguishable nature later became fundamental to Bose–Einstein statistics.

Q5. Scientist 5: Srinivasa Ramanujan. Which statement best reflects his mathematical legacy?

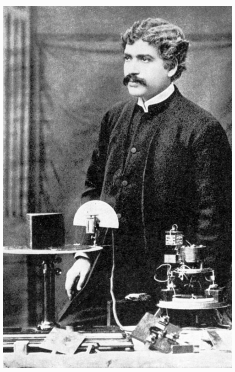
I. He formulated a general theory explaining how the distribution of prime numbers determines the convergence of physical and astronomical series.
II. He developed a unified mathematical framework connecting number theory with the statistical behaviour of quantum particles.
III. He discovered deep relationships involving numbers, infinite series and continued fractions, often producing results before formal proofs were available.
IV. He established the mathematical foundations of modern probability theory by deriving general laws for randomly distributed integers.

Q6. Scientist 6: Meghnad Saha. Which statement correctly describes his major contribution?

I. He developed a model in which the observed spectral lines of stars could be used to determine their chemical composition independently of temperature.
II. He established a quantitative relationship between temperature, pressure and ionisation that allowed stellar spectra to be interpreted in terms of physical conditions.
III. He established a quantitative relationship between stellar brightness, temperature and chemical composition that directly determined the masses of stars.
IV. He explained stellar spectra primarily by treating stars as blackbody emitters whose spectral lines arise from variations in surface temperature.

Q7. Scientist 7: Jagadish Chandra Bose. Which statement most accurately captures his scientific legacy?

I. He conducted pioneering experiments on electromagnetic waves and later developed sensitive instruments to investigate electrical responses in plants.
II. He developed the first systematic theory connecting plant growth directly with radio-wave transmission and used it to explain plant communication.
III. He primarily studied radio transmission for long-distance communication and later applied those techniques to develop biological imaging of plant tissues.
IV. He demonstrated that plants possess a nervous system comparable to animals and used electromagnetic radiation to map its structure.



Q8. Scientist 8: Subrahmanyan Chandrasekhar. Which statement correctly describes his landmark astrophysical contribution?

I. He demonstrated that the maximum mass of a star is determined primarily by the rate at which it loses heat through radiation.
II. He established that every white dwarf eventually collapses once its temperature falls below a fixed critical value, regardless of its mass.
III. He showed that neutron pressure determines the maximum mass of ordinary white dwarfs and used this to predict their transition into neutron stars.
IV. He established a limiting mass for a stable white dwarf and showed how quantum mechanics and relativity constrain the fate of massive stars.

Q9. Scientist 9: A. P. J. Abdul Kalam. Which statement most accurately describes his scientific contribution?

I. He personally designed India's first operational satellite and directed the entire scientific programme responsible for placing it in orbit.
II. He played a leading role in developing indigenous launch-vehicle and missile technologies and helped turn complex aerospace projects into national programmes.
III. He pioneered India's nuclear-reactor technology and subsequently transferred his expertise to the country's missile programme.
IV. He established India's theoretical foundations for rocket propulsion and developed the mathematical equations later used in the country's launch vehicles.

Q10. Scientist 10: Har Gobind Khorana. Which statement accurately describes the work for which he received the 1968 Nobel Prize?

I. He helped establish how specific nucleotide sequences correspond to amino acids, clarifying how genetic information is translated into proteins.
II. He discovered that mutations arise when radiation changes the chemical structure of individual DNA bases and used this to explain inherited disease.
III. He established the three-dimensional structure of DNA and showed how its geometry determines the sequence of proteins produced by cells.
IV. He demonstrated how DNA duplicates itself and established the molecular mechanism by which genetic information is copied before cell division.



*Answers can be found at the end of the issue. For an interactive version of the quiz, check out our **website***

Maths x Physics — Guess the words to complete the Linked List

This issue's linked list involves words drawn from concepts where maths and physics overlap.

Abhirup Mukherjee *[Department of Physics & Astronomy, Seoul National University]*

Linked List is a general science-based word game. The rules are straightforward:

1. The goal is to guess eleven words that have been drawn from science.
2. The first word (the seed) will be provided to you, and hints and number of letters will be provided for the remaining words.
3. You are also informed that the first letter of any word is the last letter of the previous word. So the first letter of the second word will be the last letter of the seed word, the first letter of the third word is the last letter of the second word, and so on.
4. This property goes all the way, so that the last letter of the last (eleventh) word is also the first letter of the seed word.

Find all the words!

Today's seed: **LAPLACE**

1. A direction that remains unchanged under a transformation, revealing the natural modes of a physical system. (**11**)

E									

2. A mathematical procedure for handling infinities and understanding how physical laws change with scale. (**15**)

3. A set of nonlinear differential equations describing fluid motion, and a famous unsolved mathematical problem. (**12**)

4. A mathematical relationship between quantities at different length scales, central to critical phenomena and physical universality. (**7**)

5. A mathematical freedom in how fields are described, underlying some of the fundamental interactions in nature. (**5**)

6. A property connecting the long-time evolution of a dynamical system to averages over its entire phase space. (**10**)

7. A mathematical form of interaction potential associated with a theory of nuclear forces and the particle named after its originator. (**6**)

8. The study of limiting behaviour in mathematics, essential for approximating physical systems in extreme regimes. (**10**)

9. A transformation that preserves angles mathematically and plays a central role in two-dimensional critical systems and quantum field theory. (**9**)

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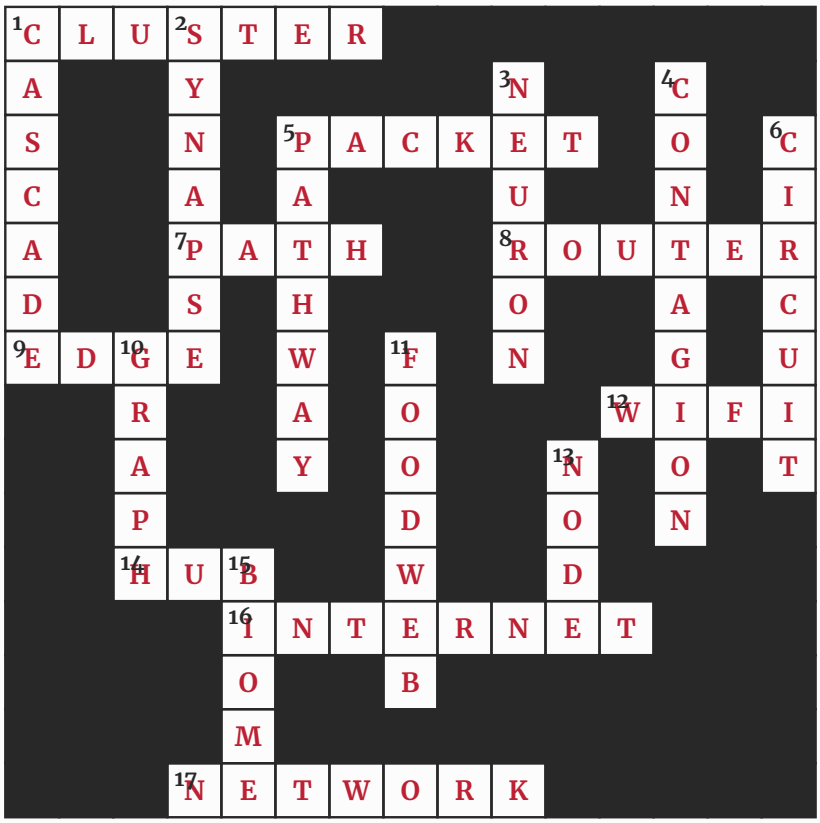
*Solution can be found at the end of the issue. For an interactive version of this game, check out our **website**.*

Solutions

Crossword



Quiz

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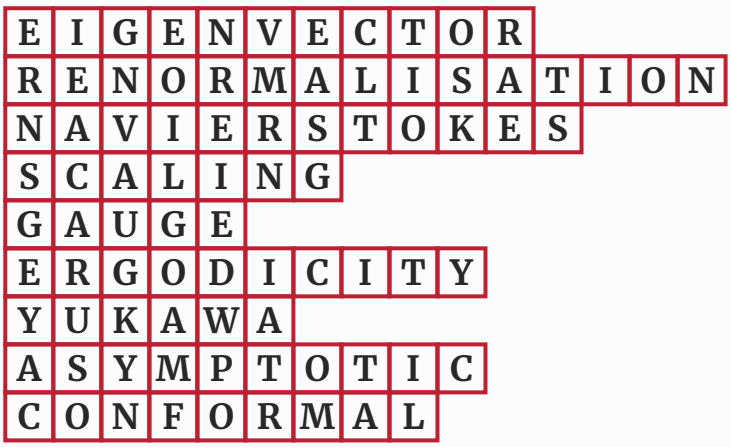
Linked List







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